

INTERACTION BETWEEN WAVE AND COASTAL STRUCTURE: VALIDATION OF TWO LAGRANGIAN NUMERICAL MODELS WITH EXPERIMENTAL RESULTS

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Summary. Numerical modeling of the interaction among waves and coastal structures is a challenge due to the many nonlinear phenomena involved, such as, wave propagation, wave transformation with water depth, interaction among incident and reflected waves, run-up / run-down and wave overtopping. Numerical models based on Lagrangian formulation, like SPH (Smoothed Particle Hydrodynamics), allow simulating complex free surface flows. The validation of these numerical models is essential, but comparing numerical results with experimental data is not an easy task. In the present paper, two SPH numerical models, SPHysics^{LNEC} and SPH^{UNESP}, are validated comparing the numerical results of waves interacting with a vertical breakwater, with data obtained in physical model tests made in one of the LNEC's flume. To achieve this validation, the experimental set-up is determined to be compatible with the Characteristics of the numerical models. Therefore, the flume dimensions are exactly the same for numerical and physical model and incident wave characteristics are identical, which allows determining the accuracy of the numerical models, particularly regarding two complex phenomena: wave-breaking and impact loads on the breakwater. It is shown that partial renormalization, i.e. renormalization applied only for particles near the structure, seems to be a promising compromise and an original method that allows simultaneously propagating waves, without diffusion, and modeling accurately the pressure field near the structure.

1 INTRODUCTION

Coastal structures are constructed to act as an obstacle to the natural wave propagation, creating areas of reduced wave activity. Their efficiency depends on their ability to dissipate wave energy. In the last decades, composite breakwaters, consisting of a rubble foundation and a vertical wall, have been design as an alternative to rubble-mound breakwater, with the main advantage of reducing costs, improving hydraulic performance and construction time. However, wave reflection, and especially wave loads, increases considerably.

Prediction of wave forces on caisson-type breakwaters is complicated, especially in the case of impact loads, which strongly depend on wave reflection and breaking. Knowledge on the hydrodynamic behaviour of these structures is necessary for their design, to guarantee both its structural safety and functionality. The forces on a composite breakwater are usually predicted using semi-empirical formulae, in part derived from physical modelling tests. However, direct application of these formulae is limited to simple structural configurations and to specific wave conditions. In practical engineering case studies, physical model testing is usually also undertaken, which permit a reliable evaluation of the structure's efficiency. The greatest disadvantages of physical model tests are the required time for their construction and exploitation, their high cost and their lack of flexibility to changing structural geometrical characteristics. Moreover, scale effects could affect the results.

In recent years, due to the continuous increase in computer processing power, numerical models have been developed further and their use is becoming increasingly attractive. However, only few numerical models allow simulating the very complex phenomena of wave breaking, reflection and overtopping, and consequently forces at vertical structures. Those models are generally based on the Navier-stokes equations and developed using an Eulerian approach.

Recently, models based on Lagrangian methods, such as the Smoothed Particle Hydrodynamics (SPH) approach, have emerged. The method is based on the Navier-Stokes equations and a completely mesh-free technique. Monaghan, in 1994 [1], shows the first application of this method for modelling free surface flows and demonstrates that SPH is a very promising alternative to simulate wave breaking. Different models have been developed, based on the SPH formulation of the Navier-Stokes equations, and there are many different numerical implementations. There are two main approaches for modelling incompressible free surface flows: the weakly-compressible SPH method - WCSPH, where fluid pressure is related to particle density using an equation of state, and incompressible SPH approach - ISPH, where fluid pressure is calculated solving a Poisson equation.

Recent studies, for analysing the influence of some critical parameters of WCSPH models, such as filtering density "renormalisation", conventional and Riemann slover-based formulations, are performed by Hughes et al. [2], Gómez-Gesteira et al. [3], and Rogers et al. [4], for dam-break problems and regular wave impacting a vertical wall.

For the case of dam-break, Hughes et al. [2] compared ISPH and density filtering WCSPH models. Both models give good predictions when compared with experimental results and authors concluded that for the optimum configuration, WCSPH performs at least as well as ISPH, and in some aspects clearly performs better. Gómez-Gesteira et al. [3], using the WCSPH SPHysics model [5, 6], for the same dam-break case-study, analysed different

methods to improve the conventional WCSPH approach, such as the use of density filters and corrections of the kernel, comparing predictions with laboratory tests. The density fields obtained using density filters are similar and less noisy than the ones obtained without density filters. Filtering methods show to be more accurate than the unfiltered one.

Rogers et al. [4] simulated forces on a caisson breakwater under the action of periodic waves impinging on the vertical wall of the structure. The weakly-compressible Riemann solver-based formulation of SPHysics model [7] is used and it was show that results are in better agreement with experimental than their obtained using the conventional filtering WCSPH SPHysics formulation [5, 6]. However no sensitivity study was made with the particle dimension even though it was pointed out that this is a crucial point for the model accuracy.

Hughes et al. [2] also compared ISPH and WCSPH models, with filtering density, for regular waves impacting on a vertical wall. Both methods are shown to be capable of simulating the generation of wave impact pressures. The predictions for the wave impacts are shown to be in qualitative and quantitative agreement with available experimental data. Results from the WCSPH appear to be significantly smoother than those from ISPH. This is particularly the case for free surface deformation and especially for wave-breaking.

The paper presents the validation of two WCSPH numerical models, SPHysics^{LNEC} and SPH^{UNESP}, to the study of wave interacting with a composite breakwater. To achieve this validation, a set of tests were performed in a wave flume at National Civil Engineering Laboratory (LNEC) with the experimental set-up and boundary conditions (wave generation) compatible with the numerical model characteristics. Therefore, the flume dimensions are the same for both numerical and physical model and the characteristics of the incident wave are identical, which allows determining the accuracy of these two numerical models, particularly regarding of two complex phenomena: wave-breaking and wave impacts on the vertical breakwater.

SPHysics^{LNEC}, based on SPHysics model [5, 6], has been used and developed at the LNEC since 2008 to obtain a useful tool for real case studies of coastal engineering. Promising agreement with experimental data has been obtained for both free surface elevation and overtopping discharge for impermeable coastal structures [8, 9, 10, 11]. However, the validation has been made with results from physical model tests not specifically designed for the model validation and with experimental set-up that differ from the numerical model.

SPH^{UNESP}, a SPH model developed by the Unesp-Ilha Solteira team, aims to solve complex flow problems, mainly due to regional characteristics. This model was validated for problems that consider the generation and breaking waves [12] and hydrodynamic impact [13].

2 NUMERICAL MODELS

SPH method consists in the integration of the hydrodynamics equations of motion on each particle in the Lagrangian formalism. The partial differential equations of continuum fluid dynamics are transformed into SPH forms, i.e. particle forms, by integral equations using integral interpolants [14, 15]. The fundamental principle is to approximate any function $A(r)$ by:

$$A(r) = \int_{\Omega} A(r') W(r-r', h) dr' \quad (1)$$

where r is the vector position, W is the weighting function, h is called the smoothing length. The interpolation function, i.e. weighting function or kernel, allows determining the interaction among neighboring particles included in the influence domain, controlled by the smoothing length h , typically higher than the initial particle spacing, do .

Figure 1 shows a typical compact support of a kernel function. The kernels should verify several conditions of positivity, compact support, Delta function behavior, etc. Different kernels were developed and can be found in the literature [16].

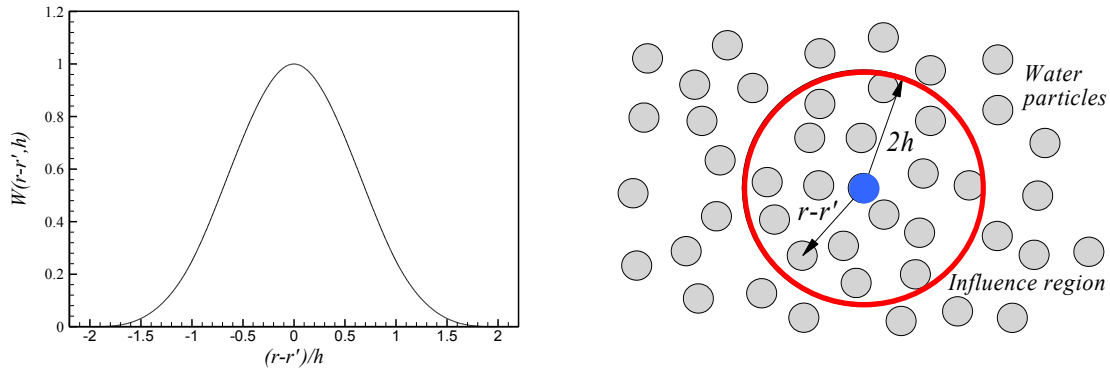


Figure 1: Typical kernel function and its compact support.

The relation given in Eq.1 is written as an approximation of the function A at a particle a , in discrete notation:

$$A(r) = \sum_b m_b \frac{A_b}{\rho_b} W_{ab} \quad (2)$$

where the summation is over all the particles within the influence region of the kernel. The mass and density are noted m_b and ρ_b respectively and $W_{ab} = W(r_a - r_b, h)$ is the kernel.

There are two main approaches for modelling incompressible free surface flows: the weakly-compressible SPH method - WCSPH [14, 17, 18], where fluid pressure is related to particle density using an equation of state, and incompressible SPH approach - ISPH [19, 20, 21], where fluid pressure is calculated solving a Poisson equation. The two SPH numerical models, SPHysics^{LNEC} and SPH^{UNESP}, are based on weakly-compressible method and inspired by the formulation of Monaghan [14] and will be briefly described on sections 2.1 and 2.2.

The bi-dimensional momentum conservation equation in a continuum field and the conservation law, in Lagrangian form, are written as

$$\frac{dv}{dt} = -\frac{1}{\rho} \nabla P + \Pi + g, \quad \frac{1}{\rho} \frac{d\rho}{dt} = -\text{div}(v) \quad (3)$$

where v refers to the velocity, P the pressure, ρ the fluid density, Π the diffusion terms and $g = (0, -9.81 \text{ m/s}^2)$ the gravitational acceleration.

Different approaches, based on various existing formulations of the pressure and diffusive

terms, can be considered in the SPH method to describe the momentum equation.

The relation-ship between pressure and density in WCSPH method is assumed to follow the equation of state (4) provided by Batchelor [22]:

$$P = B \left[\left(\frac{\rho}{\rho_0} \right)^\gamma - 1 \right], \quad B = c_0^2 \rho_0 / \gamma \quad (4)$$

where γ is a constant between 1 and 7 (although 7 is used in most of the oceanic applications), ρ_0 the reference density of water (1000kg/m³) and c_0 the speed of sound.

2.1 SPHysics^{LNEC} model

The LNEC team has mainly focused its developments, researches and applications on the wave propagation and interaction with coastal structures. The actual model SPHysics^{LNEC} is inspired by the SPHysics code, an open-source Smoothed Particle Hydrodynamics program developed jointly by researchers of several Universities [5, 6, 23, 24, 25]. The numerical model presents a modular form and a variety of features are available, such as 2D and 3D models, various kernels and viscosity models, density filter, and solid boundary conditions (dynamic boundaries, repulsive forces). Detail of numerical implementation is available in [5, 6]. The actual SPH model allows analyzing wave-structure interaction, including free surface elevation, wave overtopping, velocity profiles, pressure and forces on the structure.

For numerical simulations of wave propagation and interaction with a vertical breakwater, the quadratic kernel [26, 27] is used to determine the interaction between the particles.

Integration in time is performed by the Predictor-Corrector model using a variable time step to ensure the CFL condition. The repulsive boundary condition, developed by Monaghan [28], is used and allows preventing a water particle crossing a solid boundary.

It was shown, in previous study [9], that Sub-Particle Scale – SPS – turbulence model [20] provided better results when compared to artificial viscosity model [14] since SPS model avoids the strong dissipative effects of artificial viscosity model. So the SPS turbulence model is used in the present simulations.

Particles are usually moved using the XSPH variant due to Monaghan [29]. The method is a correction for the particle velocity, which is recalculated taking into account the velocity of that particle and the average velocity of neighbouring particles. However, it was shown in [9] that instabilities appear during wave propagation due to the XSPH correction and fluid flow exhibits unphysical behaviours. Consequently, the XSPH correction is not used.

While the kinematics of SPH simulations is generally realistic, the pressure field of the particles can exhibit large pressure oscillations. One of the most straightforward and computationally least expensive methods to smooth out pressure oscillations is to perform a filter over the density of the particles and to re-assign a density to each particle [30]. So that, in the present study, SPHysics^{LNEC} model is running with and without Shepard density filter.

2.2 SPH^{UNESP} model

The SPH^{UNESP} model uses bi-dimensional Wendland-type kernel with compact support [31] to compute interactions between particles. On the momentum equation, it is used the

classic viscous formulation [31], in order to avoid small numerical oscillations, when the particles are approaching each other. It is also added a term on the pressure gradient to avoid possible tensile instabilities [32].

Initial and boundary conditions are set in the following way. All particles are placed in a square lattice, with zero pressure, and are damped until the rest position is achieved. The boundary conditions, on the other hand, are treated as proposed by Sirovich [31], which makes force compatibility instead of cinematic one. This procedure ensures the free slip condition on the boundary.

For the particle displacement and the pressure filtering, no additional treatment is done, and are handled the same way as SPH standard.

The equations, written on the particulate form, are integrated numerically by a Verlet method. Although this method is a second order one, the Verlet integrator is conservative, which preserves the main characteristics of SPH. The time step is obtained automatically.

3 CASE STUDY AND PHYSICAL MODEL TESTS

The case study presented here is wave interacting with a coastal structure: an impermeable composite breakwater. The breakwater is composed by a berm, 0.181 m height with a 1:3 slope, and a vertical wall, located at 0.2m from the end of the slope, as it shown in the in Figure 2. The crest freeboard is 0.3m above the water level. The vertical structure was built in acrylic and the berm is impermeable and made of wood.

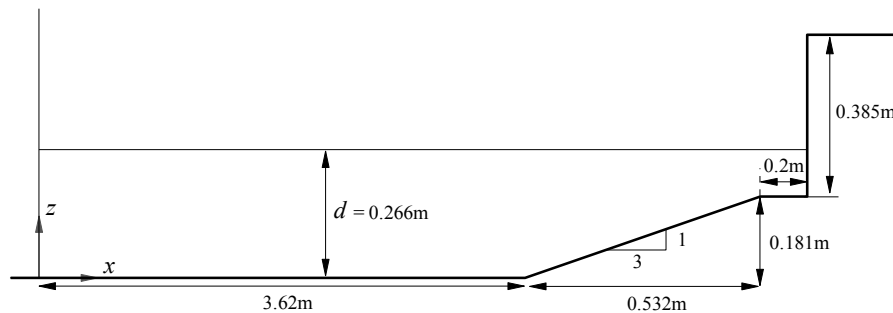


Figure 2: Schematic representation of the model cross-section, the coordinate system and the dimensions.

For this specific case, experimental data was collected recently in one of the LNEC's flume. The physical model set-up was built in such a way that was feasible the exact replication in the numerical model, taking into account the limitations of the model reproduction, as well as the wave generation. Therefore dimensions of flume and structure and wave generation conditions are the same in both cases.

Tests were conducted in 49.4m length, 1.6m wide, 1.2m high flume. The flume is equipped with a piston-type wave-maker which as active absorption of reflected waves and is cable of generating both regular and irregular waves.

For the conducted tests two resistive-type wave gauges and six pressure sensors were used. The first gauge (G1) was located at $x=2.643\text{m}$, with $x=0\text{m}$ at the wave-maker, to verify the generated wave characteristics and the second gauge (G2) was located on the slope at $x=3.943\text{m}$. The pressure sensors were located at the front face of the vertical wall with a regular vertical distance between them, center-to-center, equal to 0.055m, starting at the

bottom of the bern (sensor P1) to the top of the structure (sensor P6). Figure 3 shows the wave flume with the vertical breakwater and the position of the pressure sensors.



Figure 3: Physical breakwater model in the flume with position of the pressure sensors.

Regular waves were tested with two wave periods, T , of 1.1s and 1.3s, and wave heights, H , varying from 0.04m to 0.16m. Two water depths, d , were tested, 0.325m and 0.266m, which resulted in wave lengths, L , ranging from 2.02m to 1.50m. Test duration was chosen in order to generate at least ten waves. Active absorption of the reflected waves at the wave maker was not used in the present tests. During the interaction between wave and structure the front vertical wall experiences pressure peaks that occur in a very short time interval. Therefore a frequency of 400Hz was used for data acquisition in both surface level gauges and pressure sensors.

Each wave condition was repeated ten times. For all cases, test repetitions present a good agreement: for $d=0.266\text{m}$, $T=1.3\text{s}$ and wave heights from 0.040m to 0.140m, the maximum deviation between test repetitions, before wave re-reflexion on the wave-maker, was $\pm 0.008\text{m}$. The root mean square – *RMS* – of the free surface level deviation are very small, varying from $4.91 \times 10^{-3}\text{m}$ to $1.57 \times 10^{-2}\text{m}$, which indicates just small differences in free surface level. Overall pressure deviation shows also a good concordance despite its higher *RMS* values, ranging from $2.90 \times 10^{-2}\text{mH}_2\text{O}$ to $1.65 \times 10^{-1}\text{mH}_2\text{O}$. The phenomena involved and the high turbulent nature of the flow combined with the very short duration of the pressure peaks, which are difficult to capture, are some of the possible reasons for these higher values when compared to the *RMS* of free surface elevation.

4 DISCUSSIONS OF RESULTS

4.1 Influence of density filter for wave propagation and pressure on structures

Gómez-Gesteira et al. [3] shows that renormalization (filter density) is necessary to obtain a good representation of a dam-break flow and pressure field without oscillations. However, renormalization influence is more complex for wave propagation. Figure 4 shows the free surface elevation at gauges G1 and G2 for the physical tests together with numerical results obtained with SPHysics^{LNEC} model for the incident wave $T=1.3\text{s}$, $H=0.06\text{m}$ and depth $d=0.266\text{m}$. The wave paddle movement in numerical simulation is the same as used for

physical tests. Numerical results are obtained for the finer resolution ($do=2.66\text{mm}$) for three renormalizations: i) partial, where density filter is applied only for the particles near the structure; ii) total, where density filter is applied for all particles (classical used of renormalization); and iii) without density filter. In both gauges the reduction in wave height due to total renormalization in the full domain is very clear and numerical and experimental results are different. Renormalization acts as a numerical diffusion. Results obtained with partial renormalization and without renormalization show just slightly differences, especially at gauge G2, indicating that wave propagation is very well modeled without renormalization. However, without renormalization, pressure oscillations appear and making difficult the pressure estimation on the vertical wall. Partial renormalization, i.e. renormalization applied only for particles near the structure, seems to be a promising compromise that allows simultaneously propagating waves, without diffusion, and modeling accurately the pressure field, near the structure.

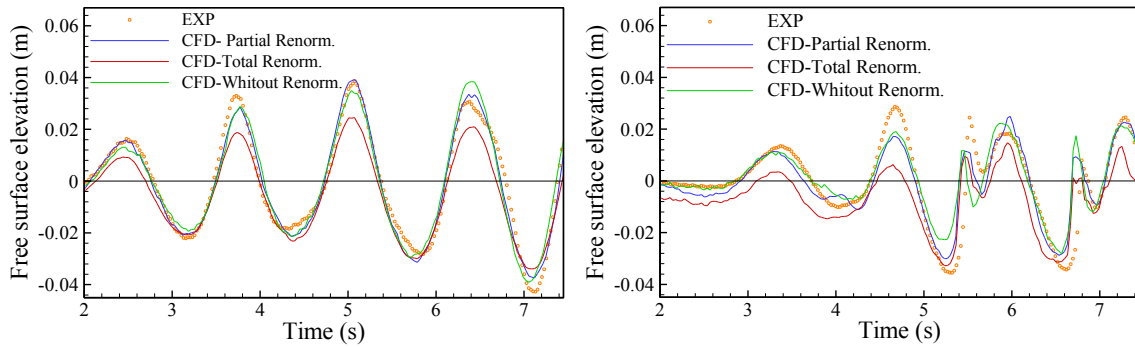


Figure 4: Influence of density filter on free surface ($T=1.3\text{s}$, $H=0.06\text{m}$): (a) Gauge G1 and (b) gauge G2.

4.2 Convergence study of SPHysics^{LNEC} with refinement

Convergence study with particle dimensions, do , varying from 8.87mm to 2.66mm (i.e. 14337 to 154032 particles) is performed for an incident wave with $T=1.3\text{s}$ and $H=0.10\text{m}$ on a depth $d=0.266\text{m}$. Figure 5 shows the free surface elevation at G1 and G2, the pressure at P1 (near the berm) and the force on the front vertical wall. SPHysics^{LNEC} results are compared with experimental data. At G1 the convergence on the free surface elevation is obtained for the finer resolutions. However, at G2, where the breaking and reflection affect the shape of the wave, the convergence is not so clear, even though it seems to globally converge to the test results for the finer resolution. Particle mass increases when resolution is coarser, resulting in changes on the kinetics of the flow and, consequently, on wave breaking and wave-reflection on the structure, which explains the results at G2. Even though, as shown in previous papers, wave propagation is not very dependent on the resolution, wave breaking is very sensitive to particle dimensions [9, 10]. The same analysis can be done for pressure and resulting force on the structure. Pressure depends greatly on the wave breaking and on the wave impact at the wall. For finer resolutions and for the first pick of load, i.e. before wave reflection, convergence of pressure and force is confirmed. However, for the second load pick, this is not so clear, due to differences on wave breaking, reflection and interaction between the breaking wave and the structure, that are enough to change the pressure at the sensor position.

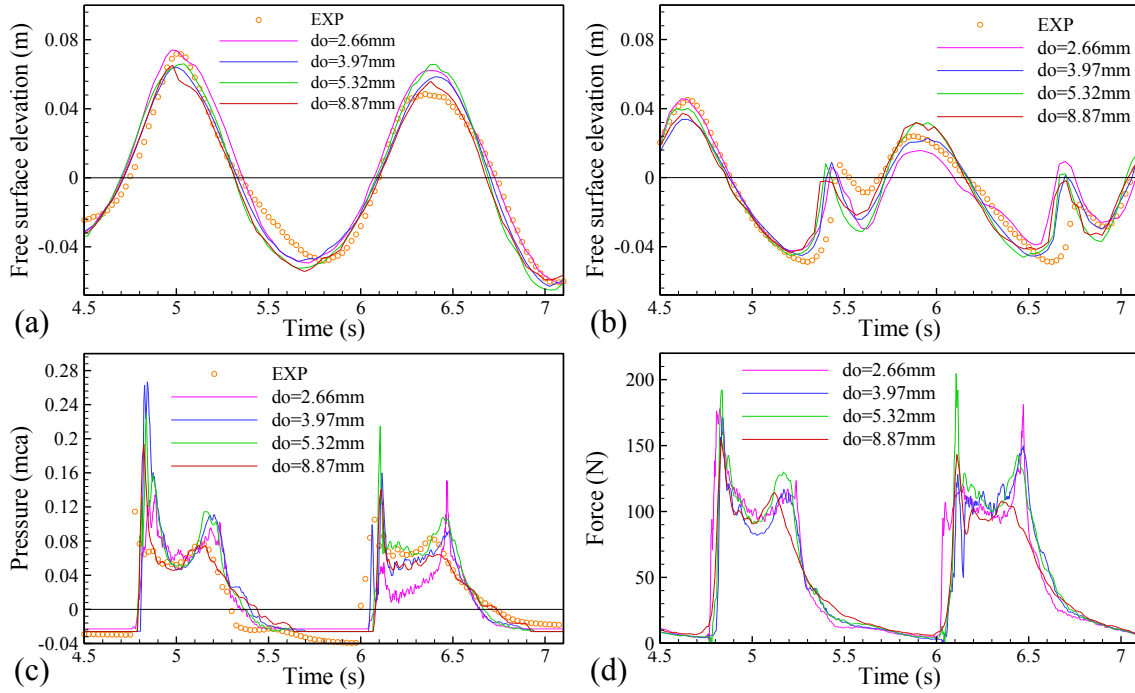


Figure 5: Convergence of free surface, pressure and force with discretization ($T=1.3s$, $H=0.10m$, $d=0.266m$): (a) Gauge G1, (b) gauge G2, (c) pressure sensor P1 and (d) force on front vertical wall.

4.3 Comparison of SPHysics^{LNEC} and SPH^{UNESP} models with experimental

Figure 6 shows the free surface elevation obtained by both SPHysics^{LNEC} ($d=2.66mm$), and SPH^{UNESP} ($do=10.4mm$) models at G1 and G2 for two incident waves with $T=1.3s$ and $H=0.06m$ and $0.10m$, respectively. Pressure at P1 and force on the vertical wall are presented on Figure 7. Model results are compared with experimental data. Overall, the SPHysics^{LNEC} model show good agreement with experimental data for both free surface elevation and pressure. Differences at G2 and P1, as explained before, are probably due to the very complex phenomena that occur near the structure, where small differences on wave breaking and wave reflection lead to significant changes on free surface elevation and pressure. SPH^{UNESP} model results are far from the experimental and the SPHysics^{LNEC} results due to the coarse resolution used, with only 10800 particles. Pressure is not presented since renormalization is not used in the actual version of the model, leading to large unphysical oscillations.

5 CONCLUSIONS

The paper presents the validation of two WSPH numerical models, SPHysics^{LNEC} and SPH^{UNESP}, to the study of wave interacting with a composite breakwater. To achieve this validation, a set of tests was performed in a wave flume at LNEC with the experimental set-up and wave generation compatible with the numerical model. Therefore, the flume dimensions are the same for both numerical and physical model and the incident wave is identically made using a wave-paddle (same motion), which allows determining the accuracy of the numerical models, particularly regarding to wave-breaking and wave impacts on the vertical breakwater.

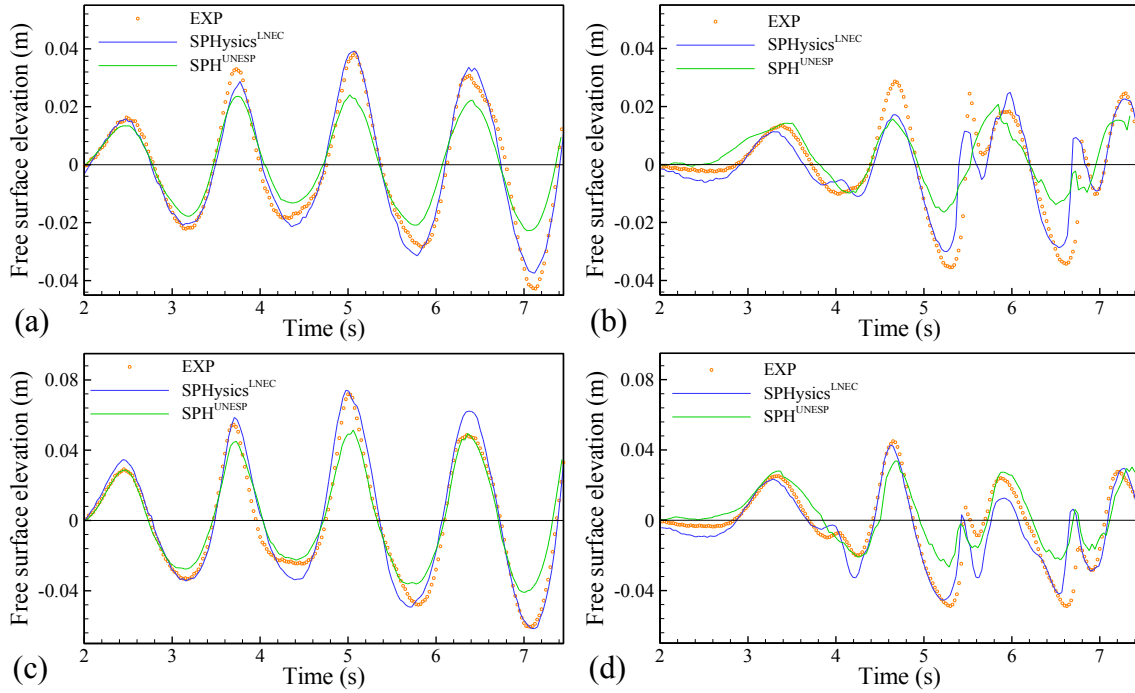


Figure 6: Comparison between numerical results and experimental data for two incident waves: $T=1.3$ s and $H=0.06$ m, (a) gauge G1 and (b) gauge G2; $T=1.3$ s and $H=0.10$ m, (c) gauge G1 and (d) gauge G2.

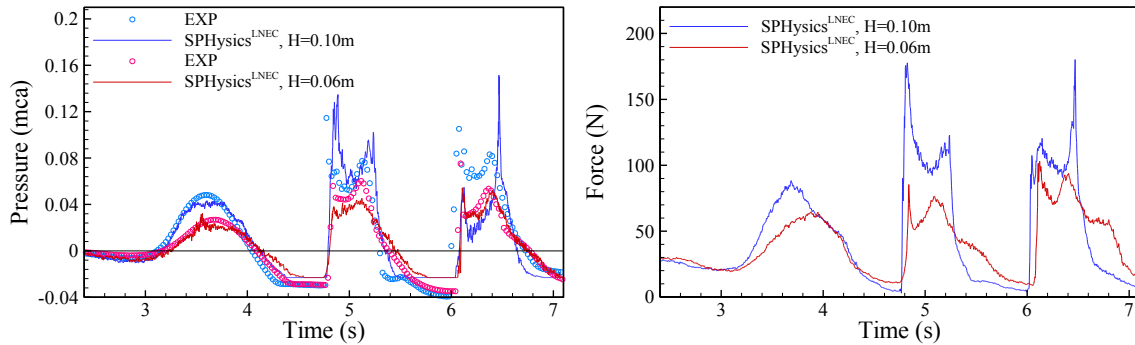


Figure 7: Pressure at sensor P1 (a) and force (b) for two incident waves with $T=1.3$ s and $H=0.06$ m and 0.10 m.

It is shown that partial renormalization, i.e. renormalization applied only for particles near the structure, seems to be a promising compromise and an original method that allows simultaneously propagating waves, without diffusion, and modeling accurately the pressure field near the structure. Without renormalization, unphysical pressure fluctuations occur and force on vertical wall of the structure cannot be calculated. On the other hand, total renormalization stabilizes the pressure field but causes a numerical diffusion of wave propagation and a reduction on wave height, incompatible with experimental results.

Overall the SPHysics^{LNEC} model shows good agreement with experimental data for both surface level and pressure results. Resolution is critical for accurately modeling the wave breaking and the resulting pressure (and force) at the structure that depends greatly on the wave breaking and on its interaction with the composite breakwater vertical wall.

SPH^{UNESP} model results were far from the experimental data. However, this model is actually in development, implementing filter density and renormalization and was used with a coarse particle dimension, which certainly affected the results.

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