Optimized approximation space for Trefftz-discontinuous Galerkin methods

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ABSTRACT

Helmholtz equation is classically encountered when modelling waves and vibrations. The numerical approximation of its solution is complex because of the generally small characteristic wavelength and of the potential sign-indefinition [1] which imply to use adapted formulations.

The Trefftz-discontinuous Galerkin methods [2] are characterized by the use of exact solutions of the governing equation inside subdomains as shape functions [3]. Therefore they are not subject to the same limitation than Finite Element methods (pollution and dispersion error). However, they lead to an ill conditioned algebraical system caused by a loss of coercivity.

In this presentation, we propose a method to solve the system in a selected approximation subspace where the coercivity and the condition number are controlled, as well as the slight loss of accuracy due to the reduction technique. If needed, that subspace can be used as the coarse level of a multigrid method.

We derive the method on a specific Trefftz-DG method: the Variational Theory of Complex Rays (VTCR) [4]. The accuracy and performance of the proposed solver are demonstrated on a set of numerical examples of acoustic wave propagation.

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