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ANALYTICAL AND COMPUTATIONAL APPROACH FOR STUDYING THE INTERACTION BETWEEN WAVES AND CYLINDRICAL WAVE ENERGY CONVERTERS OSCILLATING IN TWO MODES

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Key words: Wave Energy Converter, Cylinder, Horizontal and Vertical Oscillations, Phase Shift.

Summary. *Ocean wave energy may be recovered by oscillating wave energy converters. The energy converter studied in this work is a horizontally orientated cylinder which may be placed at different depths in the sea. The cylinder can oscillate in horizontal and vertical directions and transfer mechanical energy forward by hydraulic cylinders. To study the interaction between the waves and the converter, we have used potential flow theory separately for both the waves and the oscillating cylinder, and then combined these potential functions by using the principle of superposition. Combined potential flow fields, together with Euler's equations, enable us to obtain the pressure distribution around the cylinder. When knowing the pressure distribution, both the force upon the cylinder, and the net mechanical power transferred from the waves to the moving cylinder, can be calculated. With this model we have analyzed several interesting topics which affect the efficiency of the wave energy converter. The phase shift is the most important parameter - with the phase shift π/2 the best efficiency 0.5 was achieved. To achieve the right phase shift for different waves is essential due to the power capture. Furthermore it is shown that feedback control is necessary for keeping the phase shift constant. Also the cylinder radius has a great effect on the efficiency. The other important parameters studied in this work were the wave height and the wave period.*

1 INTRODUCTION

To define the exact interaction between the waves and the converter is a complex task, and there are no simple and unequivocal models for calculating the actual process. The issue has been approached before using linear hydrodynamic numerical models among others for flap configurations[1]. In this study, we take a different approach and perform an analytical and computational model using a cylindrical wave energy converter. There are earlier calculations of wave forces on marine structures[2,3] and especially on fixed vertical piles. This thought is utilized here; the most significant difference and renewal is the horizontally aligned cylinder oscillating in two modes. Letting the cylinder oscillate in both horizontal and vertical directions enables higher power capture from the waves and improves the efficiency of the converter.

2 SYSTEM DESCRIPTION

The energy converter studied in this work is a horizontally orientated cylinder which may be placed at different depths in the sea. The cylinder can oscillate in horizontal and vertical directions and transfer mechanical energy forward by hydraulic cylinders situated at the both ends of the converter. With the aid of hydraulic cylinders, salt water is pressurized and then feed to a reverse osmosis desalination plant which is directly coupled to the wave energy converter. One part of the pressurized water flows through the reverse osmosis membranes and becomes unsalted, even drinking water if desired, while the other part of the pressurized water can be used for power production for example by turbines. Operation of the wave energy converter in waves is shown in Fig. 1. The dimensions of the system are also included in the figure: *H* is the wave height, *L* the wavelength, *d* the water depth, *a* the radius of the cylinder and *h* the distance between the center of the cylinder and still water level.

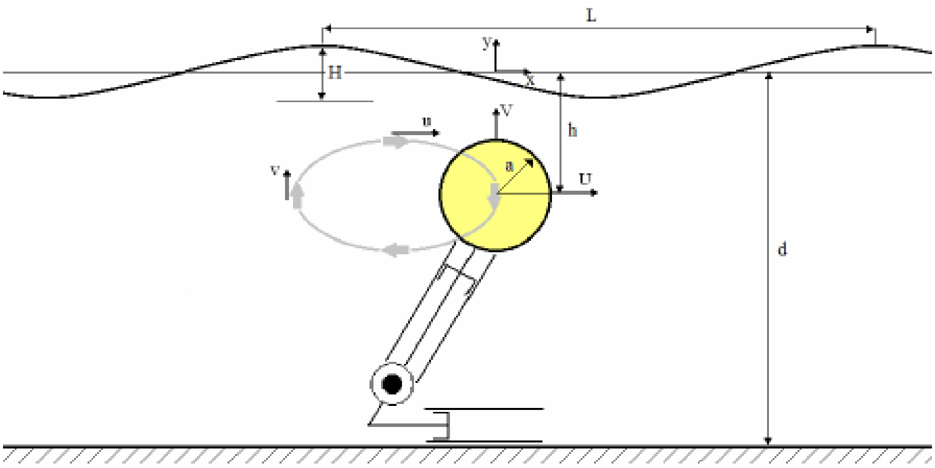

Figure 1. A wave energy converter oscillating in ocean waves.

3 INTERACTION BETWEEN WAVES AND OSCILLATING WAVE ENERGY CONVERTERS

To study the interaction between the waves and the wave energy converter, we will use potential flow theory separately both for the waves and the oscillating cylinder, and then combine these flow fields by using the principle of superposition. Combined potential flow fields together with Euler's equations enable us to obtain the pressure distribution around the cylinder. Knowing the pressure distribution, the force upon the cylinder and also the net mechanical power transferred from the waves to the moving cylinder can be solved.

We define the potential function $\Phi(x, y, t)$ as follows

$$u = \frac{\partial \phi}{\partial x} \tag{1}$$

$$v = \frac{\partial \phi}{\partial y} \tag{2}$$

where u and w are the fluid velocity components in horizontal and vertical directions.

When the fluid density is constant, the continuity equation is

$$\nabla^2 \phi = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{3}$$

The velocity flow field defined by Eqs. (1) and (2) satisfies the two-dimensional irrotationality condition

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \tag{4}$$

The Euler equations of the flow in x-y plane are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{5}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - g \tag{6}$$

where ρ is the density, p the pressure and g the acceleration of gravity.

Using the Eqs.(1), (2) and (4), the Euler equations may be rewritten as

$$\frac{\partial}{\partial x}\left[\frac{\partial \phi}{\partial t} + \frac{u^2 + v^2}{2} + \frac{p}{\rho}\right] = 0 \tag{7}$$

$$\frac{\partial}{\partial y}\left[\frac{\partial \phi}{\partial t}+\frac{u^2+v^2}{2}+\frac{p}{\rho}+gy\right]=0 \tag{8}$$

Integration of these equations with respect to *x* and *y* gives

$$\frac{\partial \phi}{\partial t}+\frac{u^2+v^2}{2}+\frac{p}{\rho}=F(y,t) \tag{9}$$

$$\frac{\partial \phi}{\partial t}+\frac{u^2+v^2}{2}+\frac{p}{\rho}+gy=G(x,t) \tag{10}$$

Subtracting Eq. (10) from Eq. (9) we obtain

$$F(y,t)=G(x,t)-gy \tag{11}$$

From Eq. (11) we see that by choosing $G(x,t)=G(t)$, Eqs. (9) and (10) are identical and can be rewritten as

$$\frac{\partial \phi}{\partial t}+\frac{u^2+v^2}{2}+\frac{p}{\rho}+gy=G(t) \tag{12}$$

which is the Bernoulli equation.

Equation (3) may be rewritten in polar coordinates as

$$\nabla^2\phi=\frac{\partial^2 \phi}{\partial r^2}+\frac{1}{r}\frac{\partial \phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}=0 \tag{13}$$

and the velocity components u_r and u_θ can be defined as

$$u_r=\frac{\partial \phi}{\partial r} \tag{14}$$

$$u_\theta=\frac{1}{r}\frac{\partial \phi}{\partial \theta} \tag{15}$$

which gives $u^2+v^2=u_r^2+u_\theta^2$ and hence from Eq. (12)

$$p(r,\theta,t)=\rho\left[-\frac{\partial \phi}{\partial t}-\frac{u_r^2+u_\theta^2}{2}-g(h+r\sin\theta)+G(t)\right] \tag{16}$$

where $y=h+r\sin\theta$ and $h<0$ is the depth of the midpoint of the cylinder. Horizontal flow around the cylinder is shown in Fig. 2.

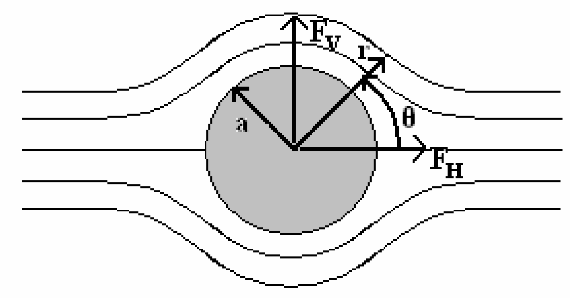

Figure 2. Potential flow around a circular cylinder.

In the potential flow theory different velocity potentials may be summed up owing to the principle of superposition. Here we divide the motion of the cylinder and water particles into horizontal and vertical parts and build up the velocity potentials respectively. The procedure of forming the Bernoulli equation and the horizontal velocity potential is quite similar to that used by Dean and Dalrymple[3]. The most important difference is the oscillating and horizontally aligned cylinder used in this model. An approximative solution for the velocity potential which satisfies the Laplace equation and the boundary conditions on the surface of the cylinder can be expressed as

$$\phi = \phi_H + \phi_V = \underbrace{ur\cos\theta + (u-U)\frac{a^2}{r}\cos\theta}_{\phi_H} + \underbrace{vr\sin\theta + (v-V)\frac{a^2}{r}\sin\theta}_{\phi_V} \qquad (17)$$

where u is the horizontal velocity of the water particle, U the horizontal velocity of the cylinder, v is the vertical velocity of the water particle, V the vertical velocity of the cylinder, a the radius of the cylinder, r the distance from the midpoint of the cylinder and θ the angle between the horizontal axis and the point. Both functions Φ_H and Φ_V satisfy Eq. (13) for any time dependent velocities $u(t)$, $U(t)$, $v(t)$ and $V(t)$, and approximately for velocities u and v which are given by Eqs. (21) and (22) and used in the calculations of Figs.3-11.

Assuming the flow to be slow around the cylinder, the term $(u_r^2 + u_\theta^2)/2$ in Eq. (16) can be neglected[2] and the pressure distribution on the cylinder may be obtained by using Equations (16) and (17):

$$p(a,\theta,t) = \rho\left[\left(-2\frac{\partial u}{\partial t} + \frac{\partial U}{\partial t}\right)a\cos\theta + \left(-2\frac{\partial v}{\partial t} + \frac{\partial V}{\partial t}\right)a\sin\theta - g(h + a\sin\theta) + G(t)\right] \qquad (18)$$

Integration of the pressure around the cylinder gives the horizontal and vertical forces on the cylinder (per unit length):

$$F_H^{'} = \int_0^{2\pi} p(a,\theta)(-a\cos\theta)\,d\theta = \pi\rho a^2\left(2\frac{\partial u}{\partial t} - \frac{\partial U}{\partial t}\right) \tag{19}$$

$$F_V^{'} = \int_0^{2\pi} p(a,\theta)(-a\sin\theta)\,d\theta = \pi\rho a^2\left(2\frac{\partial v}{\partial t} - \frac{\partial V}{\partial t} + g\right) \tag{20}$$

According the linear wave theory, the horizontal and vertical wave velocity components are[4]

$$u = \frac{HgT}{2L}\frac{\cosh\left[\frac{2\pi(y+d)}{L}\right]}{\cosh\left(\frac{2\pi d}{L}\right)}\cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) \tag{21}$$

$$v = \frac{HgT}{2L}\frac{\sinh\left[\frac{2\pi(y+d)}{L}\right]}{\cosh\left(\frac{2\pi d}{L}\right)}\sin\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) \tag{22}$$

where H is the wave height, T the wave period, L the wavelength and d the water depth. Because of the relative velocity between the cylinder and wave used in the velocity potential, we can set the values $x = 0$ and $y = h \le 0$ (the distance between still water level and the midpoint of the cylinder).

The horizontal and vertical velocity components of an oscillating cylinder are

$$U = U_0\cos\left(\frac{2\pi t}{T} + \varphi\right) \tag{23}$$

$$V = V_0\sin\left(\frac{2\pi t}{T} + \varphi\right) \tag{24}$$

where U_0 and V_0 are the maximum velocity components of the cylinder and φ the phase shift between the wave and the cylinder.

The wave power on the cylinder is the product of force and velocity:

$$P_{converter} = F_H U + F_V V \tag{25}$$

The mean energy flux of the wave energy converter can now be calculated

$$\dot{E}_{converter} = \frac{1}{T}\int_0^T P_{converter}\,dt$$
$$= \frac{\pi^2 g\rho a^2 H \sin\varphi}{L}\,\frac{U_0\cosh\left[\frac{2\pi(h+d)}{L}\right] - V_0\sinh\left[\frac{2\pi(h+d)}{L}\right]}{\cosh\left(\frac{2\pi d}{L}\right)} \quad (26)$$

The mean energy flux or power transmission per wave[2] is

$$\dot{E}_{wave} = \frac{\rho H^2 g c_g}{8} = \frac{\rho H^2 g}{8}\frac{1}{2}\frac{L}{T}\left[1 + \frac{\frac{4\pi d}{L}}{\sinh\left(\frac{4\pi d}{L}\right)}\right] \quad (27)$$

The efficiency of the wave energy converter is

$$\eta = \frac{\dot{E}_{converter}}{\dot{E}_{wave}} \quad (28)$$

4 THE EFFECTS OF PHASE SHIFT AND DIFFERENT SIZE WAVES ON WAVE ENERGY CONVERTERS

With this model we have analyzed several interesting topics which affect the efficiency of the wave energy converter. The phase shift is the most important parameter - with the optimum phase shift π/2 the best efficiency 0.5 is achieved when both horizontal and vertical oscillations are allowed. The other important parameters studied in this work are the wave height and the wave period. Also the cylinder radius has a great impact on the efficiency. Two different size cylinders are modeled: the smaller has a radius of two meters and is situated in three meter depth from still water level, while the bigger has radius of three meters and the depth four meters. In these calculations, the wave energy converter is located in the area where the water depth is 12 meters which means transitional water.

Figure 3 presents instantaneous velocity potentials Φ_H and stream function curves ψ_H around the cylinder at $t = 0$ when the phase shift is optimal, π/2. The impact of the cylinder on the stream is clear; the farthest parts of the stream are almost parallel. The water surface is also drawn in the figure. Instantaneous pressure distribution at $t = 0$ caused by the wave and hydrostatic pressure on the cylinder is presented in Fig. 4.

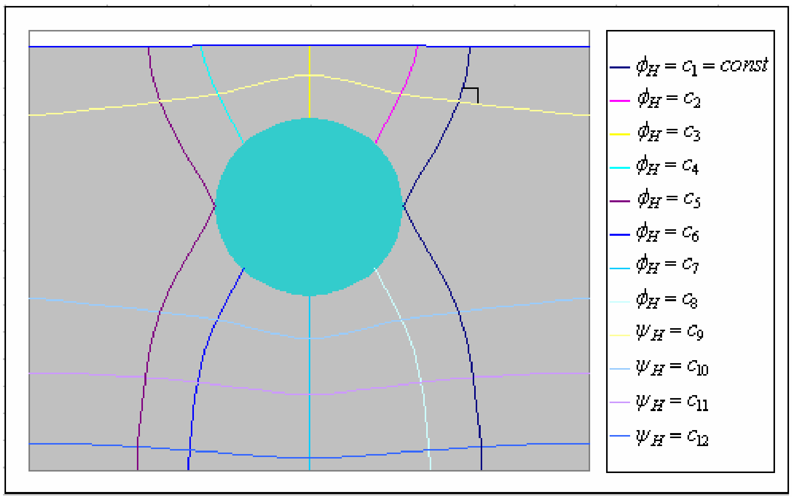

Figure 3. Constant velocity potentials Φ_H and corresponding stream function curves ψ_H around the cylinder (t = 0, phase shift = $\pi/2$).

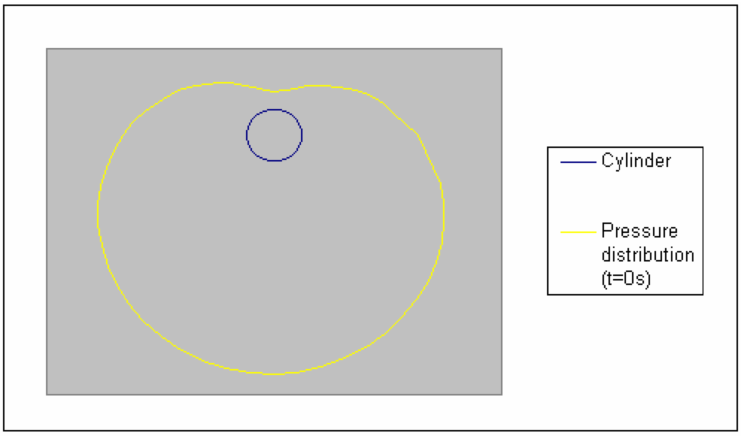

Figure 4. Instantaneous pressure distribution on the cylinder caused by the wave and hydrostatic pressure (t = 0).

The charts in figures 5-7 describe the interaction between one wave and wave energy converter. The cylinder has a radius of two meters and the depth three meters from the still water level. The wave height is 1.34 meters, the period ten seconds and the power 19 kW per wave crest meter. In all figures, H refers to horizontal movement and V to vertical movement. The phase shift between the cylinder velocity and the water particle velocity is well shown in Fig. 5. Figure 6 presents the wave forces faced by the oscillating wave energy converter. Figure 7 presents the power captured from the wave; the advantage of the two-dimensional path of the cylinder compared to the only horizontal path of the cylinder appears clearly in the graph. With optimal phase shift, power can be captured during whole wave period.

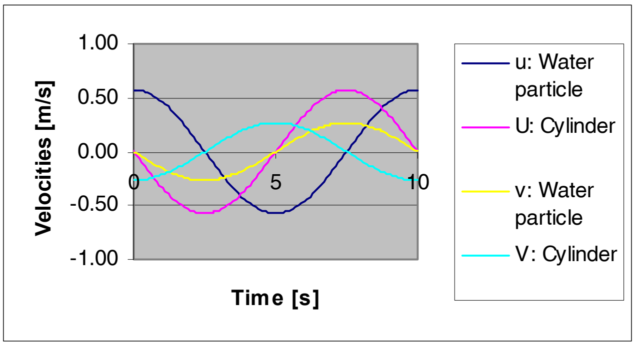

Figure 5. The velocities of the cylinder and water particles with optimum phase shift π/2 when cylinder radius is 3 m, wave height 1.34 m and wave period 10 s.

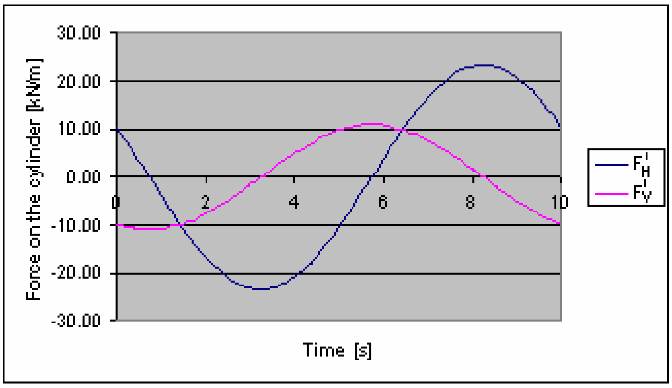

Figure 6. The force components on the cylinder when phase shift is optimal π/2, cylinder radius 3 m, wave height 1.34 m and wave period 10 s. Buoyancy is about 300 kN/m and therefore left from the chart.

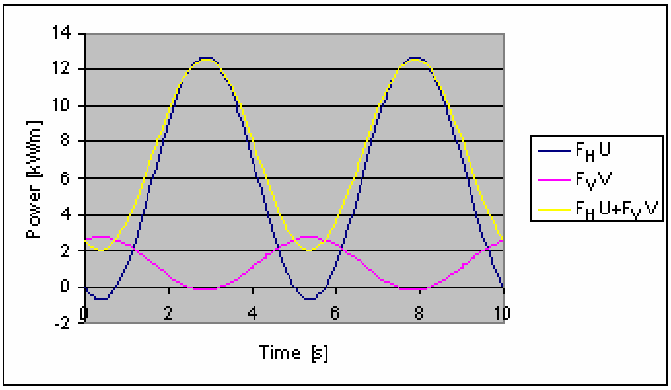

Figure 7. The power capture when phase shift is optimal π/2, cylinder radius 3 m, wave height 1.34 m and wave period 10 s.

Figures 8-11 present the impact of the phase shift between water particle velocity and cylinder velocity on mean power capture and efficiency of the wave energy converter. In Figures 8 and 9, different wave heights and cylinder sizes are modeled. According to these calculations, the efficiency of the power capture is not dependent on wave height.

In Figures 10 and 11, different wave periods and cylinder sizes are modeled. The shorter is the wave period, the bigger is the efficiency of the power transmission. The larger cylinder has predictably better efficiency than the smaller cylinder.

The importance of the right phase shift can be clearly seen in all the graphs. The implementation of the phase shift must be carefully taken into account when considered a wave energy scheme.

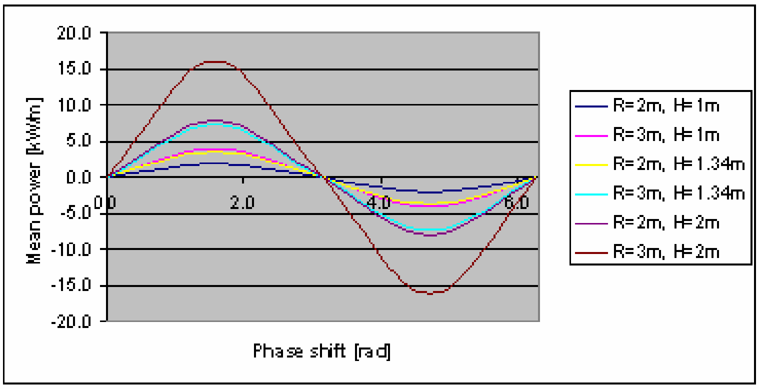

Figure 8. The impact of phase shift and different wave heights on the mean power.

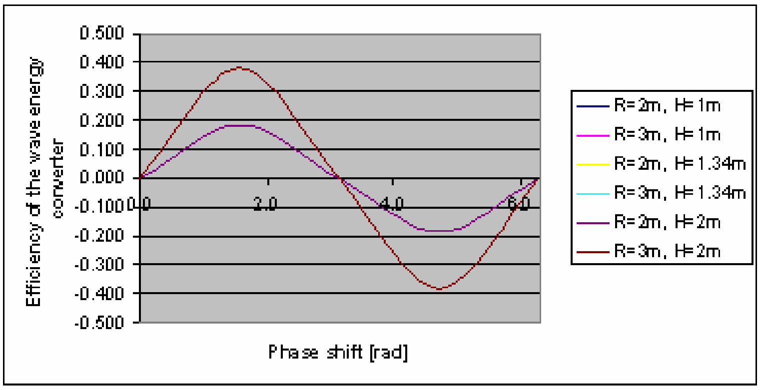

Figure 9. The impact of phase shift and different wave heights on the efficiency.

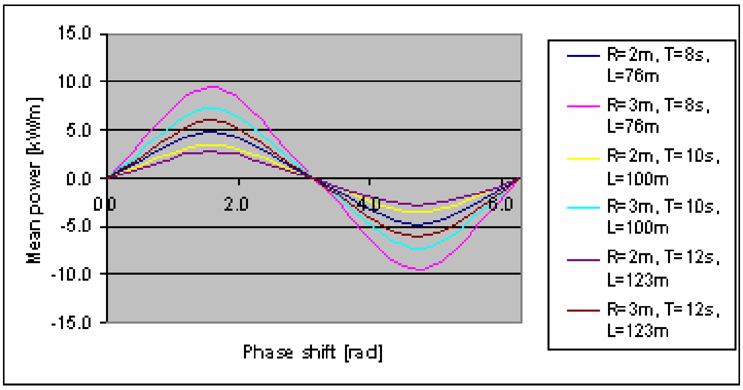

Figure 10. The impact of phase shift and different wave periods on the mean power.

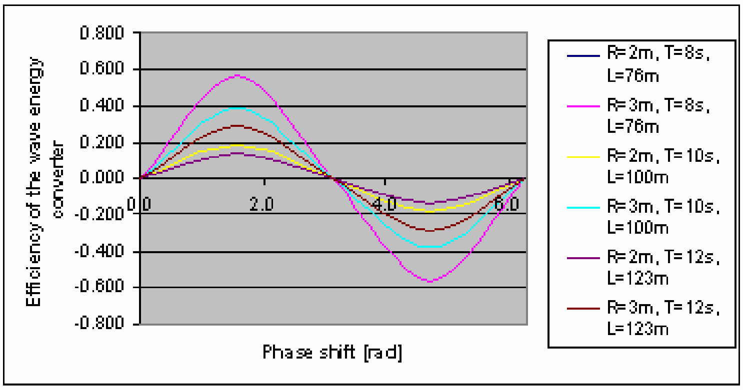

Figure 11. The impact of phase shift and different wave periods on the efficiency.

5 THE NECESSITY OF FEEDBACK CONTROL OF THE PHASE SHIFT

As shown in Section 4, a constant phase shift $\varphi = \pi/2$ between wave and cylinder is required for optimal converter efficiency. In practice, it is impossible to keep the phase shift constant without feedback control. That is direct or indirect measurement of the phase shift, and positioning of the cylinder accordingly. This can be shown as follows: If it is assumed that all the parameters in Equations (23) and (24) are known, including the initial phase shift, and the cylinder is positioned according these Equations, with $\varphi = \pi/2$ at time $t = 0$. If the actual wave period is the constant T_w, and not the assumed constant *T*, the actual phase shift φ will evolve over time according to Equation (29).

$$\begin{aligned} \varphi(t) &= \frac{\pi}{2}+\frac{2\pi t}{T}-\frac{2\pi t}{T_w} \\ &= \frac{\pi}{2}+\frac{2\pi\left(T_w-T\right)}{T_w T}t \end{aligned} \quad (29)$$

If $T_w \neq T$, which obviously will be the case in practice, the phase shift will evolve like a ramp. The phase shift is thus an integrating system with respect to the wave period, which means that it is unstable. Even a momentary deviation from the assumed wave period will result in a permanent error in the phase.

For a stable system, a momentary disturbance should only result in a momentary deviation from the initial steady state. And the only way to stabilize an unstable system is to introduce feedback[5].

6 CONCLUSIONS

The cylindrical wave energy converter enabled us to analytically study the interaction between the converter and waves. It was possible to sketch the water flow and compose the velocity potential around the cylinder.

The two-dimensional elliptical movement of the cylinder seems to be notably more efficient than only horizontal oscillations – even in transitional water depth used in the calculations. With optimal phase shift, the converter may capture energy during whole wave period.

One of the most important results in this study is the impact of the phase shift between the wave and cylinder. To achieve the right phase shift for different waves is essential due to the power capture. No energy is produced in the case of none phase shift and some energy can be lost in the case of defective phase shift. Furthermore it was shown that feedback control is necessary for keeping the phase shift constant. The implementation of such a feedback controller could be difficult in practice, and could be a topic for future studies.

The results achieved in this study are not meant to be extremely accurate. The basic purpose is to model the interaction between waves and oscillating wave energy converters, and clarify the impact of different sized waves to the power production. This model may be quite easily used when considering different circumstances for wave energy converters.

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