Annex 1. Optimization of the spar-buoy anchor position

Annex 1. Optimization of the spar-buoy anchor position

The tethers are going to be anchored to the terrain at a specified position, nevertheless, there is a range of possible positions, therefore, it is going to be studied which is the optimum position based on their cost.

Two main factors will affect the total cost, one will be the area of the cable and the other will be the length. The area will depend on the tension at the cable, which depends on the anchor position, while the length also will depend on the position of the anchor. Hence, the positioning will be quite relevant.

While the length will decrease with an anchor close to the spar-buoy it is not quite clear what will happen with the tension.

Two main situations have to be studied, one for the transversal and another for the longitudinal forces as they rarely will occur at the same time. The additional buoyancy load will be always present and therefore will always be considered. Then the studied situations will be:

- Situation 1: Vertical and transversal load
- Situation 2: Vertical and longitudinal load

These loads correspond, as previously mentioned, to:

- Vertical load: Additional buoyancy of the pontoon, corresponding to the maximum tide and live loads.
- Transversal load: Wind effect on the tower and the deck.
- Longitudinal load: Maximum load of an asymmetrical traffic load on the deck or wind effect on the tower.

For the cables design, the most unfavourable situation in ULS must be studied. As happened in the TLP design, the cables cannot support compression forces, to avoid it they will be designed for the higher vertical load and an additional tension will be applied in order to compensate the effects of tides, waves and the relaxation of the own steel.

The most unfavourable combination with respect to the vertical force for the cables will be produced with the lowest vertical load (SLS_7, self-weight with no safety factors), as the excess of buoyancy will be supported exclusively by the cables. The most unfavourable situation for the horizontal forces (both longitudinal and transversal) will occur in the ULS combinations, where the wind force and the traffic load will be maximum. Even though the combination SLS_7 and the ULS combination are not going to happen at the same time, it is going to be combined the maximum vertical unloading with the maximum horizontal loads this is because the unloading is not quite high and this solution will be on the safety side, simplifying the design and not implying an over dimensioned solution.

From the data previously found, the forces for each combination are summarized in Table 83.

		Cable-stayed bridge		Suspension bridge	
		Steel	**Concrete**	**Steel**	**Concrete**
Situation 1	**Fx**	1,54	3,36	52,6	52,6
	Fz	76,1	80,2	232,9	258,4
Situation 2	**Fy**	49,1	78,3	295,6	144,3
	Fz	76,1	80,2	232,9	258,4

Table 83: Forces on the pontoon for the cable design. [MN]

To the values showed previously, it has been added a 10% to the vertical forces in order to provide an initial tension at the cable to avoid compressions due to variations in the water height produced by the waves and tides. Then, the design forces for the design of the pontoon are shown in Table 84.

		Cable-stayed bridge		Suspension bridge	
		Steel	Concrete	Steel	Concrete
Situation 1	**Fx**	1,54	3,36	52,6	52,6
	Fz	83,7	88,3	256,1	284,3
Situation 2	**Fy**	49,1	78,3	295,6	144,3
	Fz	83,7	88,3	256,1	284,3

Table 84: Design forces on the pontoon for the cable design, considering the vertical component of the initial tension at the cable. [MN]

To solve this problem, it is going to be carried a constrained multivariable optimization, using the commercial software Matlab R2020a. To do so, it is necessary to stablish the function to optimize.

Each pontoon has 4 anchor lines, nevertheless, they are symmetrical with respect to the central deck line, reducing the variable to the position of two anchor lines. Each of these anchors will be defined by its coordinates (x_i, y_i, z_i). Some of these coordinates will be variables or data depending on each anchor line. With these 4 anchor lines it is assured a redundancy on the design, and in the case of the failure of one anchor cable, the structure should remain in its position with equilibrium.

For the definition of each anchor line some variables are going to be needed, they are schematized in Figure 92.

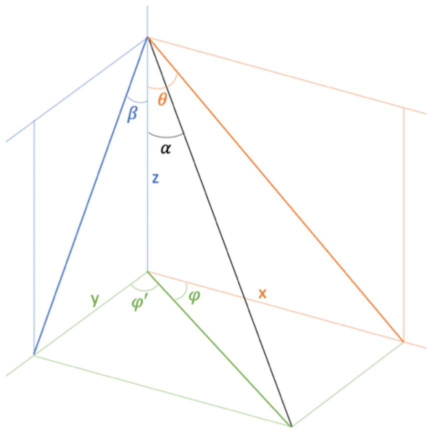


Figure 92: Different variables for the anchor line design

Therefore, the projection of the tension at the anchor line at each axis will be:

$$\begin{cases} X\ axis: T \cdot \sin(\alpha) \cdot \cos(\varphi) \\ Y\ axis: T \cdot \sin(\alpha) \cdot \sin(\varphi) \\ Z\ axis: T \cdot \cos(\alpha) \end{cases}$$

For each bridge and pontoon, different anchors lines dispositions have been proposed, and therefore each one of them require a special study.

A1.1. Suspension bridge

The suspension bridge will have only two pontoons, which will be identical, with an exception of the distance to the seabed, given that one of the towers is further from the flat seabed. These pontoons will be made of two anchor lines to the seabed and two anchor lines to the coast.

The equations that describe the tension at each anchor line, can be obtained applying equilibrium at the pontoon, and it is obtained:

$$\begin{cases} \sum F_x = (T_1 + T_1') \cdot \cos(\varphi_1) \cdot \sin(\alpha_1) - (T_2 + T_2') \cdot \cos(\varphi_2) + f_x = 0 \\ \sum F_y = (-T_1 + T_1') \cdot \sin(\varphi_1) \cdot \sin(\alpha_1) + (-T_2 + T_2') \cdot \sin(\varphi_2) + f_y = 0 \\ \sum F_z = (T_1 + T_1') \cdot \cos(\alpha_1) - f_z = 0 \end{cases}$$

T_1, T_1', T_2 and T_2' correspond to the tensions of each of the anchor lines, being T_i and T_i' the symmetrical ones, in Figures 93 and 94 can be seen at which anchor line corresponds each designation.

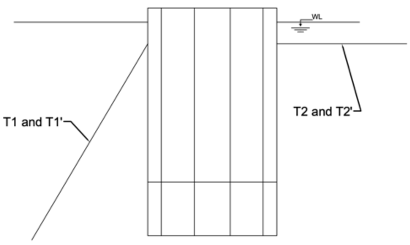


Figure 93: Scheme and nomenclature of the anchor cables front view

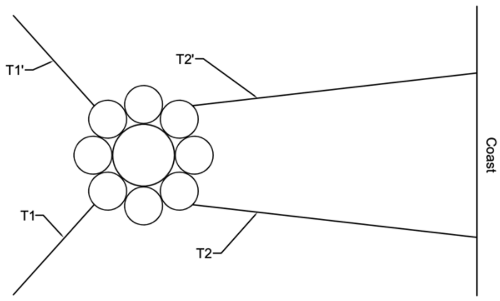


Figure 94: Scheme and nomenclature of the anchor cables plan view

From the equilibrium are obtained three equations, but there are four unknowns, what gives one additional degree of freedom, the structure is statically indeterminate. It would be possible to compute the forces on the cables by means of the compatibility equations, nevertheless an optimization of the position would be difficult.
Given that the design must be for the extreme conditions and that the structure has been designed with one additional cable to provide redundancy to the structure in case that one cable fail it is going to be studied the optimum position for two situations:

- Failure of one the cables from the pontoon to the coast.
- Failure of one the cables from the pontoon to the seabed.

In order to proceed with the optimization, it is needed to obtain the function to optimize, in order to do so, the approach has been to isolate the tension of each anchor line, obtain the required area and the length of each anchor line, with this it is possible to find the total volume of steel. If the equation for the steel volume is minimized the cost will be minimized as they are directly proportional.

In the optimization, it has been imposed, that even $T_i > T_i'$, the dimensioning of them must be $T_i = T_i'$, obtaining one design at each pontoon for each of the situations previously defined (leading longitudinal loads and leading transversal loads). This is imposed because the external loads can be in each direction, and in such case, the values of T_i and T_i' will be exchanged, so they will be designed with the maximum value of them, in other words:

$$T_{i,d} = T'_{i,d} = Max(T_i; T'_i)$$

Then the volume will be computed as:

$$V = 2 \cdot \sum_{i=1}^{2} \frac{T_i}{f_{yd}} * L_i$$

The variable being optimized is the position of the anchoring to the terrain, (x_i,y_i,z_i) of each anchor line. Given the symmetry with the central deck line, (x_i,y_i,z_i) = (x_i',y_i',z_i'). Then x_1 must be at least larger than 250m for one of the towers and larger than 680,5m for the other tower, because it needs to be anchored in the clay (see geological scheme in Figure 28), but smaller than 750m because if not it will be crossed with the other anchor line. x_2 will be equal to the distance from the pontoon to the coast, in other words, x_2=900m.The depths are also stablished, z_1 must end in the seabed at 1250m, but as it is anchored 30m below sea water level, z_1=1220m. The anchor lines "2" will be horizontal, and therefore, z_2=0m. These conditions are stablished as:

- $250m \leq x_1 \leq 750m$ (Tower 1)
- $680{,}5m \leq x_1 \leq 1180{,}5m$ (Tower 2)
- $x_2 = 900m$
- $z_1 = 1220m$
- $z_2 = 0m$

Finally, y_1 and y_2 do not have any logical boundary.

Then the values to optimize are x_1, y_1 and y_2.

For each on the situations considered will be found the optimum location of the anchoring. Then the results will be weighted 80% for the transversal loads and 20% for the longitudinal loads, as it is much more probable to have an extreme wind event, than having exclusively one span loaded at its maximum capacity. A further study of the probabilities of each event could be carried out to obtain a better optimization of the cable anchoring position but it is out of the scope of this project. This weighting of the loads will reduce the mean tension at the cable, reducing the possibility of fatigue issues on the cables.

The same procedure will be done with the failure of the cables. The failure of any of the cables may be considered as equal, then it is going to be considered equally weighted.

With this procedure is going to be found the optimum position of the anchoring considering both loads situations and the failure of any of the cables.

This procedure is applied for the different situations considered.

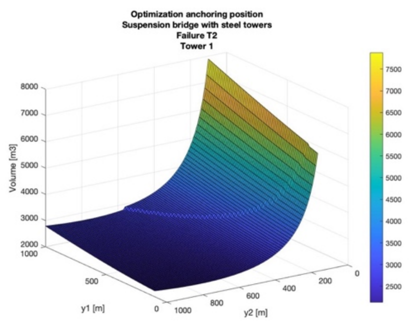
Failure of one the cables from the pontoon to the coast

Firstly, it is going to be isolated the tension of the cables:

$$\begin{cases} T_1 = \dfrac{f_z}{2 \cdot \cos(\alpha_1)} + \dfrac{f_z \cdot \tan(\varphi_2)}{2 \cdot \cos(\alpha_1) \cdot \tan(\varphi_1)} + \dfrac{f_x \cdot \tan(\varphi_2) + f_y}{2 \cdot \sin(\alpha_1) \cdot \sin(\varphi_1)} \\ T_1' = \dfrac{f_z}{2 \cdot \cos(\alpha_1)} - \dfrac{f_z \cdot \tan(\varphi_2)}{2 \cdot \cos(\alpha_1) \cdot \tan(\varphi_1)} - \dfrac{f_x \cdot \tan(\varphi_2) + f_y}{2 \cdot \sin(\alpha_1) \cdot \sin(\varphi_1)} \\ T_2' = \dfrac{f_z \cdot \tan(\alpha_1) \cdot \cos(\varphi_1) + f_x}{\cos(\varphi_2)} \end{cases}$$

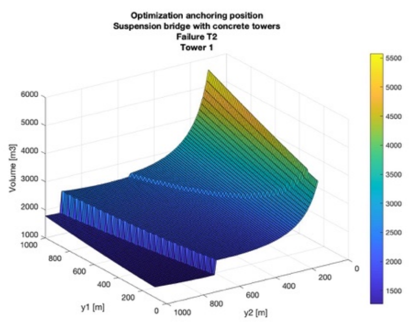
Then, these equations are introduced in the optimization algorithm, with the boundary conditions previously defined.

The optimum position for the failure of a tendon connecting the pontoon to the coast for the tower 1 is shown in Figure 95 for the solution with steel towers and in Figure 96 for the solution with concrete towers, the figures represent the volume of steel needed depending on y_1 and y_2 for x_1 = 250m.



$$\begin{cases} X_1 = (250;\ 745.7;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

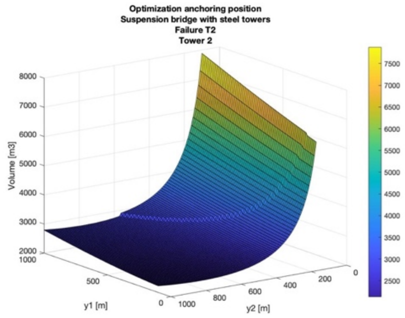
Figure 95: Optimization of the anchoring position for the suspension bridge with steel towers. Failure of T2. Tower 1. x_1=250m.



$$\begin{cases} X_1 = (250;\ 668.7;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

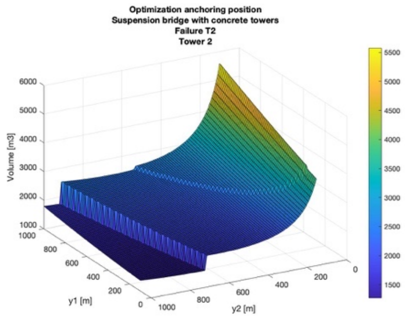
Figure 96: Optimization of the anchoring position for the suspension bridge with concrete towers. Failure of T2. Tower 1. x_1=250m.

The optimum position for the tower 2 is shown in Figure 97 for the solution with steel towers and in Figure 98 for the solution with concrete towers in this case x_1 = 680,5m.



$$\begin{cases} X_1 = (680.5;\ 816.8;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

Figure 97: Optimization of the anchoring position for the suspension bridge with steel towers. Failure of T2. Tower 2. x_1=680,5m.



$$\begin{cases} X_1 = (680.5;\ 724.8;\ 1220)\ \mathrm{m} \\ X_2 = (900, 0, 0)\ \mathrm{m} \end{cases}$$

Figure 98: Optimization of the anchoring position for the suspension bridge with concrete towers. Failure of T2. Tower 2. x_1=680,5m.

These positions will be the optimum positions for the failure of the cable connecting the pontoon with the coast, as they provide the minimum amount of steel required.

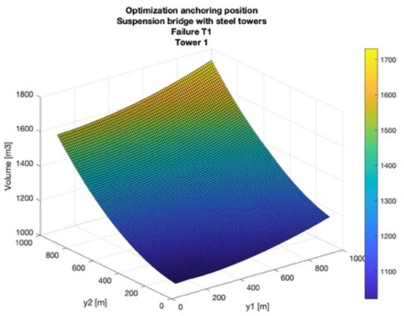
<u>Failure of one the cables from the pontoon to the seabed</u>

Firstly, it is going to be isolated the tension of the cables:

$$\begin{cases} T_1' = \dfrac{f_z}{\cos(\alpha_1)} \\ T_2 = \dfrac{f_z \cdot \tan(\alpha_1) + f_x}{2 \cdot \cos(\varphi_2)} + \dfrac{f_z \cdot \tan(\alpha_1) \cdot \sin(\varphi_1) + f_y}{2 \cdot \sin(\varphi_2)} \\ T_2' = \dfrac{f_z \cdot \tan(\alpha_1) + f_x}{2 \cdot \cos(\varphi_2)} + \dfrac{f_z \cdot \tan(\alpha_1) \cdot \sin(\varphi_1) + f_y}{2 \cdot \sin(\varphi_2)} \end{cases}$$

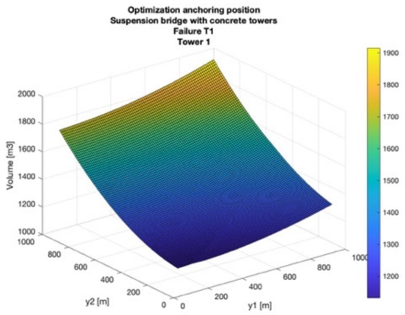
Then, these equations are introduced in the optimization algorithm, with the boundary conditions previously defined.

The optimum position for the failure of a cable connecting the pontoon to the seabed for the tower 1 is shown in Figure 99 for the solution with steel towers and in Figure 100 for the solution with concrete towers, the figures represent the volume of steel needed depending on y_1 and y_2 for x_1 = 250m.



$$\begin{cases} X_1 = (250; 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

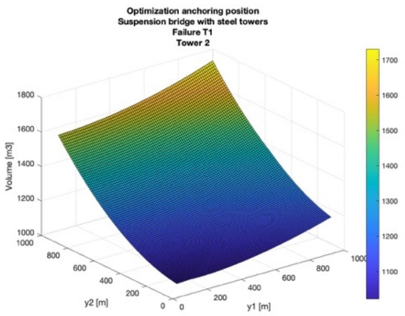
Figure 99: Optimization of the anchoring position for the suspension bridge with steel towers. Failure of T1. Tower 1. x_1=250m.



$$\begin{cases} X_1 = (250;\ 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

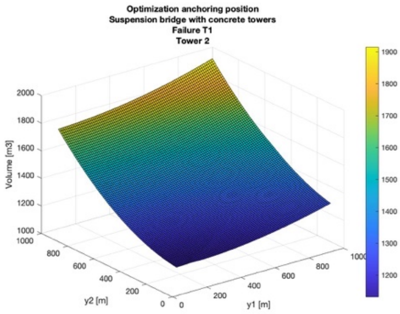
Figure 100: Optimization of the anchoring position for the suspension bridge with concrete towers. Failure of T1. Tower 1. x_1=250m.

The optimum position for the tower 2 is shown in Figure 101 for the solution with steel towers and in Figure 102 for the solution with concrete towers in this case x_1 = 680,5m.



$$\begin{cases} X_1 = (680.5;\ 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

Figure 101: Optimization of the anchoring position for the suspension bridge with steel towers. Failure of T1. Tower 2. x_1=680,5m.



$$\begin{cases} X_1 = (680.5;\ 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

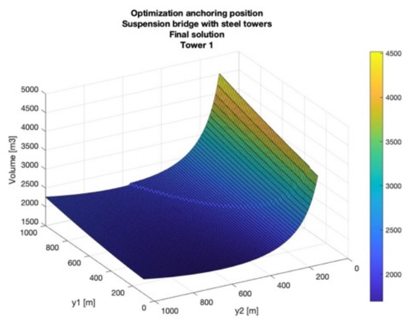
Figure 102: Optimization of the anchoring position for the suspension bridge with concrete towers. Failure of T1. Tower 2. x_1=680,5m.

These positions will be the optimum positions for the failure of the cable connecting the pontoon with the seabed, as they provide the minimum amount of steel required.

Final solution

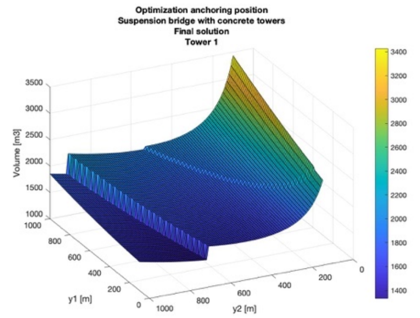
With the two previous solutions for each type of bridge it is possible to compute the optimum position of the cable anchoring considering that both situations (failure of the cable connecting the pontoon to the coast or the failure of the cable connecting the pontoon to the seabed) are equally probable.

The optimum position for the tower 1 is shown in Figure 103 for the solution with steel towers and in Figure 104 for the solution with concrete towers, the figures represent the volume of steel needed depending on y_1 and y_2 for x_1 = 250m.



$$\begin{cases} X_1 = (250;\ 593.6;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

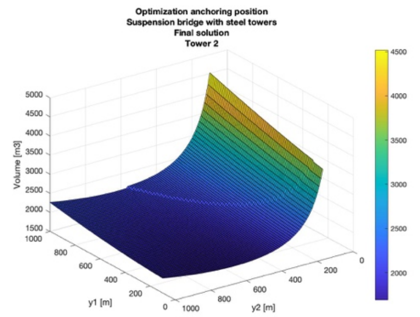
Figure 103: Optimization of the anchoring position for the suspension bridge with steel towers. Tower 1. x_1=250m.



$$\begin{cases} X_1 = (250;\ 642.5;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

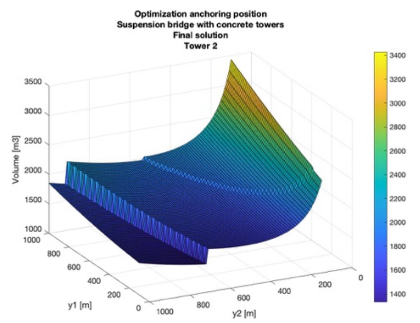
Figure 104: Optimization of the anchoring position for the suspension bridge with concrete towers. Tower 1. x_1=250m.

The optimum position for the tower 2 is shown in Figure 105 for the solution with steel towers and in Figure 106 for the solution with concrete towers in this case x_1 = 680,5m.



$$\begin{cases} X_1 = (680.5;\ 645;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

Figure 105: Optimization of the anchoring position for the suspension bridge with steel towers. Tower 2. x_1=680,5m.



$$\begin{cases} X_1 = (680.5;\ 634.9;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 0;\ 0)\ \mathrm{m} \end{cases}$$

Figure 106: Optimization of the anchoring position for the suspension bridge with concrete towers. Tower 2. x_1=680,5m.

Then the positions shown above will be chosen as the final position for the anchoring of the cables.

Note that all the solutions found above find as optimum y_2=0, in other words that the cable that connects the pontoon to the coast is parallel to the bridge. For this situation instead of installing two different cables, only one cable with the double of the area required will be installed, as they are placed at the same position.

These solutions are summarized in Table 85 for the tower 1 and in Table 86 for the tower 2.

Tower material		x_1	y_1	y_2
Concrete	Failure T1	250	0	0
	Failure T2	250	668,7	0
	Final solution	250	642,4	0
Steel	Failure T1	250	0	0
	Failure T2	250	745,7	0
	Final solution	250	593,6	0

Table 85: Optimum position of the cable anchors for the tower 1 of the suspension bridge

Tower material		x_1	y_1	y_2
Concrete	Failure T1	680,5	0	0
	Failure T2	680,5	724,8	0
	Final solution	680,5	634,9	0
Steel	Failure T1	680,5	0	0
	Failure T2	680,5	816,8	0
	Final solution	680,5	645	0

Table 86: Optimum position of the cable anchors for the tower 2 of the suspension bridge

Finally, the solution for x_1 only depend on the tower and y_2 do not vary for the different solutions. The dimension y_1 vary slightly between the different solutions between 593,6 and 645 metres. To simplify the design of the bridge all the solutions will adopt x_1=630 metres, as it is close to the optimum of all the solutions. Given the similarities between the solutions for the concrete and steel tower bridges, the same solution is adopted for both of them. Then the chosen position of the cable anchoring is shown in Table 87.

	x_1	y_1	y_2
Tower 1	250	630	0
Tower 2	680,5	630	0

Table 87: Position of the cable anchoring for the suspension bridge

A1.2. Cable-stayed bridge

The cable-stayed bridge has six towers, nevertheless only four of them are over pontoon given that the two external towers are bottom founded, in Figure 107 a scheme of the bridge with the nomenclature of the towers is shown. Although the four internal pontoons have the same forces on them the anchoring is different.

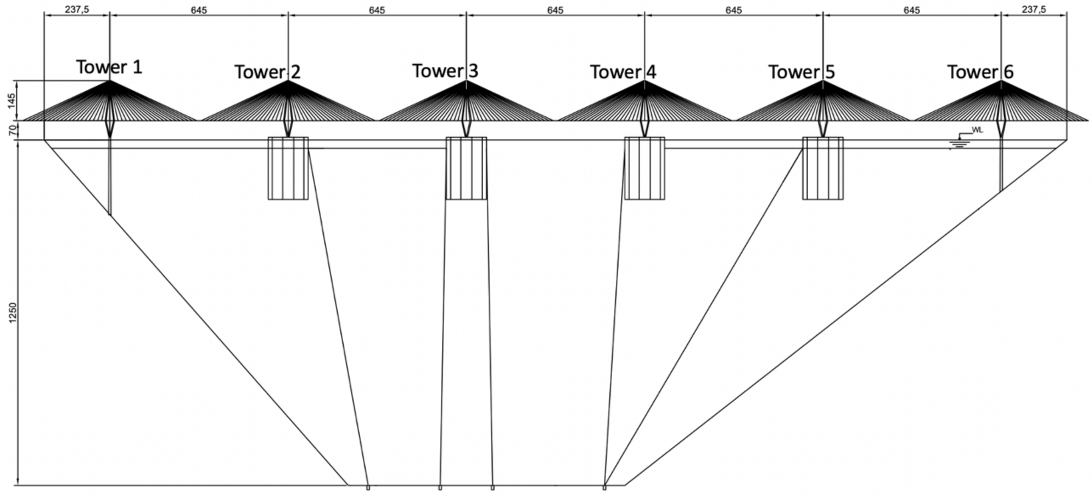


Figure 107: Scheme of the cable-stayed bridge with the tower nomenclature used in this section

Despite the bridge being symmetrical, the seabed it is not, what will result in a non-symmetrical disposition of the cable anchors.

Note that on the one hand, tower 3 has the four anchor cables to the seabed, this is because in the flat part of the seabed there is clay and therefore, the usage of suction piles is possible, this added to the efficiency of an almost vertical cable will provide a proper solution for the anchoring. On the other hand, tower 4 will have its anchoring to the seabed on one side and to the coast on the other side. This is because the slope is rock and an anchoring in the rock at this depth will be much more difficult and expensive than an anchoring to the clay or to the coast. This is the reason of the design proposed.

The spar buoy of towers 2, 4 and 5 have a similar configuration to the spars used in the suspension bridge, with two cables to the coast (which for the suspension bridge ended up being only one) and two cables to the seabed. The transversal forces can be in either direction (positive or negative in the Y axis), therefore, a symmetrical design will be proposed. The same happens with the longitudinal forces, but a non-symmetrical configuration of the cables will produce different reactions at the cables depending on the direction of the force f_x.

The spar buoy of the tower 3 will have a different configuration with the four cables anchored to the seabed. Both f_x and f_y will happen in both directions, given than the configuration of these cables is similar, the optimum design will result in a centrally symmetrical design with respect the centre of the spar-buoy.

The equations that describe the tension at each anchor line, can be obtained applying equilibrium at the pontoon. For the towers 2, 4 and 5 the equations will be:

$$\begin{cases} \sum F_x = (T_1 + T_1') \cdot \cos(\varphi_1) \cdot \sin(\alpha_1) - (T_2 + T_2') \cdot \cos(\varphi_2) + f_x = 0 \\ \sum F_y = (-T_1 + T_1') \cdot \sin(\varphi_1) \cdot \sin(\alpha_1) + (-T_2 + T_2') \cdot \sin(\varphi_2) + f_y = 0 \\ \sum F_z = (T_1 + T_1') \cdot \cos(\alpha_1) - f_z = 0 \end{cases}$$

And for the tower 3 the equations will be:

$$\begin{cases} \sum F_x = (T_1 + T_1' - T_2 - T_2') \cdot \cos(\varphi) \cdot \sin(\alpha) + f_x = 0 \\ \sum F_y = (-T_1 + T_1' - T_2 + T_2') \cdot \sin(\varphi) \cdot \sin(\alpha) + f_y = 0 \\ \sum F_z = (T_1 + T_1' + T_2 + T_2') \cdot \cos(\alpha) - f_z = 0 \end{cases}$$

The pontoon is symmetrical with respect to the deck and therefore T_i and T_i' make reference to a cable and its symmetrical one. In Figures 108 and 109 are shown a couple of schemes of these pontons and the position of each one of the cables.

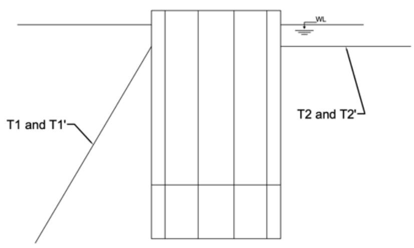


Figure 108: Scheme and nomenclature of the anchor cables front view

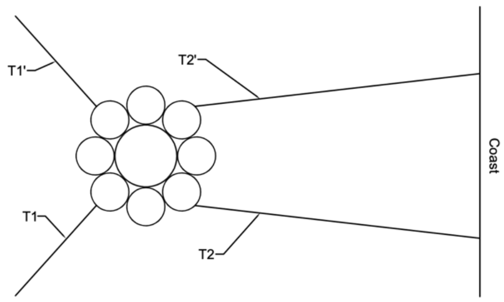


Figure 109: Scheme and nomenclature of the anchor cables plan view

As in the suspension bridge case, there are four unknowns and only 3 equations, resulting in a statically indeterminate structure. The forces at the cables could be found using the compatibility equations, nevertheless, to be able to effectuate the optimization of the cable anchoring and taking into account that there are four cables instead of three to provide redundancy to the structure. The design it is going to be based in the situation where one of the cables fails.

For the spar buoy of the towers 2, 4 and 5 two situations must be taken into account:

- Failure of one the cables from the pontoon to the coast.
- Failure of one the cables from the pontoon to the seabed.

Nevertheless, for the spar buoy of the tower 3, only the situation of the failure of one cable must be studied, as due to the central symmetry the failure of any of the cables will produce the same result.

In order to proceed with the optimization, it is needed to obtain the function to optimize, in order to do so, the same approach used for the suspension bridge has been done consisting in isolating the tension of each anchor line, obtaining the required area and the length of each anchor line and finding the total volume of steel.

For the cable stayed bridge has also been imposed the same symmetry condition in the tension of the cables:

$$T_{i,d} = T'_{i,d} = Max(T_i; T'_i)$$

Then the volume for towers 2, 3 and 5 will be computed as:

$$V = 2 \cdot \sum_{i=1}^{2} \frac{T_i}{f_{yd}} * L_i$$

While for tower 4 an easier equation is found as all the cables have the same design:

$$T_{i,d} = T'_{i,d} = Max(T_1; T'_1; T_2; T'_2)$$

$$V = \frac{4 \cdot T}{f_{yd}} * L_i$$

The variable being optimized is the position of the anchoring to the terrain, (x_i,y_i,z_i) of each anchor line. The same conditions have been evaluated for this bridge. One additional condition is added for the cable stayed bridge, the component "y" of the cable connecting the pontoon to the coast must be large enough to do not coincide with the towers between the original tower and the coast. The boundary conditions are summarized in Table 88.

Tower	**2**	**3**	**4**	**5**
x_1	≥217,5	≤427,5	≥72,5	≥717,5
y_1	≥0	≥0	≥0	≥0
z_1	1210	1210	1210	1210
x_2	882,5	-	1527,5	882,5
y_2	≥60	-	≥125	≥60
z_2	0	-	0	0

Table 88: Boundary conditions for the spar buoy cable anchoring of the cable-stayed bridge.

Then the values to optimize are x_1, y_1 and y_2.

The same weighting of the loads and the situations is considered for the cable-stayed bridge, which was 80% for the transversal loads and 20% for the longitudinal loads and equally weighted for the cable failure.

Now it is possible to evaluate the optimum position of the cable anchoring for the four different spar-buoys. For towers 2, 4 and 5 three different positions will be provided, first the optimized position for the failure of the cable connecting the spar buoy to the coast, second the optimized position for the failure of the cable connecting the spar buoy to the seabed and last the final position optimum position. For the tower 4 only the final solution will be provided as only the case of a cable connecting the spar buoy to the seabed can be evaluated.

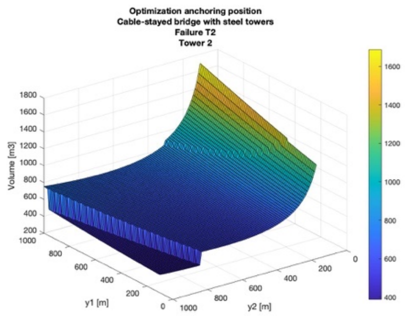
Failure of a cable connecting the pontoon with the coast

Firstly, it is going to be isolated the tension of the cables:

$$\begin{cases} T_1 = \dfrac{f_z}{2\cdot\cos(\alpha_1)} + \dfrac{f_z\cdot\tan(\varphi_2)}{2\cdot\cos(\alpha_1)\cdot\tan(\varphi_1)} + \dfrac{f_x\cdot\tan(\varphi_2)+f_y}{2\cdot\sin(\alpha_1)\cdot\sin(\varphi_1)} \\ T_1' = \dfrac{f_z}{2\cdot\cos(\alpha_1)} - \dfrac{f_z\cdot\tan(\varphi_2)}{2\cdot\cos(\alpha_1)\cdot\tan(\varphi_1)} - \dfrac{f_x\cdot\tan(\varphi_2)+f_y}{2\cdot\sin(\alpha_1)\cdot\sin(\varphi_1)} \\ T_2' = \dfrac{f_z\cdot\tan(\alpha_1)\cdot\cos(\varphi_1)+f_x}{\cos(\varphi_2)} \end{cases}$$

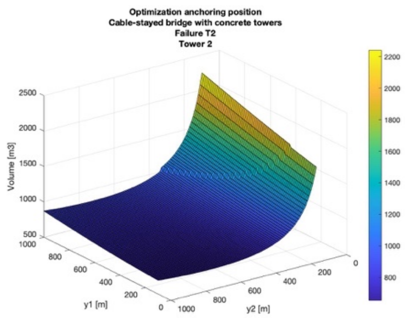
Then, these equations are introduced in the optimization algorithm, with the boundary conditions previously defined and then the optimum position is found.

The optimum position of the cable anchor for the failure of a cable connecting the pontoon with the coast for tower 2 is shown in figure 110 for the steel towers and in Figure 111 for the concrete towers. The figure represents the volume of steel required for the cables depending on the position y_1 and y_2 for a value of x_1=217,5m.



$$\begin{cases} X_1 = (217.5;\ 732.7;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

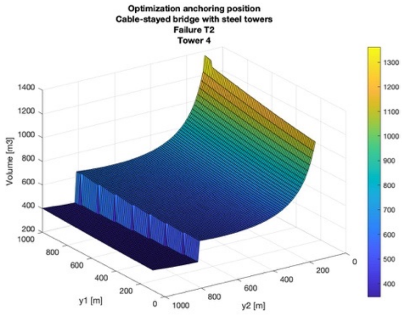
Figure 110: Optimization of the anchoring position for the cable-stayed bridge with steel towers. Tower 2. x_1=217,5m.



$$\begin{cases} X_1 = (217.5;\ 693.9;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

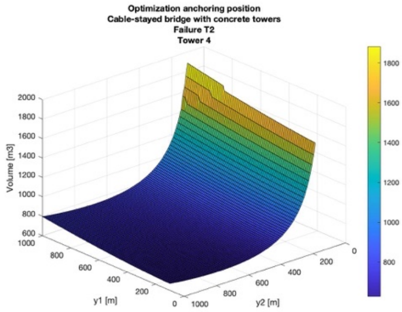
Figure 111: Optimization of the anchoring position for the cable-stayed bridge with concrete towers. Tower 2. x_1=217,5m.

The optimum position for the tower 4 is shown in Figure 112 for the solution with steel towers and in Figure 113 for the solution with concrete towers in this case x_1 = 72,5m.



$$\begin{cases} X_1 = (72.5;\ 733.5;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 125;\ 0)\ \mathrm{m} \end{cases}$$

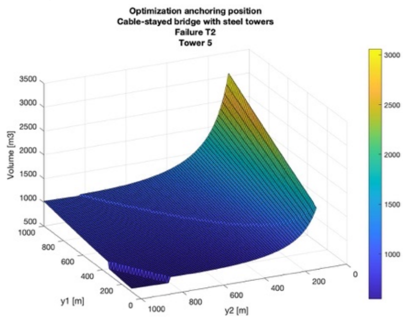
Figure 112: Optimization of the anchoring position for the cable-stayed bridge with steel towers. Tower 4. x_1=72,5m.



$$\begin{cases} X_1 = (72.5;\ 668.4;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 125;\ 0)\ \mathrm{m} \end{cases}$$

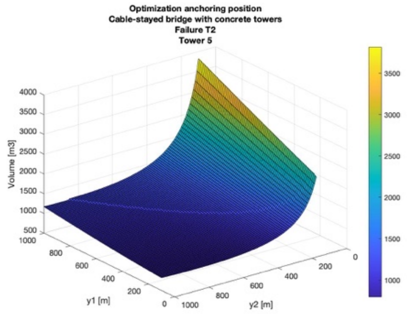
Figure 113: Optimization of the anchoring position for the cable-stayed bridge with concrete towers. Tower 4. x_1=72,5m.

Finally, the optimum position for the tower 4 is shown in Figure 114 for the solution with steel towers and in Figure 115 for the solution with concrete towers in this case x_1 = 717,5m.



$$\begin{cases} X_1 = (717.5;\ 760.2;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

Figure 114: Optimization of the anchoring position for the cable-stayed bridge with steel towers. Tower 5. x_1=717,5m.



$$\begin{cases} X_1 = (717.5;\ 770.2;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

Figure 115: Optimization of the anchoring position for the cable-stayed bridge with concrete towers. Tower 5. x_1=717,5m.

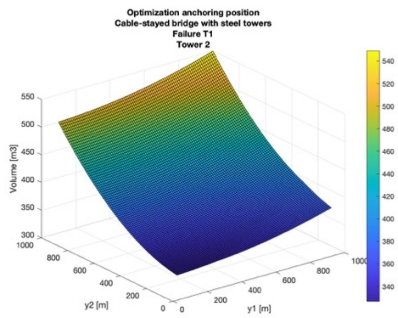
<u>**Failure of a cable connecting the pontoon with the seabed (Towers 2, 4 and 5)**</u>

Firstly, it is going to be isolated the tension of the cables:

$$\begin{cases} T_1' = \dfrac{f_z}{\cos(\alpha_1)} \\ T_2 = \dfrac{f_z \cdot \tan(\alpha_1) + f_x}{2 \cdot \cos(\varphi_2)} + \dfrac{f_z \cdot \tan(\alpha_1) \cdot \sin(\varphi_1) + f_y}{2 \cdot \sin(\varphi_2)} \\ T_2' = \dfrac{f_z \cdot \tan(\alpha_1) + f_x}{2 \cdot \cos(\varphi_2)} + \dfrac{f_z \cdot \tan(\alpha_1) \cdot \sin(\varphi_1) + f_y}{2 \cdot \sin(\varphi_2)} \end{cases}$$

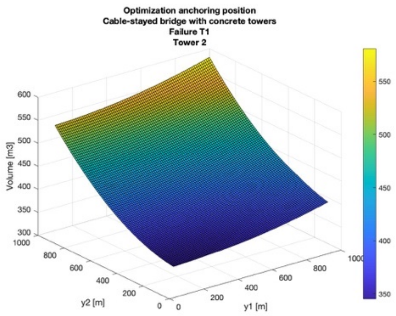
Then, these equations are introduced in the optimization algorithm, with the boundary conditions previously defined and then the optimum position is found.

The optimum position of the cable anchor for the failure of a cable connecting the pontoon with the coast for tower 2 is shown in Figure 116 for the steel towers and in Figure 117 for the concrete towers. The figure represents the volume of steel required for the cables depending on the position y_1 and y_2 for a value of x_1=217,5m.



$$\begin{cases} X_1 = (217.5;\ 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

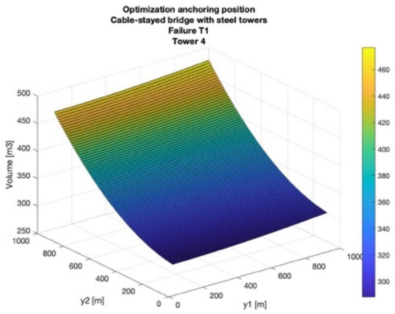
Figure 116: Optimization of the anchoring position for the cable-stayed bridge with steel towers. Tower 2. x_1=217,5m.



$$\begin{cases} X_1 = (217.5;\ 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

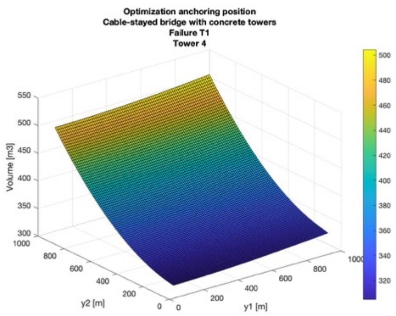
Figure 117: Optimization of the anchoring position for the cable-stayed bridge with concrete towers. Tower 2. x_1=217,5m.

The optimum position for the tower 4 is shown in Figure 118 for the solution with steel towers and in Figure 119 for the solution with concrete towers in this case x_1 = 72,5m.



$$\begin{cases} X_1 = (72.5;\ 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 125;\ 0)\ \mathrm{m} \end{cases}$$

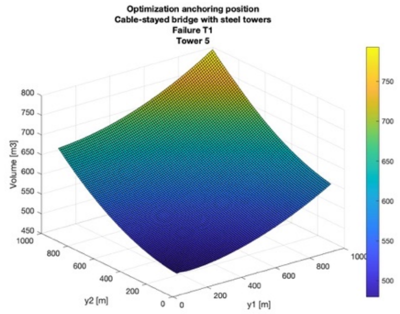
Figure 118: Optimization of the anchoring position for the cable-stayed bridge with steel towers. Tower 4. x_1=72,5m.



$$\begin{cases} X_1 = (72.5;\ 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 125;\ 0)\ \mathrm{m} \end{cases}$$

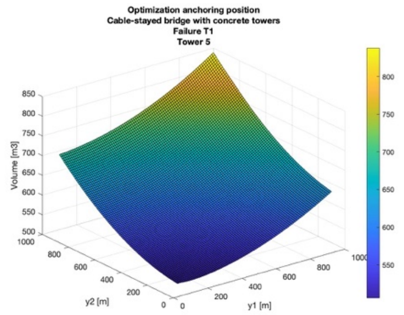
Figure 119: Optimization of the anchoring position for the cable-stayed bridge with concrete towers. Tower 4. x_1=72,5m.

The optimum position for the tower 5 is shown in Figure 120 for the solution with steel towers and in Figure 121 for the solution with concrete towers in this case x_1 = 717,5m.



$$\begin{cases} X_1 = (717.5;\ 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

Figure 120: Optimization of the anchoring position for the cable-stayed bridge with steel towers. Tower 5. x_1=717,5m.



$$\begin{cases} X_1 = (717.5;\ 0;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

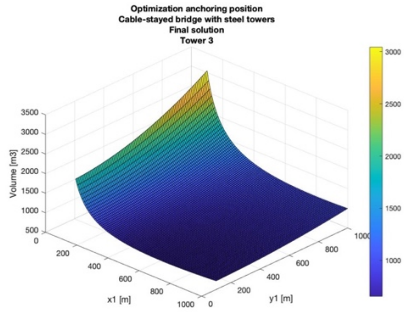
Figure 121: Optimization of the anchoring position for the cable-stayed bridge with concrete towers. Tower 5. x_1=717,5m.

<u>Failure of a cable connecting the pontoon with the seabed (Tower 3)</u>

Firstly, it is going to be isolated the tension of the cables:

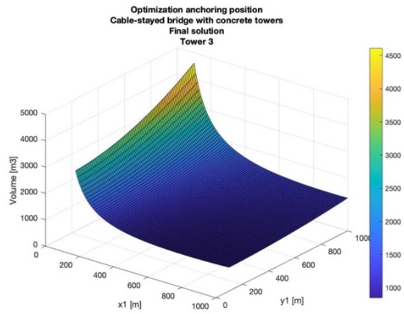
$$\begin{cases} T_1' = \dfrac{-f_x}{2 \cdot \sin(\alpha) \cdot \cos(\varphi)} \\ T_2 = \dfrac{f_z}{2 \cdot \cos(\alpha)} + \dfrac{f_y}{2 \cdot \sin(\alpha) \cdot \sin(\varphi)} \\ T_2' = \dfrac{f_x}{2 \cdot \sin(\alpha) \cdot \cos(\varphi)} - \dfrac{f_y}{2 \cdot \sin(\alpha) \cdot \sin(\varphi)} \end{cases}$$

Then, these equations are introduced in the optimization algorithm, with the boundary conditions previously defined the optimum position for each situation is found. In Figures 122 and 123 is represented the volume of steel required for the anchor cables for the tower 3 of the cable-stayed bridge with steel towers and concrete towers respectively.



$$X_1 = (22.3;\ 668.2;\ 1220)\ \mathrm{m}$$

Figure 122: Optimization of the anchoring position for the tower 3 of the cable-stayed bridge with steel towers.



$$X_1 = (46.1;\ 732;\ 1220)\ \mathrm{m}$$

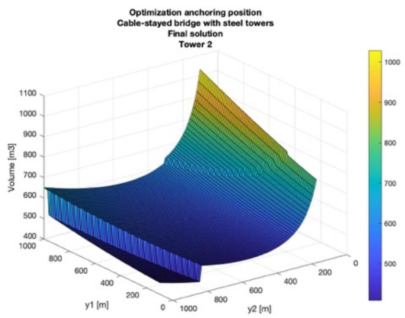
Figure 123: Optimization of the anchoring position for the tower 3 of the cable-stayed bridge with concrete towers.

These are the optimum positions for the anchoring of the pontoon for the tower 3.

<u>Final solution</u>

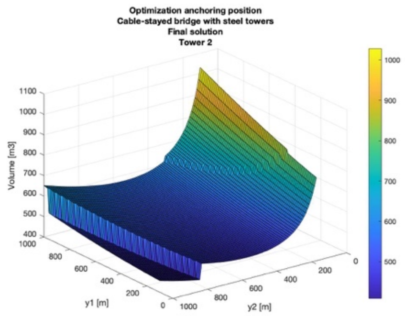
Finally, considering an equally probable failure of any cable the solutions previously shown are combined and the optimum solution for the positioning of the anchoring for the cables of the spar-buoys is found.

The optimum position of the cable anchor is shown in Figure 124 for the steel towers and in Figure 125 for the concrete towers. The figure represents the volume of steel required for the cables depending on the position y_1 and y_2 for a value of x_1=217,5m.



$$\begin{cases} X_1 = (217.5;\ 738.2;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

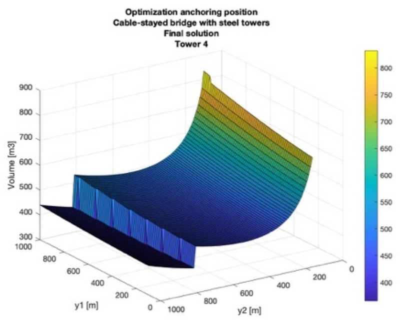
Figure 124: Optimization of the anchoring position for the cable-stayed bridge with steel towers. Tower 2. x_1=217,5m.



$$\begin{cases} X_1 = (217.5;\ 548.4;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 60;\ 0)\ \mathrm{m} \end{cases}$$

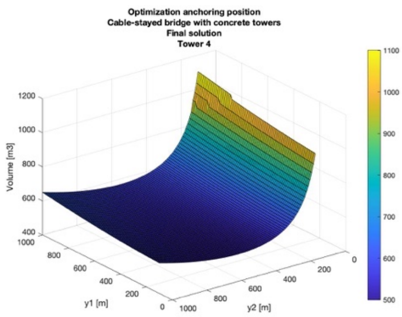
Figure 125: Optimization of the anchoring position for the cable-stayed bridge with concrete towers. Tower 2. x_1=217,5m.

The optimum position for the tower 4 is shown in Figure 126 for the solution with steel towers and in Figure 127 for the solution with concrete towers in this case x_1 = 72,5m.



$$\begin{cases} X_1 = (72.5;\ 733;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 125;\ 0)\ \mathrm{m} \end{cases}$$

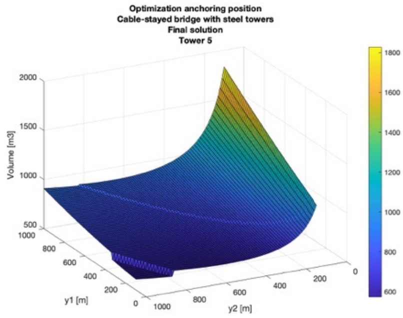
Figure 126: Optimization of the anchoring position for the cable-stayed bridge with steel towers. Tower 4. x_1=72,5m.



$$\begin{cases} X_1 = (72.5;\ 542.9;\ 1220)\ \mathrm{m} \\ X_2 = (900;\ 125;\ 0)\ \mathrm{m} \end{cases}$$

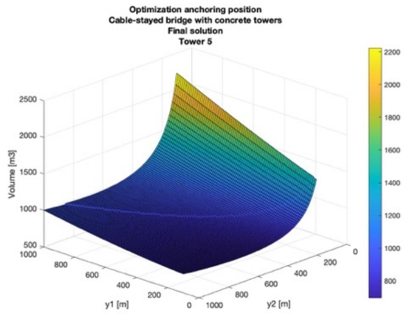
Figure 127: Optimization of the anchoring position for the cable-stayed bridge with concrete towers. Tower 4. x_1=72,5m.

The optimum position for the tower 5 is shown in Figure 128 for the solution with steel towers and in Figure 129 for the solution with concrete towers in this case x_1 = 717,5m.



$$\begin{cases} X_1 = (717.5;\ 746.6;\ 1220)\ \text{m} \\ X_2 = (900;\ 60;\ 0)\ \text{m} \end{cases}$$

Figure 128: Optimization of the anchoring position for the cable-stayed bridge with steel towers. Tower 5. x_1=717,5m.



$$\begin{cases} X_1 = (717.5;\ 603.3;\ 1220)\ \text{m} \\ X_2 = (900;\ 60;\ 0)\ \text{m} \end{cases}$$

Figure 129: Optimization of the anchoring position for the cable-stayed bridge with concrete towers. Tower 5. x_1=717,5m.

The optimum positioning of the cable anchoring for each pontoon is summarized in Table 89.

	Concrete towers				Steel towers			
Tower	**2**	**3**	**4**	**5**	**2**	**3**	**4**	**5**
x1	217,5	46,1	72,5	717,5	217,5	22,3	72,5	717,5
y1	548,4	732,0	542,9	603,3	732,7	668,2	733,5	746,6
y2	60	-	125	60	60	-	125	60

Table 89: Optimum position of the cable anchoring for the pontoons in the cable-stayed bridge. [m]

The different solutions for some towers are similar, and in order to simplify the design the final position of the anchors can be somewhere in between, especially after seeing in the plots that the optimum position is not too sensible. While for the suspension bridge the concrete and steel towers solutions had similar results, in the cable-stay they differ and therefore, different positions will be provided for every bridge solution. In Table 90 are summarized the design positions.

	Concrete towers				Steel towers			
Tower	**2**	**3**	**4**	**5**	**2**	**3**	**4**	**5**
x1	217,5	46,1	72,5	717,5	217,5	22,3	72,5	717,5
y1	550	732,0	550	550	735	668,2	735	735
y2	60	-	125	60	60	-	125	60

Table 90: Position of the cable anchoring for the suspension bridge