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ADAPTIVE MODELING AND NUMERICAL APPROXIMATION FOR A SIMPLE EXAMPLE OF MULTISCALE HYPERBOLIC RELAXATION SYSTEM

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ABSTRACT

The present work considers the mathematical and numerical analysis of a new adaptive goal-oriented strategy based on local *hpm* discontinuous Galerkin (DG) method (*h* for grid, *p* for accuracy of shape function and *m* for model). In order to perform an exhaustive analysis, we consider steady-state solutions to the one-dimensional linear hyperbolic system with multiple relaxation times $\varepsilon(x)$:

$$\begin{aligned} \partial_t u^\varepsilon + \partial_x v^\varepsilon &= 0,\ x\in[0,L],\ t>0, \\ \partial_t v^\varepsilon + a^2 \partial_x u^\varepsilon &= \frac{-1}{\varepsilon(x)}\left(v^\varepsilon - b u^\varepsilon\right), \end{aligned} \qquad (1)$$

with $L>0$, a, b given constants satisfying $0<|b|<a$ and prescribed initial and boundary conditions [1]. The above system, for which the associated equilibrium equation reads $\partial_t u + \partial_x bu = 0,\ t>0$ (2), may be viewed as a simple model of a hierarchy of PDE systems arranged according to a cascade of relaxation mechanisms. For instance, such relaxation systems are involved in the study of multiphase flows or multiscale coupling problems. In regions where ε is small, the numerical approximation of the full system (1) may become very costly and a strategy to overcome this difficulty may consist in approximating the associated steady-state equilibrium equation.

According to the main features of the flow and to the required accuracy of the description, the model, coarse (2) or fine (1), has to be locally adapted for computational efficiency. Then, these different models have to be appropriately coupled at some interfaces [2]. The automatic choice of the appropriate model requires model adaptation procedures [3] where the position of the interface has to be optimized in such a way that in the region where one computes the coarse model, the model error (expressed in term of output functionals) between the fine and coarse models does not exceed some given tolerance. Nevertheless, only an approximation of the adapted model is known in practice, thus the approximation involves both a model and a discretization error [4]. These two kinds of errors have to be localized for the model and numerical adaptation procedure.

The main goal of this work is to understand how the error of our *hpm* DG method depends on the relaxation parameter, the boundary layer effects and the coupling interface layer effects in order to validate our approach. Special emphasis is given to the theoretical (PDE level) study of the modeling error. Numerical experiments will be considered to assess the performances of the present method.

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