



UNIVERSITAT POLITÈCNICA DE CATALUNYA  
BARCELONATECH

Escola Superior d'Enginyeries Industrial,  
Aeroespacial i Audiovisual de Terrassa

Study for the computational resolution of  
conservation equations of mass,  
momentum and energy. Possible  
application to different aeronautical and  
industrial engineering problems.

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# Chapter 1

## Conductivity resolution

In the Project Report, basic information about the equations approach has been done, but the final values of each coefficient have not been specified. This will be shown in the following. The equation to be solved is the following one.

$$T_P^{n+1} = \frac{\beta(a_e \cdot T_E^{n+1} + a_w \cdot T_W^{n+1} + a_n \cdot T_N^{n+1} + a_s \cdot T_S^{n+1} + a_u \cdot T_U^{n+1} + a_d \cdot T_D^{n+1}) + b_P}{a_P} \quad (1.1)$$

Where  $n + 1$  makes reference to the temperature of the following step. The temperatures that does not have this exponent are from the values of temperature from the past time ste. Firstly, it will be shown how the coefficients  $a_{face}$  are calculated:

$$\begin{aligned} a_e &= \lambda_e \frac{S_e}{d_{PE}} \\ a_w &= \lambda_w \frac{S_w}{d_{PW}} \\ a_n &= \lambda_n \frac{S_n}{d_{PN}} \\ a_s &= \lambda_s \frac{S_s}{d_{PS}} \\ a_u &= \lambda_u \frac{S_u}{d_{PU}} \\ a_d &= \lambda_d \frac{S_d}{d_{PD}} \end{aligned} \quad (1.2)$$

Once the individual coefficients are exposed, it is the moment to develop the rest of them involved in the equation, such as  $a_P$  or  $b_P$ .

$$\begin{aligned}
 a_P = & \frac{\rho_P \cdot c_{pP} \cdot V_P}{\Delta t} + \beta(a_w + a_e + a_n + a_s + a_u + a_d + north \cdot \alpha_n S_n + \\
 & + east \cdot \alpha_s S_s + west \cdot \alpha_w S_w + south \cdot \alpha_s S_s + up \cdot \alpha_u S_u + \\
 & + down \cdot \alpha_d S_d)
 \end{aligned} \tag{1.3}$$

$$\begin{aligned}
 b_P = & \dot{q}_{vP} \cdot V_P + \frac{\rho_P \cdot c_{pP} \cdot V_P}{\Delta t} \cdot + \beta(north \cdot \alpha_n S_n \cdot T_N^{n+1} + east \cdot \alpha_s S_s \cdot T_E^{n+1} + \\
 & + west \cdot \alpha_w S_w \cdot T_W^{n+1} + south \cdot \alpha_s S_s \cdot T_S^{n+1} + up \cdot \alpha_u S_u \cdot T_U^{n+1} + \\
 & + down \cdot \alpha_d S_d \cdot T_D) + (1 - \beta) \cdot (-a_w(T_P - T_W) - a_e(T_P - T_E) - \\
 & - a_n(T_P - T_N) - a_s(T_P - T_S) - a_u(T_P - T_U) - a_d(T_P - T_D) + \\
 & + north \cdot \alpha_n (T_F - T_N) S_n + south \cdot \alpha_s (T_F - T_S) S_s + \\
 & + east \cdot \alpha_e (T_F - T_E) S_n + west \cdot \alpha_w (T_F - T_W) S_w + \\
 & + up \cdot \alpha_u (T_F - T_U) S_u + down \cdot \alpha_d (T_F - T_D) S_d)
 \end{aligned} \tag{1.4}$$

These are the coefficients that have to be calculated at each node per each iteration, with a *Gaus-Seidel* solver.

# Chapter 2

## Fluid resolution (ducts)

In the report, the equations involved in the fluid resolution have not been fully developed. Here, they will be shown in the same way as in the code.

From the momentum conservation, the pressure:

$$p[i + 1] = p[i] - \frac{1}{S} \cdot (f_i \cdot \rho_i \cdot v_i^2 \cdot P \cdot \frac{\Delta Z}{2} - \dot{m}_{in} \cdot (v[i + 1] - v[i])) \quad (2.1)$$

Where the variables  $\phi_i$  make reference to the average values of a node (between the value at the entrance and the exit) and the characteristics of each node computed with the average variable.

The "n" makes reference to the node where they are involved.  $\phi[n]$  for the value at the entrance and  $\phi[n + 1]$  for the value at the exit. For the temperature,

$$T[i + 1] = T[i] + \frac{1}{c_p} \cdot \left( \frac{Q_{convection}}{\dot{m}_{in}} - \frac{v^2[i + 1] - v^2[i]}{2} \right) \quad (2.2)$$

For the density,

$$\rho[i + 1] = \frac{p[i + 1]}{287 \cdot T[i + 1]} \quad (2.3)$$

And for the fluid velocity,

$$v[i + 1] = \frac{\dot{m}_{in}}{S \cdot \rho[i + 1]} \quad (2.4)$$

And all of this equations, solved at each iteration and compared to the previous value obtained, until the difference is the convergence criteria.

# Chapter 3

## Battery Pack

The battery pack has been designed using 4 blocks 10S5P <sup>1</sup>, which means 5 groups of 10 Li-Ion serial cylindrical cells (Panasonic NCR18650PF), 50 cells in total. Taking into account that the nominal voltage value of each cell is 3.6V, the total value for the set is 36V. The technical data of each the pack:

- Technology: Li-Ion/LiNiMnCoO<sub>2</sub>.
- Capacity: 14500mAh.
- Power: 522Wh.
- Nominal voltage: 36V.
- Charge voltage: 42V.
- Discharge voltage: 25-30V.
- Discharge current: 32A.
- Charge current (max.): 12A.
- Cells composition: LiNiMnCoO<sub>2</sub>.
- Measures: 185mm x 90mm x 65mm.
- Weight: 2360g.

The cell itself has a specific resistance, which with the pass of current, it generates heat. The internal resistance of each if the packs is  $R = 0,04\Omega$ , because of the

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<sup>1</sup><https://www.manomano.es/catalogue/p/vhbw-pack-baterias-li-ion-14500mah-36v-10s5p-para-e-bike-pedelec-12578568>

connections of each pack. Considering a discharge current of  $I = 100A$  at peak, the dissipated power is  $P = R \cdot I^2 = 400W$ .

The global battery pack will be formed by 4 of these packs connected in parallel to increase the total capacity.