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THE ACCURACY AND EFFICIENCY OF THE EFFICIENT TIME SIMULATION PROCEDURE IN DERIVATION OF THE 100-YEAR RESPONSES

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Abstract. Offshore structures are exposed to random wave loading in the ocean environment and hence the probability distribution of the extreme values of their response to wave loading is required for their safe and economical design. To this end, the conventional (Monte Carlo) time simulation technique (CTS) is frequently used for predicting the probability distribution of the extreme values of response. However, this technique suffers from excessive sampling variability and hence a large number of simulated extreme responses (hundreds of simulated response records) are required to reduce the sampling variability to acceptable levels. In this paper, three different versions of a more efficient time simulation technique (ETS) are compared by exposing a test structure to sea states of different intensity. The three different versions of the ETS technique take advantage of the good correlation between extreme responses and their corresponding surface elevation extreme values, or quasi-static and dynamic linear extreme responses. The accuracy and efficiency of an alternative technique in comparison with the conventional simulation technique is investigated.

1 INTRODUCTION

Offshore structures are expose to a variety of environmental loads all of which exhibit a high degree of statistical uncertainty. Hence, the capability to predict the response extreme values probability distribution during the service life of the structure is of great value to designers. Fatigue damage due to the effect of waves over the platform lifetime is also a very important design consideration [1]. Probabilistic procedures can account for these uncertainties by establishing the statistical properties of loads and responses and hence are necessary for risk-based assessment of these structures [2]. The main problem in establishing the probabilistic properties of extreme response is due to the nonlinearity of the wave load mechanism and/or the structural system which leads to non-Gaussian distribution for response [3-7]. The problem is further compounded by current and intermittent loading on members in the splash zone, which have a significant effect on the statistical properties of extreme responses [8,9].

Many different techniques (refer to Najafian [3] for a brief partial review) have been introduced for estimation of response statistical properties. Probabilistic properties of response can be developed in the time, frequency or probability domains. Conventional time simulation, Finite-memory nonlinear system [3,10] and NewWave theory [11] are examples of time domain techniques. In the frequency domain, Volterra series [12] for derivation of higher-order spectra of structural response have been implemented, and in the probability domain, procedures for derivation of the first four statistical moments of quasi-static response [13] from the statistical properties of water particle kinematics have been introduced.

The majority of the foregoing techniques are either very time consuming (such as the conventional time simulation technique) or limited in their application to special cases; for example, they cannot account for the load intermittency and current effect, or are only relevant to quasi-static responses [3,14]. In reality, the most versatile and reliable technique for predicting the statistical properties of the extreme response of an offshore structure to random wave loading is the time domain simulation technique. However, this technique requires very long simulations in order to reduce the sampling variability to acceptable levels. In this paper, a more efficient version of the ETS technique is introduced which takes advantage of the good correlation between the response and its corresponding linear response extreme values. This version of ETS technique had proved to be more accurate and efficient than the version based on surface elevation extreme values.

2 TEST STRUCTURE AND RESPONSES

The test structure used in this paper is a fixed platform in a water depth of 110m. The general outline of the platform is shown in Figure 1. The platform is composed of four vertical legs (similar to a jack-up platform), where the diameter of each leg is 1.5m with a wall thickness of 40mm. As shown in the figure, the distributed hydrodynamic load on each leg is represented by 30 point loads so that the total number of nodal loads on the four legs is 120. The dimensions of the platform deck are 35m*38m. The member surfaces were assumed to be rough and hence the drag and inertia coefficients were taken to be 1.05 and 1.20, respectively. The total mass of the topsides and the four legs (including the added hydrodynamic mass for the four legs of the structure) is 17665 Tonnes.

It was intended to use this general platform layout to construct three finite-element (FE) models so that their first mode natural periods will be approximately 2, 5 and 8 seconds, respectively. The dynamic effect on the responses of the first FE model is expected to be relatively small. This is because for this case, the periods of large waves are much greater than the fundamental period of the structure and hence the structure is expected to behave almost quasi-statically. On the other hand, the dynamic effects for the responses of the remaining two FE models are expected to be moderate and large, respectively. This arrangement is necessary to make sure that the conclusions of this study are valid for a wide range of structures from almost quasi-static to very dynamic ones.

JCP2, JCP5 and JCP8 are used to refer to three FE models with first mode natural periods of 2.53, 5.21 and 8.12 seconds, respectively. For JCP2, the Young's modulus of elasticity was taken to be that of normal mild steel (206000 MN/m^2). On the other hand, the modulus of elasticity for JCP5 and JCP8 were assumed to be 11220 and 3129 MN/m^2, respectively. (It should be noted that the foregoing assumptions, though unrealistic, do not have any impact on the conclusions of this study). The first ten modes have been used in the evaluation of the dynamic responses for all three FE models. Damping coefficients (inclusive of hydrodynamic damping) for all modes were assumed to be 0.05. All details of the three FE models together with results from structural analysis (modal shapes, natural frequencies, flexibility coefficients, etc.) were provided by Atkins Ltd (private communication). The foregoing test structures were subjected to various uni-directional sea-states simulated from Pierson–Moskowitz (P–M) frequency spectrum. The waves were assumed to propagate in the global Y direction (Figure 1).

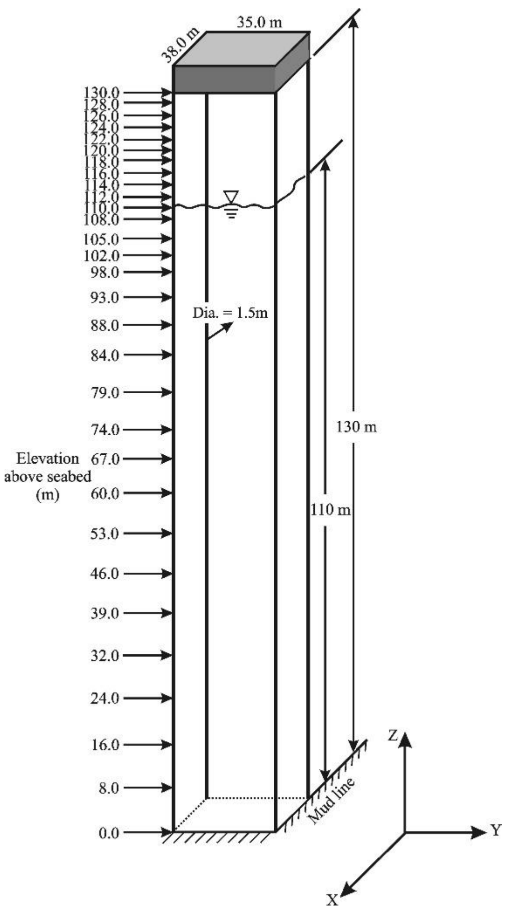

Figure 1: Schematic diagram of the test structures.

3 EVALUATION OF DYNAMIC RESPONSE OF A LINEAR STRUCTURE THROUGH MODE SUPERPOSITION PROCEDURE

In summary, the steps taken to calculate the dynamic response are as follows:

a) Convert the distributed wave load on each structural element into equivalent point loads at the two ends of the element following the standard procedure in structural engineering. (The equivalent point loads are the opposite of reaction forces at the two ends of the element when its two ends are fixed against both displacement and rotation). The equivalent point load at a particular node incorporates the contribution of wave forces on all elements which join each other to form the node.

b) Calculate quasi-static response as a linear combination of equivalent nodal loads, using the following equation.

$$\tilde{R}(t) = \sum_{k=1}^{N} S_k p_k(t) \tag{1}$$

where the first component, $\tilde{R}(t)$, is the quasi-static response determined directly from flexibility coefficients (S_k), and N is the number of nodal forces. The values of S_k for a particular response are fixed and are determined from structural analysis. Finally, $p_k(t)$ is the equivalent point load acting on node k incorporating the contribution of wave force on all the elements which join each other to form node k.

c) Calculate quasi-static modal amplitudes, $\tilde{Y}_n(t)$ as a linear combination of equivalent nodal loads,

$$\tilde{Y}_n(t) = \frac{P_n(t)}{K_n} = \frac{\sum_{k=1}^{N} \phi_{kn} p_k(t)}{K_n} \tag{2}$$

where $P_n(t)$ and K_n are the n^{th} generalised load and generalised stiffness, respectively; N is the number of nodal forces, and ϕ_n is the n^{th} mode shape vector. It should be noted that K_n is just numbers (one-by-one matrices) and that the generalised load, $P_n(t)$, is just a linear combination of all the nodal loads. The distributed wave load on each structural element is converted into equivalent point loads at the two ends of the element following the standard procedure in structural engineering (see later). Thus, $p_k(t)$ is the equivalent point load acting on node k incorporating the contribution of wave force on all the elements which join each other to form node k.

d) Calculate dynamic modal amplitudes by applying appropriate frequency response functions to the quasi-static modal amplitudes. The frequency response function to convert the n^{th} quasi-static modal amplitude, $\tilde{Y}_n(t)$, to its corresponding (dynamic) modal amplitude, $Y_n(t)$, is equal to:

$$H_n(f) = K_n H_n'(f) = \frac{1}{1 + 2i\varepsilon_n\left(\frac{f}{f_n}\right) - \left(\frac{f}{f_n}\right)^2}, \qquad i = \sqrt{-1} \tag{3}$$

where $H_n'(f)$ is the frequency response function of the single-degree-of-freedom system, ε_n and f_n are the n^{th} mode damping ratio and natural frequency, respectively. In practice, the Discrete Fourier Transform (DFT) of the dynamic modal amplitude would be determined by multiplying the DFT of the quasi-static modal amplitude by the foregoing frequency response function. The dynamic modal amplitude is then calculated by taking the Inverse Fourier Transform of the DFT of the dynamic modal amplitude.

e) Calculate the difference between the dynamic and its corresponding quasi-static response from modal analysis.

$$[R(t) - \tilde{R}(t)] = \sum_{n=1}^{NM} \delta_n \{Y_n(t) - \tilde{Y}_n(t)\} \tag{4}$$

$$[\delta]_{1xNM} = [\xi]_{1xN} [\Phi]_{NxNM}$$

where *NM* is the number of significant modal amplitudes. The values of ξ_n for a particular response are fixed [15] and are determined from structural analysis. Φ, which is referred to as the mode shape matrix, is an *N*-by-*N* matrix whose n^{th} column is the n^{th} mode shape vector of the structure.

(f) The total dynamic response would then be equal to the sum of its quasi-static response and the difference between the dynamic and its corresponding quasi-static response from modal analysis.

$$R(t) = \tilde{R}(t) + [R(t) - \tilde{R}(t)] \tag{5}$$

Further details of the evaluation of linear quasi-static and dynamic response procedure can be found in Abu Husain et al [16].

4 DERIVATION OF PROBABILITY DISTRIBUTION OF RESPONSE EXTREME VALUES BY THE CTS PROCEDURE

For short-term distribution, use the procedure in Section 3 to simulate a response record from a simulated surface elevation record and determine its extreme value. Then repeat the process many times to generate a large sample of response extreme values. Rank all the simulated extreme values from smallest to largest. Then use the following plotting position equation for the Gumbel distribution to estimate the value of the probability distribution for each of the ranked extreme values.

$$Prob(R_{max} < q_n) = P_{R_{max}}(q_n) \approx \frac{n - 0.44}{N + 0.12}, \qquad n = 1, 2, 3, \dots, N \tag{6}$$

where R_{max} denotes the response extreme value, q_n is the n^{th} smallest simulated extreme value, and finally *N* is the total number of simulated extreme values.

5 DERIVATION OF PROBABILITY DISTRIBUTION OF RESPONSE EXTREME VALUES FROM THE ETS PROCEDURE

An efficient time simulation (ETS) method was introduced [14] which takes advantage of the correlation between extreme responses and their corresponding extreme surface elevations. The method has proved to be very efficient for high-intensity sea states; however, the correlation between extreme responses and extreme surface elevations, and hence the efficiency of the technique, reduces for low-intensity sea states. In this section, a more efficient version of the ETS technique is introduced which takes advantage of the correlation between linear and nonlinear response extreme values. It will be shown that this new version of the ETS method is more accurate than the surface elevation version in all cases. It should be noted that a somewhat similar approach has been discussed in Torhaug et al [17], where advantage was taken of the good correlation between the extreme values of linear and nonlinear mid-span moment of a fast-moving ship to reduce the required time simulation for accurate derivation of the hourly maximum ship response statistics.

5.1 Theoretical background

The theoretical probability distribution of the linear response extreme values during period T (based on the assumption that the linear response is a Gaussian random process) is derived from the following relationship.

$$Prob(r_{L,max} < q) = P_{r_{L,max}}(q) = exp\left[-M * \exp\left(-\frac{q^2}{2\sigma_{r_L}^2}\right)\right] \quad (7)$$

where $r_{L,max}$ denotes the linear response extreme value, $M = T/T_z$ is the expected number of zero-upcrossings of the linear response during period T and σ_{r_L} is the standard deviation of the linear response. As an example, the probability density function of the extreme values of the linear quasi-static overturning moment, for H_s = 5m, T_z = 7.94sec and T = 128sec is shown in Figure 2. It is clear from the figure that the extreme values of the great majority of simulated records are between 7MNm and 28MNm, and that linear quasi-static response records with very high extreme values are very rare. Hence, considering the high correlation between the extreme values of linear and nonlinear quasi-static responses, it can be concluded that a large number of surface elevation records must be simulated and then converted to response records to get a fairly small number of response (both linear and nonlinear) records with high extreme values. This is the main problem with the CTS procedure.

To overcome the foregoing deficiency in the conventional time simulation (CTS) method, the ETS procedure has been introduced. In this method, the simulated response records are divided into a number of groups depending on the magnitude of the extreme value of their associated linear response record. In this study, seven groups (refer to Figure 2) and hence six different values of extreme linear response have been considered corresponding to the following probability distribution values.

$$P_{r_{L,max}}(q_i) = [0.100\ 0.500\ 0.800\ 0.9500.990\ 0.999] \quad (8)$$

The corresponding q_i values for the above probability distributions can be determined from Eq. (7). For example, for H_s = 5m, T_z = 7.94sec and T = 128sec, the linear quasi-static overturning moment $(r_{L,i})$ values would be equal to

$$q_i = [12.83\text{MNm}\ 16.32\text{MNm}\ 19.03\text{MNm}\ 22.06\text{MNm}\ 25.00\text{MNm}\ 28.64\text{MNm}] \quad (9)$$

The expected number of simulated linear response records and hence nonlinear response records for each group can then be calculated by multiplying the total number of simulated records by the probability of occurrence of each group. As an example, for 20000 simulated records, the expected number of linear quasi-static response records and hence nonlinear quasi-static response records belonging to Group 1 would be 20000*0.1 = 2000 (refer to Table 1). It is clear from Table 1 that the great majority of simulated records belong to the first few groups and that only 20 records are expected to belong to the last group (the group with the highest linear quasi-static response extreme values and hence the highest nonlinear quasi-static response extreme values).

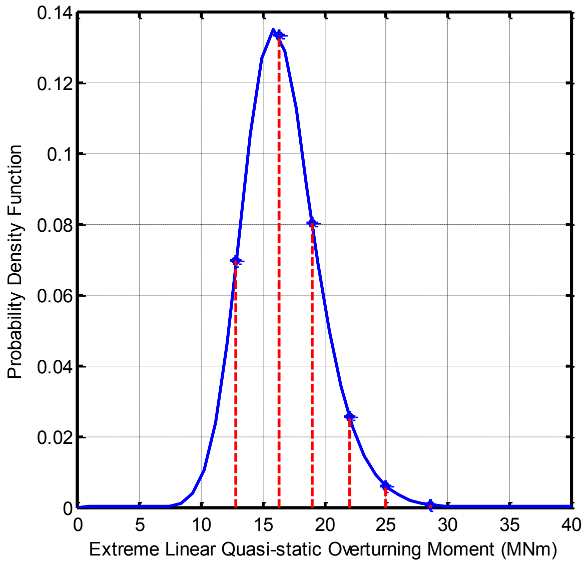

Figure 2: Probability density function of extreme values of linear quasi-static overturning moment divided into 5 segments, H_s = 5m, T_z = 7.94s, T = 128s.

Table 1: Number of simulated records used in efficient time simulation (ETS) procedure.

Group Number (i)	Probability of $r_{L,max}$ belonging to each group (P_i)	Expected number of simulated response records practically used for each group in the CTS procedure (based on 20000 simulations)	Number of simulated response records used for each group (N_i) in the ETS procedure
1	0.100	2000	20
2	0.400	8000	60
3	0.300	6000	60
4	0.150	3000	60
5	0.040	800	60
6	0.009	180	40
7	0.001	20	20
Total	1.000	20000	320

For the linear response version of the ETS technique, a surface elevation record is simulated and then converted to a linear response record. Then, this record is allocated to a particular group depending on the magnitude of its extreme value. The surface elevation record is then used to simulate a nonlinear response record and its extreme value will be allocated to the same group. The process is repeated. When a group’s target (for example, 20 for Group 1) has been achieved, then, further surface elevations records belonging to that group are discarded and are not converted into nonlinear response records. The process is repeated until all groups have achieved their target numbers. This means that in practice, the great majority of simulated surface elevation records are not converted into nonlinear response records. In contrast, in the conventional time simulation (CTS) technique, simulated records are not divided into groups, and as a result, all the simulated surface elevation records must be converted to nonlinear response records to have an unbiased sample of response extreme values. This explains why the ETS procedure is efficient than the CTS procedure.

Let A be the set of all response extreme values divided into *NG* (mutually exclusive) groups based on the magnitude of the extreme values of their corresponding linear quasi-static response records (as previously explained). That is A = {A_1, A_2, A_3, … A_i, … A_{NG}}, where A_i is the subset of nonlinear response extreme values belonging to Group i. Now using the total probability theorem, the probability distribution of the response extreme values can be calculated from the following relationship.

$$Prob(r_{max} < q) \quad = P_{r_{max}}(q) = \sum_{i=1}^{NG} P_{r_{max}}^{(i)}(q) * P_i \tag{10}$$

where $P_{r_{max}}^{(i)}(q)$ is the probability of the response extreme value being less than q given that its corresponding linear response extreme value belongs to Group i. In other words, $P_{r_{max}}^{(i)}(q) = P_{r_{max}}\big(q|\ r_{L,max} \in \text{Group i}\big)$, and P_i is the probability of occurrence of Group i (i.e. probability of linear response extreme value belonging to Group i). It should be noted that P_i values are already known; hence, according to Eq. (9), all that is required for accurate estimation of the probability distribution of the response extreme values is the accurate estimation of $P_{r_{max}}^{(i)}(q)$ for each group.

5.2 Derivation of the response extreme values probability distribution for each group

The main point of the ETS technique is that there is no reason to calculate $P_{r_{max}}^{(i)}(q)$ based on vastly different number of simulated extreme values for different groups (refer to Table 1, column 3). This is, in effect, what is happening in the CTS procedure. Instead, from each group a limited number of surface elevation records (N_i) are converted into nonlinear response records (Table 1, column 4). The probability distribution of response extreme values for each group is then estimated from the following equation.

$$P_{r_{max}}^{(i)}\big(q_n^{(i)}\big) = \frac{n - 0.44}{N_i + 0.12}\,, \qquad n = 1, 2, 3 \dots, N_i \tag{11}$$

where $q_n^{(i)}$ is the n^{th} smallest response extreme value belonging to Group i. However, in order to calculate $P_{r_{max}}(q)$ from Eq. (10), the values of $P_{r_{max}}^{(i)}(q)$ must be known at all q values, where q is the set of all simulated response extreme values belonging to all the different groups. $P_{r_{max}}^{(i)}(q)$ is calculated from the following equation.

$$\begin{aligned} &P_{r_{max}}^{(i)}\big(q|\ q < q_1^{(i)}\big) = 0\,, \\ &P_{r_{max}}^{(i)}\big(q|\ q > q_{N_i}^{(i)}\big) = 1 \\ &P_{r_{max}}^{(i)}\big(q|\ q_1^{(i)} < q < q_{N_i}^{(i)}\big) \ = \text{determine by interpolation from Eq. (11)} \end{aligned} \tag{12}$$

6 COMPARING THE ACCURACY AND EFFICIENCY OF THE THREE ALTERNATIVE VERSIONS OF THE ETS PROCEDURE IN DERIVATION OF THE 100-YEAR RESPONSES

In this investigation, the CTS procedure was first used to calculate the probability distribution of the extreme responses based on 20000 simulated records, each of duration T = 128 seconds. The method of moments was then used to fit a Gumbel distribution to the simulated extreme values. The 100-year response, determined from the fitted Gumbel distribution, is then taken as the reference (accurate) value of the 100-year response ($R_{100\text{-ref}}$). Furthermore, both CTS and ETS procedures have been used to calculate the probability distribution of the extreme responses based on 320 simulated records (T = 128 seconds). To show the level of sampling variability, the foregoing exercise is repeated 100 times (100 runs). For each run, the 100-year response (R_{100}) has been calculated from the fitted Gumbel distribution. Then, the ratio between the 100-year response from each run and the reference 100-year response is calculated (denoted by r). The mean and the standard deviation of the foregoing ratios are then determined for both CTS and ETS procedures. A mean ratio close to unity indicates that the procedure does not suffer from any systematic error. On the other hand, the sampling variability of a statistic is inversely proportional to the sample size. Therefore, the efficiency of the proposed technique is determined from following equation:

$$\text{Level of efficiency} = \left[\frac{\sigma_{r_{CTS}}}{\sigma_{r_{ETS}}}\right]^2 \tag{13}$$

where σ stands for the standard deviation.

The accuracy and efficiency of the proposed techniques have been investigated for different structures and environmental conditions. As an example, the distributions for the total JCP5 base shear for H_s = 15m and zero current are presented in Figures 3 and 4 for the CTS and ETS-Linear Quasi-static Response (ETS-LQR) methods, respectively. Examination of the foregoing figures reveals that the sampling variability is much higher for the CTS procedure. Therefore, the CTS method would be computationally more demanding as the number of extreme values in the sample must be increased substantially to reduce the sampling variability of R_{100} to the same level as those from the ETS methods.

The accuracies and efficiencies for the ETS procedures are summarised in Table 2 for the ETS methods based on surface elevation, linear quasi-static and linear dynamic extremes. It is observed that in all cases the mean 100-year response ratios are very close to unity and it can therefore be concluded that these methods do not suffer from any systematic error. Furthermore, it is observed that the maximum efficiencies occur for the total base shear responses (dynamic JCP5) with the following values 10.41, 20.74 and 14.19 for the ETS-Surface Elevation (ETS-SEL), ETS-LQR and ETS-Linear Dynamic Response (ETS-LDR) methods, respectively. Again, the ETS-LQR is the most efficient method for this high-intensity sea estate in agreement with previous conclusions.

Table 2: Accuracy and efficiency of ETS and CTS procedures in calculating (short-term) 100-year responses for different sea states
ETS: Number of runs = 100, number of records for each sea state in each run = 320, T = 128sec, U = 0.00 m/s.
CTS: Number of runs = 100, number of records for each sea state in each run = 320, T = 128sec, U = 0.00 m/s.

		ETS-SEL				ETS-LQR				ETS-LDR			
H_s (m)	**Structure**	**Correlation coefficient (extreme responses, extreme surface elevations)**	**ETS Efficiency**	**Mean 100-year response Ratio (CTS)**	**Mean 100-year response Ratio (ETS)**	**Correlation coefficient (extreme responses, linear quasi-static extremes)**	**ETS Efficiency**	**Mean 100-year response Ratio (CTS)**	**Mean 100-year response Ratio (ETS)**	**Correlation coefficient (extreme responses, linear dynamic extremes)**	**ETS Efficiency**	**Mean 100-year response Ratio (CTS)**	**Mean 100-year response Ratio (ETS)**
15	**Total base shear**												
	QS	0.9172	7.00	0.983	1.007	0.9634	14.85	0.983	0.993	N/A	N/A	N/A	N/A
	JCP2	0.9149	6.78	0.983	1.007	0.9576	13.98	0.983	0.993	0.9573	10.57	0.983	1.013
	JCP5	0.9056	10.41	0.984	1.011	0.9447	20.74	0.984	0.999	0.8551	14.19	0.984	1.017
	JCP8	0.9165	10.24	0.987	1.012	0.9221	13.38	0.987	0.999	0.8174	10.09	0.987	1.016
	Overturning moment												
	QS	0.9131	7.50	0.983	1.007	0.9599	16.82	0.983	0.986	N/A	N/A	N/A	N/A
	JCP2	0.9058	6.86	0.983	1.007	0.9487	13.78	0.983	0.985	0.9474	11.33	0.983	1.013
	JCP5	0.8820	9.57	0.982	1.009	0.9298	18.35	0.982	0.988	0.7246	15.82	0.982	1.017
	JCP8	0.8944	9.46	0.985	1.009	0.8947	13.82	0.985	0.986	0.6748	11.06	0.985	1.016

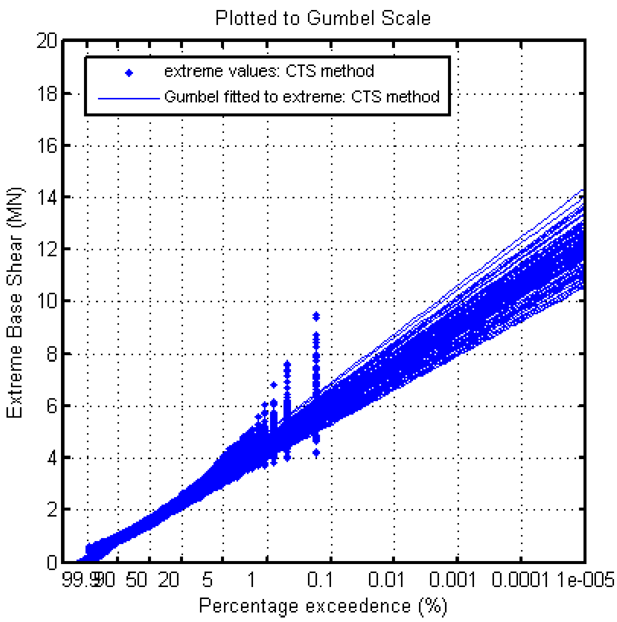

Figure 3: Sampling variability of the probability distribution of extreme base shear from CTS method. Number of runs = 100, number of response records for each run = 320, T = 128sec.

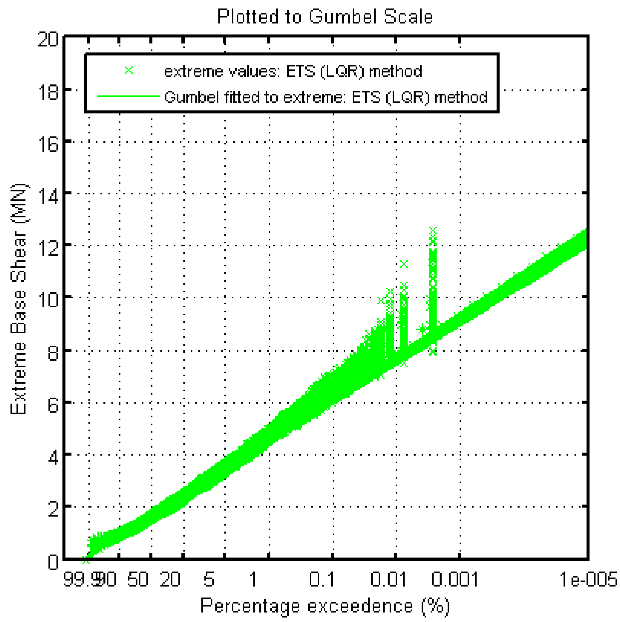

Figure 4: Sampling variability of the probability distribution of extreme base shear from ETS-LQR method. Number of runs = 100, number of response records for each run = 320, T = 128sec.

7 CONCLUSIONS

- The conventional time simulation (CTS) procedure is frequently used for derivation of the probability distribution of the extreme values of offshore structural response due to wave and current loading. However, this procedure is computationally very demanding as a large number of simulations is required to reduce the sampling variability to acceptable levels.
- In this paper, two new versions of the ETS technique has been introduced, which take advantage of the good correlation between extreme responses and the linear quasi-static and dynamic extremes.
- The three versions of the ETS procedure have been tested by comparing the short-term probability distributions of extreme responses from them with corresponding distributions from the CTS method (based on a very large number of simulated extreme values). It has been concluded that the ETS-LQR is the most efficient one.
- It has been shown that the ETS procedures do not suffer are not prone to any systematic error.

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