Dynamic modelling of mooring for floating offshore structures

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Key words: Mooring dynamics, Offshore structures, Lumped mass, Coupled analysis.

Abstract. There is a demand to develop and evaluate new concepts for offshore structures for harnessing ocean energy. Tools capable of calculating the behaviour of this kind of structures are also requested by the industry. An accurate analysis comprises fully coupled simulations and the use of complex models for non-linear dynamics analysis, including the modelling of mooring lines on floating offshore structures. In opposite of quasi-static modelling of mooring lines such as catenary lines [1], time - dependent models [2, 3] present some advantages.

This work presents a dynamic model of mooring lines for deep-water structures using LMM (Lumped Mass Method) [4, 5] formulation and its application on different kinds of floating devices. First, the mathematical model for mooring lines is presented. Then an application tool based on that mathematical model is showed. This tool is linked to the time-domain seakeeping solver SeaFEM [6]. The resulting model is able to obtain a fully coupled analysis of seakeeping and mooring dynamics of floating offshore structures. A preliminary study of mooring dynamics is presented over different solutions and configurations of mooring lines for several floating devices. Furthermore, an optimal design of mooring line configurations for different solutions of floating structures can be achieved.

This work also presents the study of different floating devices under several sea wave conditions. Finally some conclusions are remarked, and the future research lines are presented.

1 INTRODUCTION

The purpose of this work is to present a code for analysing the dynamics of mooring systems. Today, it is well known that the increase of developments of large offshore structures, (floating offshore wind turbines, for instance), requires a precise study of mooring arrangements. It is possible to find a wide variety of platform and mooring design concepts for offshore floating devices. Mooring systems are made by a set of cables, chains or wire
ropes, which are attached to offshore structures at different points with lower ends of these cables anchored at the seabed.

Several authors [7, 8] have proposed algorithms to analyse the dynamic behaviour of mooring arrangements in offshore structures. Usually, floating offshore simulators can be found, which tend to use a quasi-static solution for mooring models. This approach has the advantage of computational efficiency. So, for cases where the waves are small and the movements of offshore devices and mooring arrangements are minimal, these solutions provide a good estimation of the line tension. However, for cases where the platform and mooring line motions are significant, quasi-static models neglect dynamic effect that may be important. The primary effect of mooring line dynamic is the increasing of platform damping. This effect improves the stability of the platform.

Thus, mooring lines should be investigated when the inertia of the line is important. The code employs a variation of Lumped Mass Method (LMM) for inelastic line [4, 5] and uses the Finite Difference method, as well as the implicit Newmark’s average acceleration method [9, 3] to solve the dynamic behaviour of mooring lines. Several authors have proved that LMM is an effective algorithm to calculate dynamic behaviour of different mooring configurations.

2 PROBLEM DESCRIPTION

2.1 Mathematical formulation

The mooring line is discretized by a finite number of nodes, which are called ‘lumped masses’ and all forces are applied in each node. So, the line is divided in a finite number of segments, which are considered as massless springs.

The equations of motion for the dynamic problem of the mooring line in two dimensions in local tangential, (t) and normal (n), directions are:

\[
m \begin{pmatrix}
\frac{\partial \mathbf{u}}{\partial t} \\
\frac{\partial \mathbf{v}}{\partial t}
\end{pmatrix}
- \frac{\partial \mathbf{\tau}}{\partial t}
= -\mathbf{w} \sin \theta + \mathbf{F}_n (1 + e),
\]

\[m + \alpha \frac{\partial \mathbf{v}}{\partial t} + m u \frac{\partial \mathbf{\theta}}{\partial s} = \mathbf{T} \frac{\partial \mathbf{\theta}}{\partial s} - w \cos \theta + \mathbf{F}_n (1 + e),
\]

\[
\frac{\partial \mathbf{e}}{\partial t} = \frac{\partial \mathbf{u}}{\partial s} - v \frac{\partial \mathbf{\theta}}{\partial s},
\]

\[
\frac{\partial \mathbf{\theta}}{\partial t} (1 + e) = \frac{\partial \mathbf{v}}{\partial s} - u \frac{\partial \mathbf{\theta}}{\partial s},
\]

where \(s\) is the unstretched length of the cable; \(u\) and \(v\) are the component of velocity vector; \(m\) is the mass per unit length; \(T\) is the tension vector; \(\mathbf{F}\) is vector of external forces; \(e\) is the stretched length; \(w\) is the submerged weight per unit length of the cable; \(\alpha\) is the two-dimensional added mass of the cable, and \(\theta\) is the angle formed between the horizontal and the local tangential direction of the cable.

Material damping, bending and torsional moments are normally neglected. Discretization assumptions are shown in Figure 1.
Usually, mooring line is connected with floating offshore structures, and the motions of these structures on irregular waves are normally not affected by mooring line tension [5]. Applying the Newton’s law to discretized line in global coordinates, it can be written,

\[
(M_j + AM_j(t)) \{\ddot{x}_j(t)\} = \{F_j(t)\},
\]

where \(M_j\) is the inertia matrix, \(AM_j\) is the hydrodynamic inertia matrix, \(t\) is the time, \(\ddot{x}_j\) is the acceleration and \(F_j\) are the loads applied in the considered lumped mass. Note that drag forces and elastic stiffness are also included in the analysis.

The nodal force loads applied in each lumped mass are respectively given by:

\[
\{F_j(t)\} = \{F_W\} + \{F_B\} + \{F_S\} + \{F_T\} + \{F_D\},
\]

where \(F_W\) is the weight of lumped mass, \(F_B\) is the buoyancy of lumped mass, \(F_S\) is the force caused by soil interaction, \(F_T\) is the tension of the line in each node and, and \(F_D\) is the drag forces in global coordinates.

The forces caused by seabed interaction can be modeled using Coulomb friction model to obtain the forces in horizontal direction, or using the Drucker-Prager friction model [10]. Some authors modeled the seabed as an elastic foundation with linear stiffness and damping properties [11]. In this case, the soil interaction is modeled through Coulomb friction,

\[
\{F_S\} = \mu \overrightarrow{N_j},
\]

where \(\mu\) is the coefficient of friction, and \(N_j\) the normal force.

On the other hand, drag forces are calculated in normal and tangential directions in each element of the line. Then, directional matrices are used to transform the local forces and velocities into global drag forces and velocities,

\[
\begin{align*}
F_{Dn}^j(t) &= \frac{1}{2} \rho D_j \|l_j\|^2 C_{Dn} u_j(t) u_j(t), \\
F_{Dt}^j(t) &= \frac{1}{2} \rho D_j \|l_j\|^2 C_{Dt} u_j(t) u_j(t),
\end{align*}
\]
where $\rho$ is the water density, $D_j$ the characteristic length of the element, $l_j$ the segment length, and $C_D$ the drag coefficient in normal and tangential directions (typical values are 2.0 for normal direction and 0.2 for tangential direction). Finally, $u^n_j$ and $u^t_j$ are the normal and tangential components of velocity of the fluid acting on the regarded $j^{th}$ line segment.

2.2 Boundary conditions

To obtain an accurate analysis of mooring line, it is necessary to establish boundary conditions. These are related to position of the line, line extension and seabed contact.

As previous remarked, the seabed interaction is modeled by using Coulomb friction model. The value of the coefficient of friction $\mu$ will depends of soil type. In the treated cases a value of 0.4 it is considered for this parameter.

Regarding the line position, it is obvious that the line has not displacement in the lower end position. In each time step the movements of the line are checked to fix the position of the line in the lower end position and upper end position.

On the other hand, it is necessary to fix the line extension to avoid instabilities in numerical simulation; for this reason a predictor-corrector method based on Newmark method and Central Finite Difference method to predict the behaviour in each time step is employed. Then, line extension is calculated and checked to apply basic mechanical laws. So, new position is estimated in each time step.

3 DYNAMIC ANALYSIS OF THE BEHAVIOUR OF THE LINE

In order to solve the dynamic behaviour of mooring lines, some numerical methods are used. In this work, a predictor-corrector method based on the Central Finite Difference and the Newmark’s average acceleration methods is proposed. The procedure for solving the dynamic behaviour of mooring line is explained in a more detailed manner.

3.1 Proposed numerical scheme

The method is initialized using quasi-static solution of catenary equations. This solution supply an accurate initial position at $t = 0$ s for mooring line. Then, the Finite Difference Method is used to launch the calculation at $t = \Delta t$. The steps are the following:

I. Selection of the interval time $\Delta t < \Delta t_{\text{critic}}$.

II. Calculation of the position at $t = -\Delta t$.

$$\{\vec{x}_{t=-\Delta t}\} = \{\vec{x}_{t=0}\} - \Delta t \{\vec{x}_{t=0}\} + \frac{\Delta t^2}{2} \{\vec{x}_{t=0}\}. \quad (9)$$

III. Estimation of the position at $t = \Delta t$, using Central Finite Difference scheme.

$$\{\vec{x}_{t=\Delta t}\} = \left[\frac{1}{\Delta t^2} M\right]^{-1} \left[\{\vec{F}_{t=0}\} + \frac{2}{\Delta t} M\{\vec{x}_{t=0}\} - \frac{1}{\Delta t^2} M\{\vec{x}_{t=-\Delta t}\}\right]. \quad (10)$$

IV. Evaluation of the velocities and acceleration of the line at $t = 0$ s.
\[
\{\ddot{x}_{t=0}\} = \frac{1}{2\Delta t} \{[\dot{x}_{t=\Delta t}] - [\dot{x}_{t=-\Delta t}]\},
\]
\[
\{\ddot{x}_{t=0}\} = \frac{1}{\Delta t^2} \{[\dot{x}_{t=\Delta t}] - 2[\dot{x}_{t=0}] - [\dot{x}_{t=-\Delta t}]\}.
\]

V. Calculation of the stretched line \( e \) at \( t = \Delta t \).
\[
e = (\ddot{x}_{t=\Delta t}) - (\ddot{x}_{t=0}),
\]

VI. Calculation of the effective tension at \( t = \Delta t \).
\[
T_j = \left( \frac{E A_j}{L_j} \right) e.
\]

VII. Recalculation of the tension in each lumped mass.

VIII. Correction of the initial position at \( t = \Delta t \), applying boundary conditions.

IX. Selection of the correct values of \( \gamma \) and \( \beta \) for the Newmark method.

X. Estimation of the position at \( t = \Delta t \), using Newmark method,
\[
\{\ddot{x}_{t=\Delta t}\} = \left[ \frac{1}{\beta(\Delta t)^2} M \right]^{-1} \left\{ \ddot{F}_{t=\Delta t} + M \left( \frac{1}{\beta(\Delta t)^2} (\ddot{x}_{t=0}) \right) + \frac{1}{\beta \Delta t} \{\ddot{x}_{t=0}\} + \left( \frac{1}{2\beta} - 1 \right) \{\ddot{x}_{t=0}\} \right\}.
\]

XI. Determination of the velocity and acceleration vectors for the current time step.
\[
\{\ddot{x}_{t=\Delta t}\} = \frac{1}{\beta(\Delta t)^2} \{ (\ddot{x}_{t=\Delta t}) - (\ddot{x}_{t=0}) \} - \frac{1}{\beta \Delta t} \{\ddot{x}_{t=0}\} - \left( \frac{1}{2\beta} - 1 \right) \{\ddot{x}_{t=0}\},
\]
\[
\{\ddot{x}_{t=\Delta t}\} = \{\ddot{x}_{t=0}\} + (1 - \gamma) \Delta t \{\ddot{x}_{t=0}\} + \gamma \Delta t \{\ddot{x}_{t=0}\}.
\]

XII. Repeating the procedure from III, to XII until the end of the simulation.

The employed procedure is shown in a simplified manner in Figure 2.

**Figure 2:** Proposed numerical scheme for solving mathematical model
3.2 Stability and discretization aspects

In order to obtain useful results, several aspects such as stability and accuracy of the employed methods must be considered. The number of lumped masses (nodes) should be sufficient to get an accurate estimation of the mooring line position. Furthermore, Van der Boom [5] observed that LMM might insert parasitical motions into the simulation. Increasing the numbers of elements of the line can prevent these parasitical movements; it is to say, by means of reducing the lumped mass for each node.

Usually, the reader can find in the bibliography other methods to solve the dynamic behaviour of mooring lines. For instance, Houbolt method [4, 5] is used to solve the specific problem. In this work is used the Newmark method, since dynamics of mooring line may be considered as typical vibration problem of structural dynamics.

As later remarks, the stability and error of this method depends basically on $\gamma$ parameter. This method is unconditionally stable to values of $\gamma \geq 0.5$ and $\beta \geq 0.25(\gamma + 0.5)^2$. If the value of $\gamma$ is equal to 0, a self-excited vibration gets into numerical procedure. However, if $\gamma$ is equal to 0.5, a damping is added into the numerical scheme. So, for undamped cases the choice of these values leads to unconditionally stable time-integrator operator of maximum accuracy.

For damping cases, Newmark scheme remains stable as long as $\varepsilon < 1.0$, where $\varepsilon$ is the modal damping coefficient. In general, damping has a stabilizing effect for moderate values of $\varepsilon$.

The knowledge of natural frequencies $\omega_n$ of continuous system results necessary to ensure the stability of numerical integration by Newmark method. In each case, the continuous system is discretized by LMM, so a simple one-dimensional bar can be considered for each segment line. The time step can be calculated as,

$$\Delta t = L_j/c,$$

where $c$ is the longitudinal wave velocity [11], and $L_j$ is the length of $j$-th element. The wave velocity is given by,

$$c = \sqrt{E/\rho},$$

being $E$ the Young Modulus and $\rho$ the density per unit length.

It is important to avoid instabilities in numerical simulation due to discretization aspects. The touchdown point on seabed of mooring system should be locate in all time of the simulations, for this reason the authors have introduced an adaptive algorithm to divided the mooring line taking into account this point. The number of elements of the portion of the line, which rest on seabed may be different depending the upper end position of the line.

4 LINKING WITH SEAFEM

Fluid structure interaction is a topic of great interest in engineering, more specifically in offshore engineering. The interest in this field is growing due to renewable energy field in recent years. The coupled dynamic problems in offshore industry may be important in several applications like offshore wind energy or wave energy converters. These problems require a complex study, which involves several specialties; for instance, offshore wind energy implies aeroelastic calculations, wave-interaction problems, and mooring calculations.

Most of computer programs used in offshore engineering are based in frequency domain,
since the computational cost is cheapest than time domain simulations. Nowadays, the increasing on computer capabilities makes possible to carry out simulations in time domain. A great advantage of time domain simulations is to make easier the coupling with other phenomena, non-linear effects, etcetera.

So, it is possible to find some computer programs, which are capable of solving wave-structure interaction in time domain [6]. The reader can find bibliographic resources dedicated to solve fluid structure interaction in presence of waves surface using FEM formulation [13]. However, these works requires high computational costs. The use of potential flow theory with Stokes perturbation approximation allows to save computational costs in opposite of previous ones. SeaFEM is a useful program capable of solving wave-interaction problem using Finite Element method and unstructured meshes.

Borja et al. [6] have based SeaFEM on Stokes perturbation theory. This procedure is more efficient, and no re-meshing or moving mesh techniques are needed, which keeps computational costs and times low. These authors have adapted the algorithm to include non-linear external forces, like those used to define mooring systems, and variations on the pressure over the free surface. This motivates the development of an algorithm to calculate the forces and tensions due to mooring arrangements.

The governing equations for the first diffraction-radiation of a floating body are:

\[ \nabla^2 \phi = 0, \quad \text{in } \Omega \]  
\[ \partial_t \phi + g \eta = -\frac{P_a}{\rho} + C, \quad \text{in } z = 0 \]  
\[ \partial_t \eta - \partial_z \phi = 0, \quad \text{in } z = 0 \]  
\[ \partial_z \phi = 0, \quad \text{in } z = -H \]  
\[ \nabla \phi \cdot n_B = v_B \cdot n_B, \quad \text{in } \Gamma_B \]  

where \( \phi \) and \( \eta \) are the first order potential and free surface elevation, \( \Omega \) is the fluid domain bounded by \( z = 0 \); \( P_a \) is the atmospheric pressure; \( \rho \) is the water density; \( C \) is a constant value; \( \Gamma_B \) represents the wetted surface of a floating body; and \( H \) is the water depth.

5 VALIDATION AND APLICATION EXAMPLES

5.1 Validation

The validation of the obtained results constitutes a primary objective to know the effectiveness of the code presented. Some bibliographic resources have been employed to get the validation of code. Mavrakos et al. [12] presented experiments, which were compared with numerical simulation using, both time and frequency domain computer codes. They showed the beneficial effects of buoys in reducing the mooring line dynamic tension. The Figure 3 shows a comparison between numerical results offered by Mavrakos et al. [12] and the results obtained by numerical procedure. These authors [12] established an experimental set-up, as well as the data acquisition procedures, and compared the experimental results with their numerical results.
Figure 3 shows that the results obtained are in accordance with those experimentally obtained by Mavrakos et al. [12]. In this case, the line is divided in 100 nodes. The time step is established at $\Delta t = 1.0 \times 10^{-3}$ s to avoid stability problems (in fact, $\Delta t_{\text{critic}} = 2.3 \times 10^{-3}$ s). The simulation established a wave period of 3.33 s. It is remarkable that the maximum (peak) deviation between numerical and experimental results is less than 10%.

The second test case is based in the simulation of a cable subjected to the action of its self-weight and simply supported at its ends [14]. It is compared with analytical solution of the problem. The cable has the following characteristics: section area $A = 0.0005$ m$^2$; Young modulus of the material $E = 5.01 \cdot 10^5$ N/m$^2$; length of the cable $L = 7.07105$ m; weight per unit length 0.49 N/m. The analytical reaction in the end of the cable is 34.681 N and the obtained vertical tension of the line is 34.684 N. The slightly difference may be caused by discretization errors.

The accuracy of the code is also verified applying classical test [15]. It consists of a horizontally suspended cable with one support free to slide laterally. For a cable of an unstretched length of $L = 200$ m, a weight per unit length of $w = 1$ kg/m, an extensional stiffness of $EA = 10^5$ N, and a horizontal load of $F = 5.77$ N applied at the free end, the theoretical static-equilibrium solution is for a horizontal span of $X = 152.2$ m and a vertical sag of $Z = 58.0$ m. It can be observed in Figure 4.

Figure 4: Benchmark problem for checking the accuracy of the program

This benchmark test showed that the displacement converged to analytical solution (see
Figure 5). The influence of several aspects may be important to obtain a stable and accuracy method to analyze the behaviour of mooring lines.

Figure 5: Solution for the benchmark problem

Figure 6 shows the time evolution of line tension at upper end position depending on the wave amplitude. This test is based on mooring system of monohull semisubmersible for North Sea oil production, which operates at 375 m water depth [12]. It can be noted an increase in the line tension with the wave amplitude, and the reader can observe that the average tension is the same in all cases.

Figure 6: Response of the tension line at upper end position with incrementing in wave amplitude

We can analyse the displacement of upper end line position with the increment on current velocity. The characteristics of the line are the same as previous one. Table 1 shows the variation of the upper end position of the line with the current velocity before 30 s of
simulation.

Table 1: Horizontal displacement of the upper end line position with the increment in current velocity

<table>
<thead>
<tr>
<th>Current velocity (m/s)</th>
<th>Position (m)</th>
<th>Increment (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1830.00</td>
<td>0.000</td>
</tr>
<tr>
<td>1.0</td>
<td>1830.04</td>
<td>0.002</td>
</tr>
<tr>
<td>5.0</td>
<td>1831.01</td>
<td>0.055</td>
</tr>
<tr>
<td>10.0</td>
<td>1834.05</td>
<td>0.221</td>
</tr>
<tr>
<td>15.0</td>
<td>1839.12</td>
<td>0.498</td>
</tr>
</tbody>
</table>

6 CONCLUSIONS AND FURTHER RESEARCHES

The easiness to implement non-linear external forces and moment acting on floating structures into SeaFEM program leads the authors to develop a code for mooring system. A solver for dynamic behaviour of mooring systems has been presented. The solver has been based on a Lumped Mass Method adding other features and capabilities. Central Difference Finite in combination with an implicit Newmark’s average acceleration methods are used to carry out the time domain integration.

The boundary conditions are related to seabed interaction, drag forces applied, mechanical characteristics of the line material and the wave-structure interaction. The solver has been compared with available results for a mooring line, and benchmark test. The agreement between the solutions shows that the implemented solver in this work performs well.

The algorithms are also linked with SeaFEM to obtain a full solver of dynamic behaviour of marine structures.

Finally, it can be noticed the importance of mooring analysis in offshore engineering. However, it is necessary to continue in the development of new model for mooring system, which include material damping, non-linear effects, and seabed interaction.

REFERENCES