CNOIDAL WAVE INDUCED FORCES ON A SUBMARINE PIPELINE

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Key words: Cnoidal water wave; Biot equation; Submarine pipeline; Pore pressure

Abstract. Forces acting on a submarine pipeline by cnoidal water waves is analyzed with Biot equation. Parametric studies are carried out to examine the influence of air content in pore water and the soil hydraulic conductivity on the forces. It has been shown that the air content and the soil hydraulic conductivity can significantly affect the pore pressure, horizontal and vertical forces acting on the pipeline. An increase in the air content and a reduction of the soil hydraulic conductivity as well as an increase of the soil hydraulic anisotropy will reduce the pore pressure on the surface of the pipeline. The maximum uplift force acting on the pipeline approaches 0.2 times of the pipeline buoyancy and, a high soil hydraulic anisotropy can make the magnitude of the horizontal force approaches 0.15 times of the pipeline buoyancy.

1 INTRODUCTION

Numerous researches had been focused on ocean waves induced transient response on a seabed and submarine pipelines. Mei and Foda [1] gave an analytical approximation on stresses in a semi-infinite seabed with a pipeline fixed on the seafloor. In the study on waves induced pore pressure around a pipelines, Spierenburg [2] gave the uplift force on the pipeline based on the potential theory. Furthermore, a physical experiment was set up to model the breakout of the half buried pipeline in sand by Foda, Law and Chang [3]. Cheng and Liu [4] investigated waves induced seepage force on a rigid pipeline buried in a seabed with the boundary integral equation method. Magda et al. [5-7] analyzed the pore pressure around the pipeline and the uplift force with a finite element method approach on the base of Biot equation. In addition, analogous laboratory and numerical studies have been published [8-10].

In view of that waves are usually nonlinear especially in shallow water, Mostafa and Mizutoni [11]; Gao and Wu [12] investigated the nonlinear wave force on marine pipeline with a coupled FEM-BEM model. Naturally, the seabed in coastal frequently become unstable before the ultimate design conditions of pipeline, Teh et al. [13, 14] and Sumer et al. [15, 16] studied the stability of submarine pipeline laid in a liquefied seabed.

The preceding researches present sufficient, multi-perspective and comprehensive knowledge about the response of the the seabed and pipelines to waves. The linear wave theory is frequently applied in the preceding analytical analysis. Whereas, waves are usually nonlinear in the coastal region, and the linear wave is not adequately suitable. Cnoidal wave theory is an explicit theory mainly applicable for shallow water. Much work has been devoted to facilitate the application of that wave theory [17-19]. This paper considers a poro-elastic seabed embedding a pipeline under cnoidal waves herein.

Consequently, the natural seabed is very complex in composition and texture. Properties about the air content in pore water and soil hydraulic conductivity are very difficult to measure. In this paper parametric studies are carried out to indicate the influence of the air content and soil hydraulic conductivity on the forces on the pipeline.

2 PROBLEM FORMULATION

Problem analyzed in this study is illustrated in Fig. 1. A pipeline is fully buried in a semi-infinite seabed. The seafloor is horizontal and the pipeline is weightless. Interaction between the pipeline and the seabed is not considered. Biot equation on poro-elastic media is employed to describe the response of the seabed, which has been generally accepted as the governing equation for the soil displacement and pore pressure. Adopting the sign of compression positive for stress which is frequently used in geotechnical engineering, the governing equations may be written as

$$G\nabla^2 u + (\lambda + G)\frac{\partial \varepsilon_V}{\partial x} = \frac{\partial p}{\partial x} \tag{1}$$

$$G\nabla^2 v + (\lambda + G)\frac{\partial \varepsilon_V}{\partial z} = \frac{\partial p}{\partial z} \tag{2}$$

$$k_x \frac{\partial^2 p}{\partial x^2} + k_z \frac{\partial^2 p}{\partial z^2} = n\beta\gamma_w \frac{\partial p}{\partial t} + \gamma_w \frac{\partial \varepsilon_V}{\partial t} \tag{3}$$

in which *u(x, z, t), v(x, z, t)* denote the soil displacement in horizontal and vertical direction, and *p(x, z, t)* represents the pore pressure. ε_V is the bulk strain (i. e. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$), γ_w is the specific weight of water. *t* is the time, *n* is the soil porosity. k_x, k_z are the hydraulic conductivity in x and z directions. β denotes the compressibility of the pore water including a small amount of air, i. e.

$$\beta = \frac{1-d_a}{K_w} + \frac{d_a}{P_{abs}}$$

where K_w is the bulk elasticity modulus of the pure water (taken as 2.3×10^9Pa), d_a is the percentage of air content in pore water, P_{abs} is the absolute pore pressure. The pore water in seabed is not always completely saturated and contains a small quantity of bubbles. Whereas, even a very few amounts of bubbles in the water can decrease the effective bulk modulus of the pore water significantly [20-22].

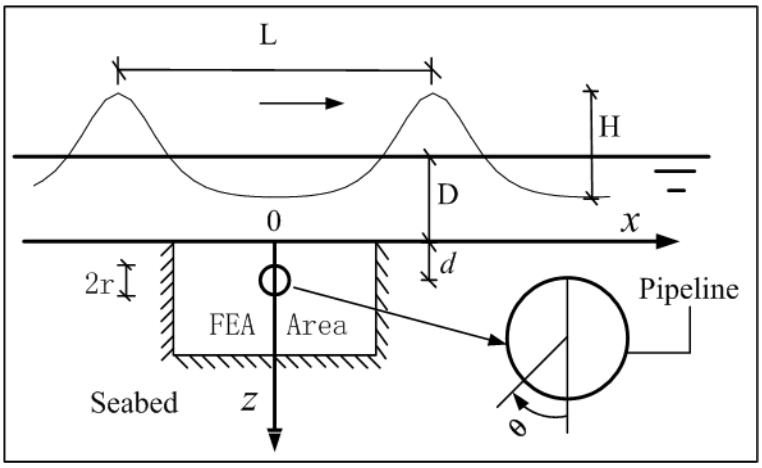


Fig. 1 Problem analyzed.

When cnoidal waves pass over the seabed, the oscillatory water pressure on the surface of seabed can be approximated as

$$p(x,t) = p_0 + g(x,t) \tag{4}$$

where $p_0 = \gamma_w H\left(\frac{1}{m^2} - \frac{E}{m^2 K} - 1\right)$, $g(x,t) = \gamma_w H cn^2\left[2K\left(\frac{x}{L} - \frac{t}{T}\right)\right]$, H is the wave height, L is the wave length, T is the wave period, m is the modulus. K and E are the first and second kinds of complete elliptic integral.

The boundary conditions on the seafloor are that the pore pressure is given by Eq. (4) and there is no normal and shear stress. Referring to Fig. 1, we consider a finite seabed which is bounded from below and sides by rigid and rough surfaces. The boundary conditions for the pore pressure flux is $\frac{\partial p}{\partial n} = 0$ in which n is the outward normal of the boundary. And for the soil skeleton, the boundary condition is $U_n = 0$ where U_n is the normal component of the displacement. In general, there is a soft impermeable layer on the surface of a pipeline which can prevent the pipeline from corrosion by the sea water. Then the boundary condition for the pore pressure is no water flux and the soil skeleton is free along the pipeline surface.

The initial conditions of Eqs. (1-3) are given with the assumption that the soil displacement and the pore pressure are negligible at the beginning, i. e. $u(x,z,0) = 0$, $v(x,z,0) = 0$ and $p(x,z,0) = 0$.

The horizontal and vertical forces acting on the pipeline can be found by integrating the pore pressure along the pipeline outer contour Γ, i. e.

$$F_h = \int_{\Gamma} p n_x dl \tag{5}$$

$$F_v = -\int_{\Gamma} p n_y dl \tag{6}$$

in which F_h is the horizontal seepage force (right positive) and F_v is the vertical force (upward positive).

3 NUMERICAL CALCULATION

3.1 Verification

Pore pressures on the surface of the pipeline was measured by Cheng and Liu [4] in an laboratory test. A numerical modeling of the problem is accomplished with COMSOL 4.3 package and compared with Cheng and Liu (Fig. 2). A linear water wave is employed with wave height H=0.143m, wave period T=1.75s and water depth D=0.533m, the corresponding wave length is L=3.54m. Parameters for the seabed are: shear modulus G=640kPa, Poisson constant $\upsilon = 0.33$, soil porosity n=0.42, soil hydraulic conductivity k_z=1.1×10^{-3}m/s, air content in the pore water d_a=5%. Radius of the pipeline is $r = 0.084\text{m}$ and the buried depth d=0.167m. Fig. 2 shows that the agreement of the current solution and the results of Cheng and Liu is excellent.

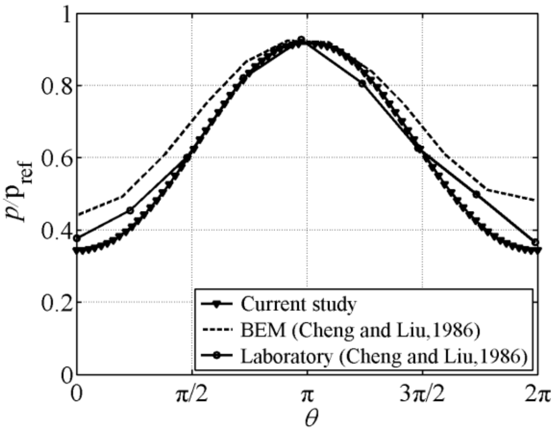


Fig. 2 Pore pressures on the surface of the pipeline. Notation: $p_{ref} = \frac{1}{2}\frac{\gamma_w H}{\cosh(2\pi D / L)}$

3.2 Solution

This section shows the seabed response under cnoidal waves. The input data for the wave are as followed: the wave height H=2.0m and wave period T=10s travelling in a water depth of D=6.5m. The calculated modulus of the cnoidal wave is m=0.979876 with a wave length of L=80.03m. The normalized wave pressure on the seabed with respect to time is shown in Fig. 3. It indicates that the normalized wave crest approaches 0.7 and the wave trough is near -0.3. Properties about the seabed are given as: the soil shear modulus G=20MPa, the Poisson constant $\upsilon = 0.3$, the porosity of the soil n=0.5, the soil hydraulic conductivity $k_x = k_z = 2.0 \times 10^{-4} m/s$ and the air content in pore water d_a=1%.

Fig. 4 gives the soil displacement along the pipeline under waves. Fig. 4(a) and Fig. 4(b) represent respectively the displacement under the wave crest and wave trough. It is clear that the soil horizontal displacement is negligible compared to the vertical. The maximum normalized value of the soil vertical displacement is approximate to 0.001, corresponding to the maximum vertical displacement $v = 2$ mm. Fig. 5 shows the pore pressure on the pipeline. It states that the magnitude of the pore pressure on the top varies more sharply than that on the bottom. Fig. 6 gives the horizontal and vertical forces acting on the pipeline. It is clear that the horizontal force is much smaller that the vertical. The maximum vertical force is approximate to $0.2F_{buo}$, which is in accordance with the analysis of Cheng and Liu [4].

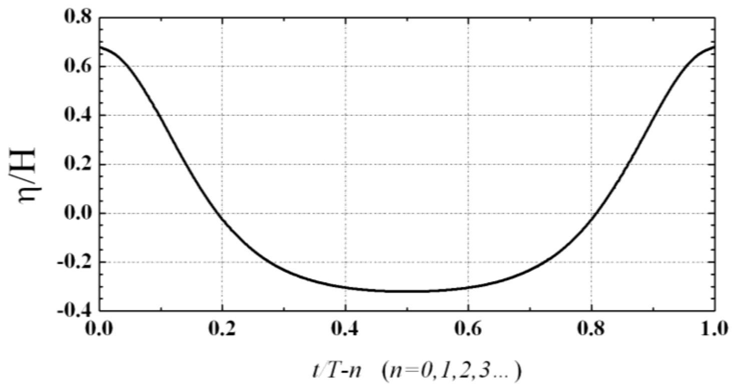
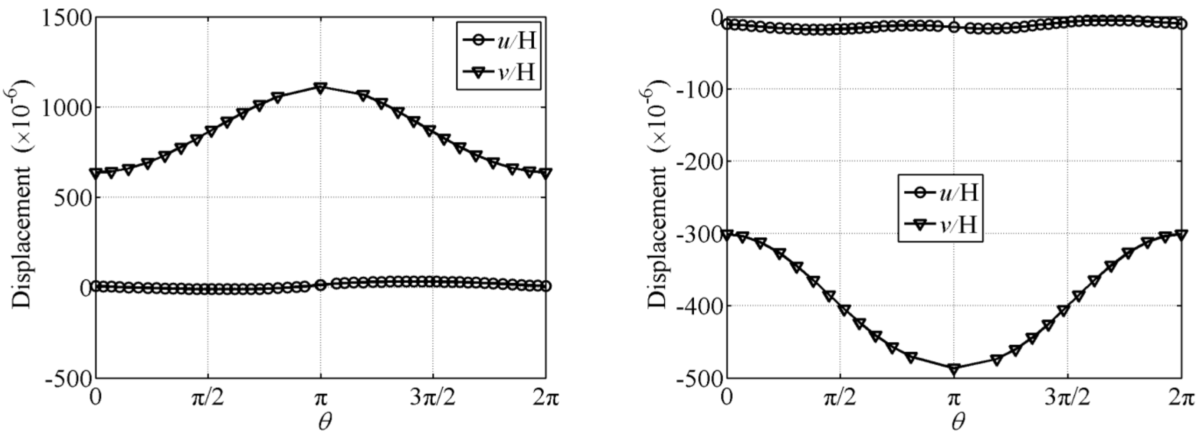


Fig. 3 Profile of the cnoidal water wave



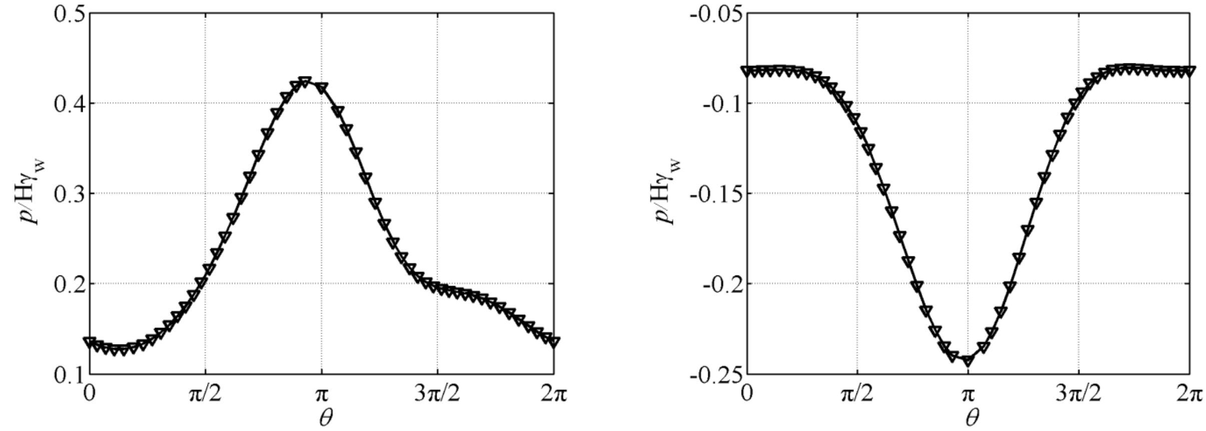
(a) Under the wave crest

(b) Under the wave trough

Fig. 4 Soil displacement along the pipeline

4 PARAMETRIC STUDIES

Naturally, the seabed is layered with complex composition and distinct hydraulic anisotropy. However, it is difficult to determine the exact values of the air content in pore water and the soil hydraulic conductivity as well as the soil hydraulic anisotropy in engineering practice. Therefore, parametric studies are necessary to provide useful insight on the sensitivity of forces on a pipeline with these parameters. In this section, parametric studies are carried out to investigate the influence of the air content in the pore water, as well as the soil hydraulic conductivity and the soil hydraulic anisotropy. The air content d_a varies from 0 to 2% and the hydraulic conductivity k_x=k_z varies from 1.0×10^{-4} to 2.5×10^{-3} m/s, consequently k_x/k_z varies from 1 to 25. Parameters about the seabed and the wave are the same as given in section 3.2.



(a) Under the wave crest (b) Under the wave trough

Fig. 5 Pore pressure around the pipeline

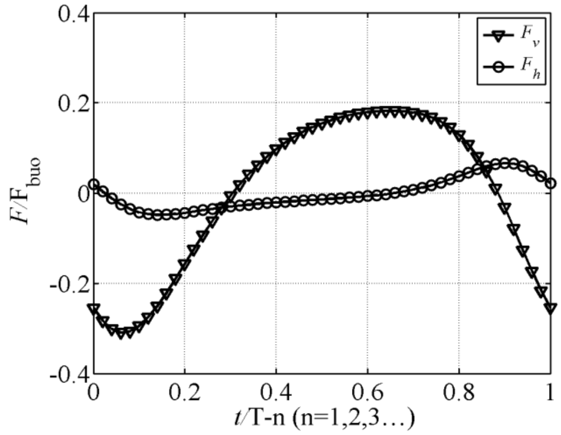
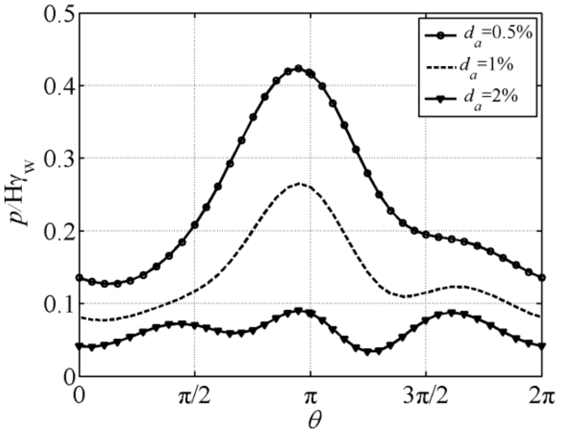


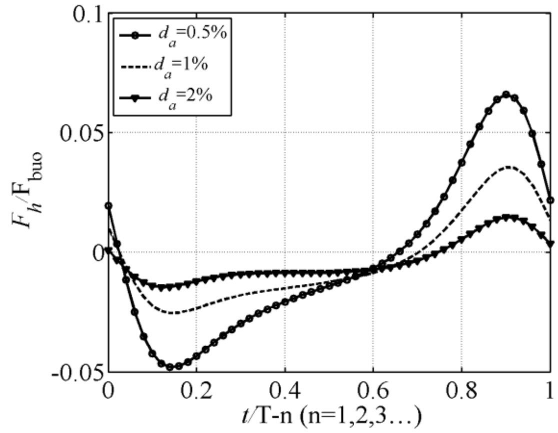
Fig. 6 Forces acting on the pipeline. Notation: $F_{\text{buo}} = \gamma_w \pi r^2$

4.1 Influence of the air content

Fig. 7 illustrates the influence of the air content on the pore pressure along the pipeline (Fig. 7a) and forces acting on the pipeline (Fig. 7b, c). For simplicity, this section only gives pore pressure under the wave crest. The soil hydraulic conductivity in the study is $k_x = k_z = 1.0 \times 10^{-4}$ m/s. It is clear that the pore water pressure is very sensitive to the air content. An increase of the air content can decrease the magnitude of pore pressure and forces acting on the pipeline. Nevertheless, the maximum of the vertical force on the pipeline is near $0.2F_{\text{buo}}$.



(a)



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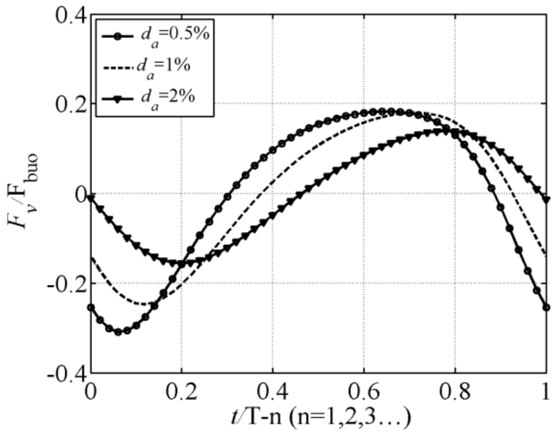


(c)

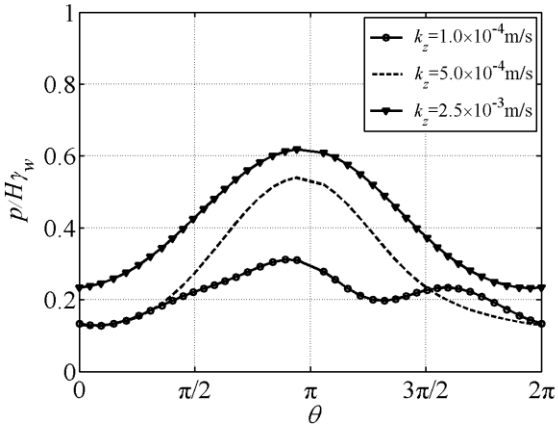
Fig. 7 Influence of the air content

4.2 Influence of the soil hydraulic conductivity

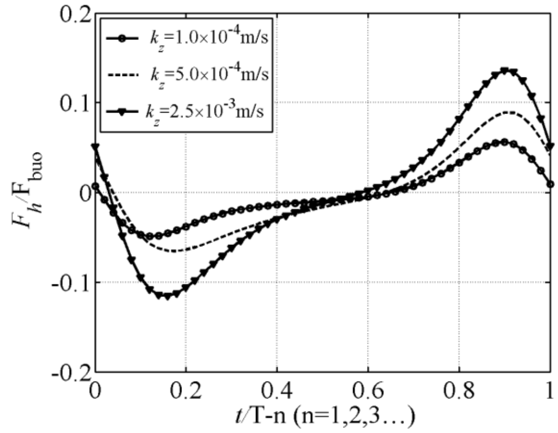
Fig. 8 illustrates the influence of the soil hydraulic conductivity on the pore pressure (Fig. 8a), forces on the pipeline (Fig. 8b, c). The air content in the calculation is $d_a = 1\%$. It is clear that the soil hydraulic conductivity can affect the pore pressure significantly. An decrease of k_z can decrease the magnitude of the pore pressure and forces on the pipeline. Similarly as above, the maximum of the vertical force on the pipeline approaches $0.2F_{buo}$.

4.3 Influence of the soil hydraulic anisotropy

Fig. 9 illustrates the variation of the pore pressure (Fig. 9a) and forces (Fig. 9b, c) acting on the pipeline when k_x/k_z varies from 1 to 25. It is clear that the magnitude of the pore pressure decrease as the value of k_x/k_z increases, whereas, an increase of k_x/k_z will increase the horizontal force acting on the pipeline with a maximum of $0.15F_{buo}$. Nevertheless, the vertical force on the pipeline varies slightly with k_x/k_z and the maximum is close to $0.2F_{buo}$.



(a)



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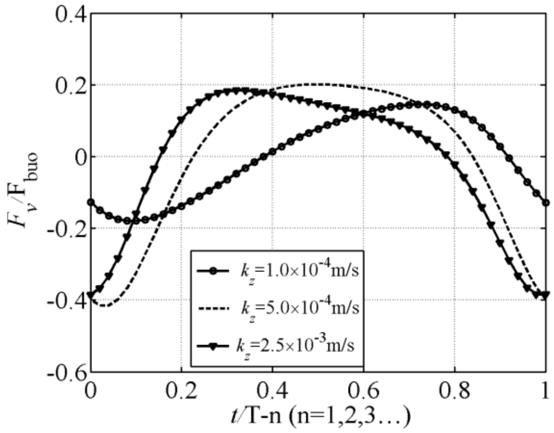
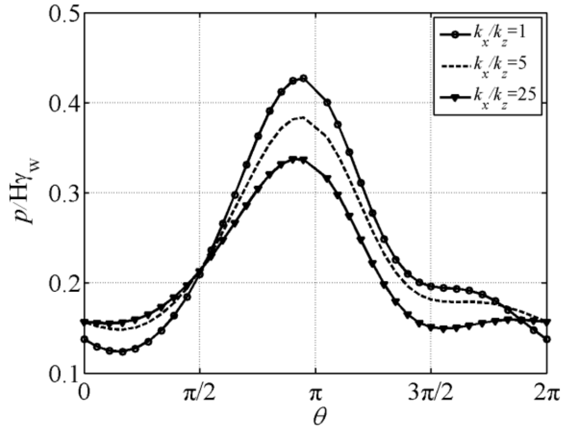


(c)

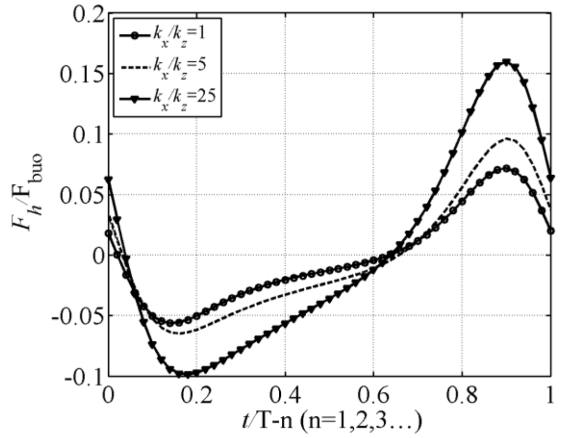
Fig. 8 Influence of the soil hydraulic conductivity on pore pressure

5 CONCLUSIONS

Biot equations can be used to evaluate forces acting on a submarine pipeline under cnoidal water waves without the consideration of the interaction between the seabed and the pipeline. The air content in pore water and soil hydraulic conductivity can significantly affect the forces. When the air content in the pore water varies from 0 to 2% and the soil hydraulic conductivity varies from 2.0×10^{-5} to 2.5×10^{-3} m/s, an increase of the air content in pore water and a decrease in the soil hydraulic conductivity as well as an increase of the soil hydraulic anisotropy can decrease the magnitude of the pore pressure on the pipeline. The maximum vertical force acting on the pipeline is near $0.2F_{buo}$, which is independent of the air content and the soil hydraulic conductivity. Nevertheless, a high anisotropy ratio k_x/k_z can make the magnitude of the horizontal force approaches 0.15 F_{buo}.



(a)



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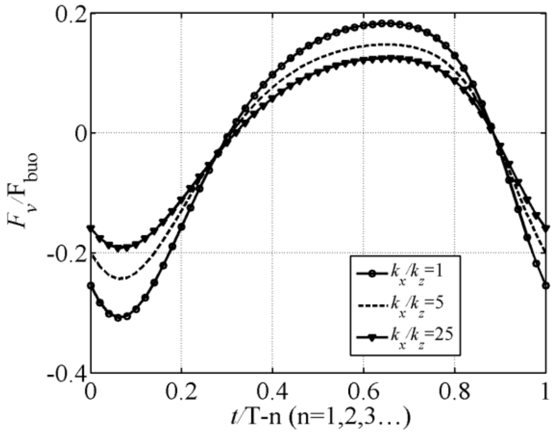

(c)

Fig. 9 Influence of the soil hydraulic anisotropy

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