THE EFFECTIVE AXIAL FORCE CONCEPT FOR OFFSHORE LINED AND CLAD PIPES

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1 INTRODUCTION

Pipelines, risers and piping systems are subject to internal and external pressures as well as variations in temperature due to natural variations in the surrounding environment and variations in content temperature. Pressures and, depending on the boundary conditions, temperature will cause stresses in the pipe wall. Integrating the axial stresses over the cross section of a pipe gives the so-called true wall axial force [1]. The global bending behavior of pipes is affected by the axial force through 2nd order effects. However, due to the effect of the internal and external pressures it is the so-called effective axial force that governs the global response of the pipe. By using Archimedes’ law, Sparks [2] demonstrated that the overall effects of pressure on a pipe may be expressed by an equivalent force system where equivalent axial forces replace complicated pressure integrals over doubly curved surfaces. Note that these equivalent axial forces do not cause axial stresses in a pipe, hence the distinction between true wall axial force and effective axial force, but they do, however, govern static and dynamic beam bending behavior and buckling [1-3].

The somewhat confusing definitions of the effective axial force and the distinction between the effective and true wall axial force concepts elude many, and historically have caused open disagreements between engineers and researchers in mechanics alike [4, 5]. Even recently, the concept of effective axial force has been questioned [6], demonstrating that there is a need to solidify the theoretical background for the effective axial force concept.

Pipelines, risers and piping systems are often composed of layered cylinders. The dominating material is carbon manganese (CMn) steel for its cost and capacity to bear pressure and bending loads, but other materials are often applied for various other reasons. Thin stainless steel liners are sometimes applied to avoid corrosion in the CMn steel [7,8]. Variations in the Poisson’s ratio and thermal expansion coefficients in the layers will alter the stress configurations through the thickness of the combined cross-section as compared to a monolithic cross-section, and ultimately result in axial reactions between the layers. The effective axial force concept for a layered cross-section will consequently be different from that of a monolithic cross-section. In this paper, a novel approach to the effective axial force concept is presented accounting for layered pipe cross-sections. The importance of variation in stiffness, Poisson’s ratios and thermal expansion coefficients to the effective axial force and true wall axial force will be illustrated by applying the novel theory on typical lined and
clad pipeline cross-sections.

2 THEORETICAL BASIS FOR THE EFFECTIVE AXIAL FORCE

Archimedes’ law states that “the effect of the water pressure on a submerged body is an upward directed force equal in size to the weight of the water displaced by the body”. Formulated mathematically, Archimedes’ law states that

\[ \int_S -p_e n dS = \rho_w V g k = b k \] (1)

where \( S \) is the closed surface surrounding the submerged body, \( n \) is the normal vector to the surface \( S \), \( p_e \) is the external pressure from the water, \( \rho_w \) is the density of the water, \( V \) is the volume of the body, \( b \) is the buoyancy, \( g \) is the gravitational constant and \( k \) is the unit normal vector in vertical direction. The mathematical formulation is further illustrated in Fig. 1a)-b).

![Figure 1](image)

**Figure 1**: Archimedes’ principle applied to a general body submerged in water and a cylinder.

In Fig. 1c)-d), Archimedes’ law is restated for a vertical cylinder with end caps. If we adjust the dry weight \( w_d \) of the vertical cylinder to be exactly equal to the buoyancy \( b \), the net force acting on the cylinder according to Figure 1d) is zero. Hence, if we apply Archimedes’ law, we can express mathematically that, unless influenced by other forces, the cylinder will remain still in the water. If we only consider the load case illustrated in Fig. 1d), i.e. replacing the external pressure with a concentrated load acting on the center of gravity, the stresses in the cylinder wall are necessarily zero. From the load case in Fig. 1c) we can easily establish, however, the mean non-zero hoop stresses in the cylinder by equilibrium

\[ \sigma_{\theta \theta} = -\frac{p_e D}{2t} \] (2)

where \( \sigma_{\theta \theta} \) is the mean hoop stress, \( D \) is the outer diameter of the cylinder and \( t \) is the cylinder wall thickness.

Furthermore, the approximate axial stresses in the cylinder wall must balance the pressure force acting on the end-caps, i.e.

\[ \sigma_{zz} = -\frac{p_e A_e}{A_s} \] (3)

where \( \sigma_{zz} \) is the axial stress, \( A_e \) is the area of the end caps and \( A_s \) is the cross-sectional area of the cylinder wall, and \( p_e \) is approximated as constant over the height of the cylinder.
We can observe that Archimedes’ law will predict an equivalent vertical force equaling the buoyancy, which can replace a pressure integral. We can also, importantly, observe that this equivalent force cannot replace the actual pressures over the surfaces when establishing stresses and strains in the body. If we, on the other hand, allow the dry weight to be smaller than the buoyancy for the case described in Fig. 1d), the cylinder will move upwards towards the surface of the water if unconstrained. If we place a hand on the cylinder, the force we will feel in our hand is the difference between the dry weight and the buoyancy. Consequently, even if the force from the Archimedian upthrust cannot be applied to understand the stresses and strains in the cylinder, this is still the force we can measure when a body is submerged in water, and the force which will govern its displacements, albeit not its deformations. Hence we postulate the following observation or corollary to Archimedes’ law:

The Archimedian upthrust is a measurable force equal to integrating the external pressure acting on a submerged body. It governs the displacements of the body, but not the stresses and strains on the interior of the body.

When we look at a pipe exposed to internal and external pressure, we find that we can add and subtract the end cap pressures to achieve an equivalent load system, as shown in Figure 2.

\[ N + p_e A_e - p_i A_i \]

\[ N + w_{cont} \]

**Figure 2:** Archimedes’ law applied twice to an internally and externally pressurized cylinder.

In Fig. 2a) an original cutout of a pipe is shown, and the forces acting on it are the pressures, the true wall axial force and the dry weight of the steel wall cross-section, drawn as \( w_{d,i} \). In order to apply Archimedes’ law we need pressures acting over closed surfaces. To make the pressure surfaces continuous and closed, we add (and subtract) the external and internal pressures to the external area \( A_e \) and the internal area \( A_i \) respectively, as shown in Fig. 2b) and 2c). By Archimedes’ law, the pressures over the closed surfaces can be replaced by the buoyancy \( b \) in terms of the external pressure, and the weight of the content fluid \( w_{cont} \) in
terms of the internal pressure. Thus the weight of the pipe itself $w_d$, plus the weight of the content and subtracting the buoyancy gives the submerged weight $w_s$. Thereby the submerged weight can replace the initial pipe weight and the pressures over the closed surfaces, as shown in Fig. 3d). The only loading which remains is the submerged weight and the opposite axial loadings, including the contributions added to create pressures over closed surfaces. Thus the effective axial force in a cylindrical structure, free to expand axially, subject to internal and external pressure is defined by:

$$S_{\text{eff}} = N - p_i A_i + p_e A_e$$

(4)

It is, however, important to note that since we have utilized Archimedes’ law to arrive at the expression for the effective axial force, the effective axial force cannot be used to express stresses and strains in the pipe, according to our previous observation. For an axially free pipe, the axial stresses must, consequently, be based on the true wall axial force $N$, and the pressures will contribute to radial and hoop stresses, but not axial stresses for the case described in Figure 2, and Eq. (4).

3 GENERAL ASSUMPTIONS

Exact displacement fields for multi-layer cylinders exposed to temperature, internal and external pressure, as well as axial forces, have been determined by Vedeld and Sollund [9].

$$u_r = \frac{C_{r1}}{r} + C_{r2}r, \quad u_\theta = 0, \quad u_z = \frac{C_z}{L}$$

(5)

In Eq. (5), $u_r$ is the displacement in radial direction, $u_\theta$ is the displacement in hoop direction, $u_z$ is the displacement in axial direction, and $C_{r1}$, $C_{r2}$ and $C_z$ are undetermined coefficients. Temperature expansion coefficients, Young’s moduli and Poisson’s ratios of the two layers in the cylinders are assumed to not be equal. Consequently, due to loading, finite cylinder length and differences in thermal expansion coefficients, the cylinders will be exposed to axial forces, and stresses and strains will be coupled by Poisson’s ratio effects.

The boundary condition is shown in Figure 3, i.e., the pipe is exposed to an axial force $N$, as well as to internal pressure $p_i$, external pressure $p_e$ and a uniform temperature change $\Delta T$.

![Figure 3](image)

Figure 3: Boundary conditions for a segment of the heated, pressurized pipeline

The layers in Fig. 3 are assumed axially coupled, i.e. no axial sliding is accounted for.
4 DEDUCTION OF THE EFFECTIVE AXIAL FORCE

Based on full three dimensional elasticity, the stress fields in the two layers can be derived from Eq. (5).

\[
\begin{bmatrix}
\sigma_{rr} \\
\sigma_{\theta\theta} \\
\sigma_{zz}
\end{bmatrix} = \hat{E} \begin{bmatrix}
-(1-2\nu)\frac{C_{r1}}{r^2} + C_{r2} + \nu \frac{C_z}{L} - (1+\nu)\alpha\Delta T \\
(1-2\nu)\frac{C_{r1}}{r^2} + C_{r2} + \nu \frac{C_z}{L} - (1+\nu)\alpha\Delta T \\
2\nu C_{r2} + (1-\nu)\frac{C_z}{L} - (1+\nu)\alpha\Delta T
\end{bmatrix}
\]

(6)

\[
\begin{bmatrix}
\sigma_{rr,b} \\
\sigma_{\theta\theta,b} \\
\sigma_{zz,b}
\end{bmatrix} = \hat{E}_b \begin{bmatrix}
-(1-2\nu)\frac{C_{r1,b}}{r^2} + C_{r2,b} + \nu \frac{C_{z,b}}{L} - (1+\nu)\alpha_{b}\Delta T \\
(1-2\nu)\frac{C_{r1,b}}{r^2} + C_{r2,b} + \nu \frac{C_{z,b}}{L} - (1+\nu)\alpha_{b}\Delta T \\
2\nu C_{r2,b} + (1-\nu)\frac{C_{z,b}}{L} - (1+\nu)\alpha_{b}\Delta T
\end{bmatrix}
\]

In Eq. (6), \(\sigma_{rr}\) is the radial stress, \(\sigma_{\theta\theta}\) is the hoop stress and \(\sigma_{zz}\) is the axial stress for the stainless liner or clad part of the pipe, and the subscript \(,b\) has been assigned to equivalent properties of the CMn backing steel. \(E\) is the Young’s modulus where

\[
\hat{E} = \frac{E}{(1+\nu)(1-2\nu)}
\]

(7)

In order to derive the expression for restrained effective axial force in a lined or clad pipe, we will first illustrate the solution technique on the simpler case of a pipe with a monolithic (CMn) cross-section.

Eq. (6), now applying only to the single CMn layer, contains three unknown displacement coefficients. These quantities can be obtained by applying the known relations for pressure balance on the inner and outer pipe boundaries, together with force balance in the longitudinal direction, given by

\[
\begin{align*}
\sigma_{rr}(r_i) &= -p_i \\
\sigma_{rr}(r_e) &= -p_e \\
\sigma_{zz}A_i &= N
\end{align*}
\]

(8)

Applying Eq. (6) to solve for the three displacement coefficients gives

\[
\begin{align*}
C_{r1} &= \frac{1+\nu}{E} \frac{r_i^2 r_o^2 (p_i - p_e)}{r_o^2 - r_i^2} \\
C_{r2} &= \frac{1-\nu}{E} \frac{p_i r_i^2 - p_e r_o^2}{r_o^2 - r_i^2} - \frac{\nu}{E} \frac{N}{A_i} + \alpha\Delta T \\
C_z &= \frac{NL}{EA_i} - 2 \frac{vL}{E} \frac{p_i r_i^2 - p_e r_o^2}{r_o^2 - r_i^2} + L\alpha\Delta T
\end{align*}
\]

(9)
The strain in the longitudinal direction can be found based on Eq. (9).

\[
\varepsilon_{zz} = \frac{\partial u_z}{\partial z} = \frac{C_z}{L} = \frac{N}{EA_z} - \frac{2v}{E} \frac{p_i r_i^2 - p_e r_e^2}{r_o^2 - r_i^2} + \alpha \Delta T
\]  

(10)

At lay down, the effective axial force \(H_{\text{eff}}\) is equal to the lay tension and the true wall axial force \(N_1\) is given by:

\[
N_1 = H_{\text{eff}} + p_{i,1} A_i - p_e A_e
\]

(11)

The effective axial force will change when the pipe goes into operation when heat and a new internal pressure is applied. However the effective axial force expression, Eq. (4), still holds generally and consequently

\[
N_2 = S_{\text{eff}} + p_{i,2} A_i - p_e A_e,
\]

(12)

where \(N_2\) is the true wall axial force in operational condition, \(p_{i,2}\) is the internal pressure in operational condition and \(S_{\text{eff}}\) is the effective axial force in operational condition. Since the pipe is assumed fully fixed axially in operational condition, for instance due to pipe-soil axial friction between the pipe and the seabed, the longitudinal strain remains constant between the laying and the operational conditions.

\[
\varepsilon_{zz,1} = \varepsilon_{zz,2} \Rightarrow \frac{C_{z,1}}{L} = \frac{C_{z,2}}{L}
\]

(13)

In Eq. (13), \(\varepsilon_{zz,1}\) is the longitudinal strain after laying and \(\varepsilon_{zz,2}\) is the longitudinal strain in operation. The longitudinal strain is assumed constant over the steel cross-section and can be found by inserting for \(N, p_i, p_e\) and \(\Delta T\) in Eq. (10) for the laying and operational conditions respectively.

\[
\frac{H_{\text{eff}} + p_{i,1} A_i - p_e A_e}{EA_s} - \frac{2v}{E} \frac{p_{i,1} r_i^2 - p_e r_e^2}{r_o^2 - r_i^2} + \alpha (T_i - T_0) = \frac{S_{\text{eff}} + p_{i,2} A_i - p_e A_e}{EA_s} - \frac{2v}{E} \frac{p_{i,2} r_i^2 - p_e r_e^2}{r_o^2 - r_i^2} + \alpha (T_2 - T_0)
\]

(14)

Solving Eq. (14) for the effective axial force \(S_{\text{eff}}\) yields

\[
S_{\text{eff}} = H_{\text{eff}} - \Delta p_i A_i (1 - 2v) - EA_s \alpha \Delta T,
\]

(15)

where \(\Delta p_i = p_{i,2} - p_{i,1}\) and \(\Delta T = T_2 - T_1\). Eq. (15) is identical to the equation for the effective axial force in a fully axially restrained pipe on the seabed, as found in DNV-OS-F101 [3].

The aim is, however, to solve for two-layer cylinders rather than the simple monolithic case. For two-layer cylinders, the same displacement field as was determined for the single layer case can be applied, i.e. the Lamé field, Eq. (5), can be applied successively for each layer [11]. The necessary solutions for the present problem are for axially unrestrained pipes of finite length, exposed to temperature and pressure. The solutions for temperature and pressure, derived in [11], are shown separately due to their significant length. The effects of
temperature and pressure may be superimposed, since we assume linear material behavior.

The solution for a lined pipe exposed to a uniform temperature change $\Delta T$ over the cross-section is:

$$
C_{r1,b}^T = \frac{R_1^T K_{22}^T - R_2^T K_{12}^T}{K_{11}^T K_{22}^T - K_{12}^T K_{21}^T} \quad \land \quad C_{r2,b}^T = \frac{-R_1^T K_{22}^T + R_2^T K_{12}^T}{K_{11}^T K_{22}^T - K_{12}^T K_{21}^T}
$$

$$
C_z^T = \frac{L}{v_b} \alpha_b \Delta T (1 + v_b) + \frac{L(1 - 2v_b)}{r_o^2 v_b} C_{r1,b}^T - \frac{L}{v_b} C_{r2,b}^T
$$

where

$$
K_{11}^T = \frac{\hat{E}_A (1 - 2v_b) - \hat{E} (1 - 2v)}{r_o^2} + (1 - 2v_b) \frac{\hat{E} v - \hat{E}_b v_b}{r_o^2 v_b} - \frac{c_A (1 - v)}{v A_i} \left( 1 - \frac{2v_b}{r_o^2} \right)
$$

$$
K_{12}^T = -\hat{E} \left( 1 - 2v + \frac{v}{v_b} \right) - \left( 2v_b \hat{E}_b A_{s,b} - c_A \right) \left( 1 - v \right)
$$

$$
K_{22}^T = \frac{v(1 - 2v_b)}{v_b r_o^2} - \frac{1 - 2v}{r_i^2} - \left( 1 + (1 - 2v) \frac{A_o}{A_i} \right) \frac{c_A (1 - 2v_b)}{2v \hat{E} A_i r_o^2}
$$

$$
R_i^T = \Delta T \left( \hat{E} A_i (1 + v) - \frac{\hat{E} v \alpha_b (1 + v_b)}{v_b} - \frac{1 - v}{v A_i} \left( \hat{E} A_i \alpha (1 + v) - c_b \right) \right)
$$

$$
R_z^T = \Delta T \left( \alpha (1 + v) - \frac{v}{v_b} \alpha_b (1 + v_b) - \left( 1 + (1 - 2v) \frac{A_o}{A_i} \right) \frac{\hat{E} A_i (1 + v) - c_b}{2v \hat{E} A_i} \right)
$$

$$
c_A = \frac{\hat{E} A_i (1 + v) - \hat{E}_b A_{s,b} (1 + v_b)}{v_b} \quad \land \quad c_B = \alpha_b (1 + v_b) \hat{E} A_i \alpha (1 + v) + \hat{E}_b A_{s,b} (1 - 2v_b)
$$

For the pressure solution we get the following expression:

$$
C_{p1}^{i} = \frac{K_{11}^p R_{11}^p - K_{12}^p R_{22}^p}{K_{11}^p K_{22}^p - K_{12}^p K_{21}^p} \quad \land \quad C_{p2,b}^r = \frac{-K_{12}^p R_{11}^p + K_{11}^p R_{22}^p}{K_{11}^p K_{22}^p - K_{12}^p K_{21}^p}
$$

$$
C_p^{r} = \frac{L}{v_b - v} \left( \frac{p_i}{E} - \frac{p_o}{E_b} - C_{r1,b}^r r_i^2 + r_o^2 (1 - 2v) \right) + C_{r1,b}^r r_o^2 \frac{r_o^2}{r_o^2} \left( 1 - \frac{2v_b}{r_o^2} \right)
$$

where
\[ K_{11}^p = \hat{E}(1-2\nu)\left(\frac{1}{r_i^2} - \frac{1}{r_o^2}\right) \quad \wedge \quad K_{21}^p = 2vA_s\hat{E}\frac{1-2\nu}{r_i^2} - c_L\frac{r_i^2 + r_o^2(1-2\nu)}{r_o^2}\]

\[ K_{12}^p = \hat{E}_b(1-2\nu_b)\left(\frac{1}{r_o^2} - \frac{1}{r_{o,b}^2}\right) \quad \wedge \quad K_{22}^p = 2v_bA_{s,b}\hat{E}_b\frac{1-2\nu_b}{r_{o,b}^2} + c_L\frac{r_{o,b}^2 + r_o^2(1-2\nu_b)}{r_o^2}\]

\[ R_i^p = p_i - p_e \quad \wedge \quad R_f^p = N + 2v_pA_s + 2v_bA_{s,b}p_e - c_L\left(\frac{p_i - p_e}{\hat{E}} - \frac{p_e}{\hat{E}_b}\right)\]

\[ c_L = \frac{EA_s + E_bA_{s,b}}{v_b - v}\]

Inserting for Eqs. (16) and (18) into (13) and solving for the effective axial force yields:

\[ S_{\text{eff}} = \left(\frac{C}{A}\right)\left(\frac{p_{i,2} - p_{i,1}}{\hat{E}} - C_{r_{i,b}^1}S_1 + C_{r_{i,b}^2}S_2 + \frac{v_b - v}{v_b}(1 + v_b)\alpha_b(\Delta T_1 - \Delta T_2)\right)\]

\[ + \frac{1-2v_b}{r_{o,b}^2v_b}\left(C_{r_{i,b}^2} - C_{r_{i,b}^1}\right) - \frac{v_b - v}{v_b}\left(C_{r_{i,b}^2} - C_{r_{i,b}^1}\right)\]

where

\[ S_1 = \frac{r_i^2 + r_o^2(1-2\nu)}{r_i^2 r_o^2} \quad \wedge \quad S_2 = \frac{r_{o,b}^2 + r_o^2(1-2\nu_b)}{r_{o,b}^2 r_o^2}\]

\[ A = K_{12}^p S_1 + K_{11}^p S_2 \quad \wedge \quad B = -(K_{22}^p S_1 + K_{21}^p S_2) R_{i,2,\text{rem}}^{p_2} + AR_{2,\text{rem}}^{p_2} \quad \wedge \quad C = K_{11}^p K_{22}^p - K_{12}^p K_{21}^p\]

\[ R_{2,\text{rem}}^{p_2} = p_{i,2}A_s - p_eA_e + 2v_{p_{i,2}}A_s + 2v_bA_{s,b}p_e - \frac{EA_s + E_bA_{s,b}}{v_b - v}\left(\frac{p_{i,2} - p_e}{\hat{E}} - \frac{p_e}{\hat{E}_b}\right)\]

The rather complex formulae expressed in Eq. (20) will be proposed substituted with a much simpler expression:

\[ S_{\text{eff,approx}} = H_{\text{eff}} - \Delta p_iA_s\left(\frac{(1-2\nu)A_s + (1-2\nu_b)A_{s,b}}{A_s + A_{s,b}}\right) - \Delta T\left(E\alpha A_s + E_b\alpha_b A_{s,b}\right)\]

A simple and common way to estimate the effective axial force for a lined or clad pipe, is to ignore the material properties of the liner or clad layer and apply the expression from DNV-OS-F101, Eq. (15), on the combined cross-section, which then is assumed to consist of CMn steel only, but with the liner thickness included in the CMn steel thickness. This manner of calculating the effective axial force will in the following be termed \(S_{\text{eff,OS-F101}}\), but it should be noted that this procedure is not described in DNV-OS-F101 [3]. This methodology has been applied in the present context to create a comparative basis to the novel formulae in Eqs. (20) and (22) and making it possible to isolate the effects of variation in material properties between the layers.
\[ S_{\text{eff}, \text{OS}, -P_101} = H_{\text{eff}} - \Delta p_A (1 - 2\nu_b) - \Delta T E_b \alpha_b (A_s + A_{s,b}) \]  

(23)

5 RESULTS AND COMPARISONS

Pipelines with outer diameters in the range of 6 inches to 40 inches, and \( D/t \) ratios from 8 to 40 have been systematically studied for three separate loading cases. In all cases, a constant liner thickness of 3 \( \text{mm} \) has been assumed. The loading cases are presented in Table 1.

<table>
<thead>
<tr>
<th>Load case</th>
<th>( \Delta p_i )</th>
<th>( \Delta T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>260</td>
<td>120</td>
</tr>
<tr>
<td>2</td>
<td>220</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 1: Load cases for comparisons of effective axial force formulae.

Load case 1 represents a high pressure, high temperature pipeline (HTHP), load case 2 represents a standard pipeline operating at ambient temperature and load case 3 represents a depressurized pipe which has not had time to cool off yet. Load case 1 is included since it represents a pipe for which the effective axial force is a critical design parameter whereas loading cases 2 and 3 are included to investigate the effect of the liner or clad on the effective axial force as a function of pressure and temperature individually. In Table 2, the material properties for the corrosion-resistant alloy (CRA) and the CMn steel are given.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus ( E ) [GPa]</th>
<th>Poisson’s ratio ( \nu ) [-]</th>
<th>Temperature expansion coeff. ( \alpha ) [°C⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liner (CRA)</td>
<td>191</td>
<td>0.29</td>
<td>1.70 \times 10⁻⁵</td>
</tr>
<tr>
<td>Backing steel (CMn)</td>
<td>207</td>
<td>0.30</td>
<td>1.17 \times 10⁻⁵</td>
</tr>
</tbody>
</table>

Table 2: Material properties for liner and backing steel.

Figure 4: Effect of liner on effective axial force for different pipe diameters and steel diameter-to-thickness ratios. Both internal pressure and temperature have been applied (load case 1).
Fig. 4 shows how the accuracy of Eq. (23) varies with steel $D/t$ ratio for HTHP pipes with three different outer diameters of 6 inches, 14 inches and 32 inches, respectively. The figure clearly shows that the effect of the liner on the compressive effective axial force is increasing with $D/t$ for the steel, and also that the effect declines with increasing outer diameter. Since the effect of the liner on the effective axial force depends on both the $D/t$ ratio and the outer diameter as individual parameters, a different, more general parameter is desirable in order to isolate the effect of the liner to a single representative variable. The $A_{s,b}/A_s$ ratio is proposed, since one would expect that the effect of ignoring the liner material properties decreases monotonically for increasing values of this ratio, as shown in Figure 5 for load cases 2 and 3.

![Figure 5: Effect of liner on effective axial force as a function of the cross-sectional ratio $A_{s,b}/A_s$. Load case 2 (pressure only) and load case 3 (temperature only) have been applied.](image)

From Figure 5, it is observed that the effect of temperature is much more significant than the effect of pressure for the relative change in effective axial force from the liner. This is to be expected, since the difference in temperature expansion coefficients is much more pronounced than the differences in Poisson’s ratios between CMn steel and stainless steels.

In Figure 6, below, the effect of the liner or clad on the effective axial force is demonstrated for the HTHP pipeline. It is observed that the functional relationship is no longer one-to-one, but it is also seen that the accuracy of applying Eq. (23) rather than the exact expression, Eq. (20), still may be determined with good precision based on the $A_{s,b}/A_s$ ratio.

Based on the observations of the relation between cross-sectional areas of the steels and the outcome to the effective axial force, Eq. (22) is tested to determine its accuracy. It is reasonable to expect a good relation due to the clear dependence on the $A_{s,b}/A_s$ parameter to the effect of the liner. The results for load case 1 are presented in Figure 7.
Figure 6: Effect of liner on effective axial force as a function of the cross-sectional ratio $A_{s,b}/A_s$. Load case 1 (pressure and temperature) has been applied.

Figure 7: Accuracy of the proposed simplified expression, Eq. (22), compared to the exact expression, Eq. (23), as a function of the cross-sectional ratio $A_{s,b}/A_s$.

As shown in Figure 7, Eq. (22) gives an excellent prediction of the effect of the liner on the effective axial force. Load cases 2 and 3 (not shown) exhibit the same accuracy for the proposed Eq. (22). It is also positive that the slight (negligible) inaccuracy is consistently conservative in terms of the effective axial force. Moreover, it should be noted that the alternative approximate expression, Eq. (23), was not only found to be less accurate than Eq.
(22), but was also consistently non-conservative for the cases examined in the present study.

### 12 CONCLUSIONS

- The effect of liner and clad materials on the effective axial force has been determined by an exact analytical deduction.
- Liner and clad materials increase the compressive effective axial force since their thermal expansion coefficients are higher and their Poisson’s ratios lower.
- Particularly for small diameter pipes with moderate to high D/t ratios it is important to include the effects of the liner or clad materials on the effective axial force.
- A (much) simplified formula to predict the effective axial force for lined and clad pipes has been proposed, and its accuracy is excellent.

### REFERENCES


