

## SIGNIFICANCE OF PARAMETRIC HULL FORM DEFINITION ON HYDRODYNAMIC PERFORMANCE OPTIMIZATION

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**Abstract.** Hydrodynamic performance optimization of ship hulls by CFD methods is becoming popular in modern naval architecture. From local or partial parametric shape optimization, the current frontline of the research is to solve the global hull shape optimization by automatic computational procedures. Key of these procedures is in the parametric hull form modification. Two different techniques are considered in this study: Full parametric hull form definition as opposed to the Free Form Deformation technique. After a general definition, pro and contra of both approached are discussed on the basis of the results obtain in a real hydrodynamic hull optimization example: a high speed round bilge monohulls optimized by means of a panel method for the minimum wave resistance at design speed.

### 1 INTRODUCTION

Hydrodynamic performance optimization of ship hulls by CFD methods is becoming popular in modern naval architecture. From local or partial parametric shape optimization, the current frontline of the research is to solve the global or full parametric hull form optimization by automatic computational procedures. If well integrated into the design synthesis model, this global shape optimization would actually solve one of the most difficult tasks of a naval architect: find the best shape of the hull that is able to achieve the best hydrodynamic performance while meeting all the particular design specifications (constraints).

From a systemic point of view, in general, global hull shape optimization requires four main components:

1. a computational fluid dynamic (CFD) solver,
2. a parametric definition of the hull geometry,
3. a numerical method to create the computational mesh, once the hull geometry is given
4. a single or multi-objective optimization algorithm.

Large efforts are currently taken to improve and verify the accuracy and robustness of CFD solvers (1.) to predict the ship hydrodynamic performance of interest for the naval architect for different types of ships and operating conditions. Although well-validated solvers exist (at least for a number of hull types and operational conditions), potential improvements in this field are still desirable, from the accuracy and robustness on one hand and from the computational efficiency on the other.

Optimization algorithms (4.) need to allow for complex multi-valued objective functions, typically with unknown gradient, defined in the space of multiple free variables and subject to several design constraints (sometimes implicit functions of the objective functions and of the free variables). Different algorithms have been proposed, but typically evolutionary or stochastic types of algorithms are preferred due to their desirable global convergence property, at the cost of a slower convergence rate. Potential improvements in this field are still many, especially when hybrid minimization algorithms using adjoint-equations CFD solvers are considered, or when response surface techniques and reduced order models are used. Anyhow, also in this field, if computational efficiency is not the main issue, solutions currently exist to effectively converge onto the optimum hull shape also with rather complex objective functions and constraints.

The true essence of a ship hull form design by optimization, though, resides in the parametric variation of the surface of the hull. In fact, once the capability of the CFD solver and the numerical optimization algorithm are cleared up, the success of the parametric optimization procedure to converge on the solution that a designer would be able to reach with his experience and traditional (unlimited) means, depends basically the capability to reproduce the various shapes that the experienced naval architect would design.

This can be done at a higher level of abstraction, i.e. on the mathematical definition of the hull surface (2.) or directly on the discrete representation of the hull surface, i.e. the computational mesh (3.). Keeping in mind the goal of the current study, i.e. to find effective ways of optimizing the global shape of the hull, in the first case we deal with full parametric (or global) hull form definition (3.1), while in the second we deal with numerical methods to modify or deform an initial (given) mesh geometry (3.2).

The scope of the paper is to compare these two “philosophically” different approaches that currently seem to divide the researchers working on hull form optimization. Reference is made to a practical design case recently considered by MIT-iShip lab, i.e. the design by optimization of the hull for a notional high speed round bilge hull form. The two different geometry modification techniques (3.1 vs. 3.2) are compared and put in competition in a real optimization case. Results obtained are pro and contra are highlighted from the direct comparison of the results obtained from a series of optimization runs. The full parametric hull form definition (3.1) is based on a new minimalistic definition devised for destroyer vessels, at MIT-iShip, which has proven to be very effective in reproducing typical shapes of modern navy combatant hull forms. The direct geometry deformation (3.2) is based on a multiple Free Form Deformation (FFD) technique directly applied on the reference hull form geometry.

Minimum total resistance at one or more speeds is considered, with constraints from the internal arrangements, a given initial intact stability index and given main dimensions and displacement. The critical analysis of the results will highlight the expected flexibility of the full parametric definition and the limitations of the FFD technique as well as ways to bring the latter close to the first one, at an expense of an increase of free variable number. Differences

between the optimum shapes are also finally discussed.

## 2 REFERENCE HULL DESCRIPTION

The considered hull is presented in Figure 1. It is a fast, 48m long, round bilge, deep-V hull with a wide transom stern and with flat bottom. Main sections, as in fast slender mononulls is aft of midship. The reference displacement is 275 metric tons (corresponding to about 1.75m draft even keel) and the design speed is about 25 knots. So the reference  $F_{nL} = 0.59$  and  $F_{nV} = 1.61$ , so well in the semi-displacement regime which can be still successfully addressed with Rankine panel methods [1]. The top speed of 38 knots, corresponds to  $F_{nL} = 0.90$  and  $F_{nV} = 2.45$ , at the limit of the planing regime. The panel method used in this study would be less adequate to predict the residuary resistance at this higher speed, where a considerable portion of resistance induced by hydrodynamic lift acting on the hull is expected.

The original hull design is topologically divided in two portions: one below and the other above the spray rail at the chine. Only the hull portion below the spray rail has been parametric modeled, fixing the shape of the chine line (profile and plan view) during the optimization. So each variant of the lower part will match with the part above the chine. This assumption is quite realistic, as in fact the lines obtained projecting the chine on the profile and waterline planes are usually designed and set first on the basis of hydrodynamic criteria related to the planing regime. In addition, nobody prevents the shape of the chine itself to be object of optimization: this would involve a slightly more complex parametric model, in which these two lines are described in a parametric fashion, as for instance it has been done for the keel profile in the current model.

An even keel draft has been imposed to create the mesh of the hull and run the calculation with the panel method. This draft is the maximum even keel draft before intersecting the spray rail at the chine at midship.

The lower part of the hull has been defined based on minimum number of geometric elements that come from classical naval architecture design practice. In this case, the selection of basic parameters was based on the shape of keel, the longitudinal distribution of the deadrise angle, of the flare angle at chine, the profile of the chine itself and the fullness coefficient of the cross sections.

A 3D view of the parametric model of the hull created in Friendship is given in Figure 2, in which the free portion of the hull has colored in cyan with the representation of the buttocks for a rapid check of the fairness of the generated hull surface. Indeed the parametric model is able to generate quite faired shape variants even with a large variation of shape.

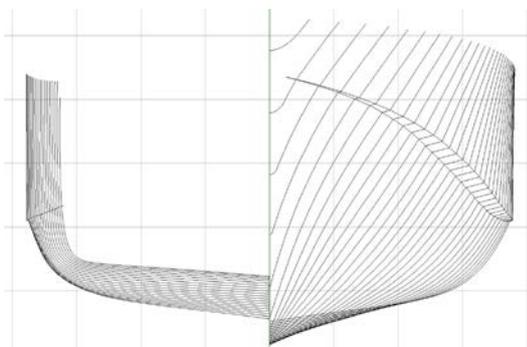


Figure 1: Body plan of the Reference Hull

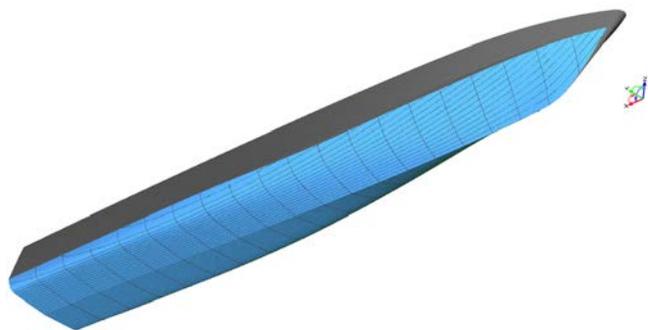


Figure 2: Parametric surface (blue) modeled under the chine

The hull in Figure 2 which is the result of the full parametric model finds an excellent correspondence with the original hull form defined by the designer (maximum deviation is in the order of the centimeter). The matching is a result of the consistent definition of the parametric model which in turn needs a proper definition of the parameters that define the shape of the basic curves. A dedicated optimization procedure that finds the values of these parameters that are able to minimize the geometric deviation of the hull generated hull with the reference one, has been implemented, following the method defined in [2]. The resultant hull becomes the reference (initial) hull for the following optimization studies.

### 3 HULL FORM GENERATION METHODS AND SHAPE MODIFICATION APPROACHES

Two different methods and modeling philosophies are considered to accomplish the hydrodynamic optimization of the proposed hull. Both methods are developed within *CAESES* parametric modeling environment [3] following two different new modeling techniques.

The first is a Full parametric Approach (FPA), recently developed at MIT-iShip for navy hull forms [2] based on the definition of a minimal set of *Basic Curves* by which the surfaces are created. This method is detailed in the following section. Surface shape variations are obtained by changing imposed to the *Basic Curves* (controlled in turn by a number of free design variable).

The second method is a modern implementation of the classic Free Form Deformation (FFD) [4] that acts directly on a given surface (defined for example using the *iges* format) by one or more consecutive transformations that are superimposed onto the original hull form. A BSpline control volume whose control points coordinates are used as free variables defines the functional relations of such transformations.

In the following two chapters a description of the application of both methods to the proposed hull is provided.

#### 3.1 Full parametric Approach

The transverse round sections below the chine have been parametric modeled through a NURBS curve defined through its control polygon, having a uniform weight of all the point except the mid point, whose NURBS weight can be changed in order to control the radius of curvature of the section. This is the parameter that controls the fullness (and hence the SAC) of the transverse section, together with the position of the double points at the flare and keel needed to impose the start and end tangents to the curve.

The parametric model has been created in order to minimize the number of basic curves that define the variation along the length of significant geometric properties of the transverse sections. The properties chosen are: the longitudinal profile at the keel (Figure 4) defined by two joint f-splines connecting at the keel-rise point, the profile and plan-view of the chine (Figure 4 and Figure 5), the longitudinal distribution of the fullness factor for the transverse sections (Figure 5), defined as a cubic spline curve through 5 points, the position of the second point the longitudinal distribution of the deadrise and flare angles (Figure 7) of the transverse sections, again defined by cubic spline curves interpolating 5 points. These control points of these basic curves are found by an automatic optimization procedure to best fit the reference

hull form, following the method described in [2].

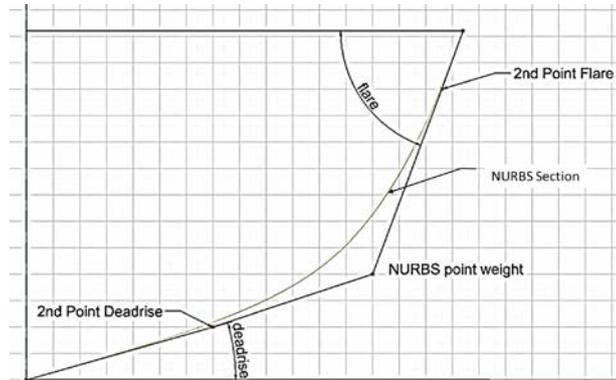


Figure 3: Parametric definition of the transverse sections

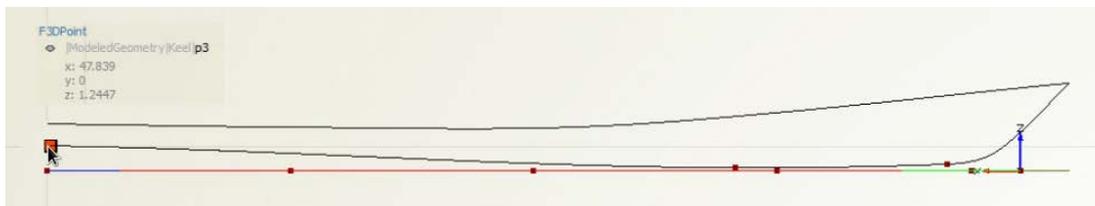


Figure 4: Longitudinal Basic Curves: Chine and Keel profiles

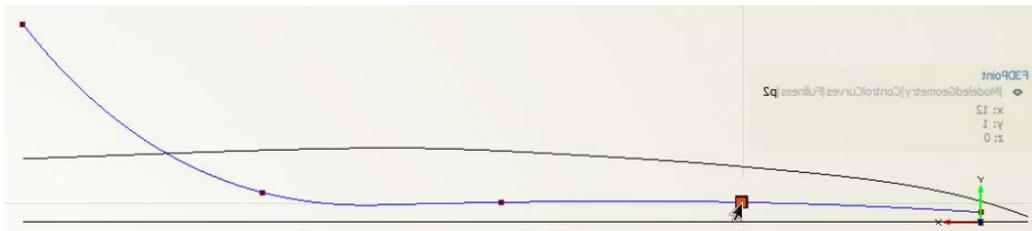


Figure 5: Longitudinal Basic Curves: Chine planview and Fulness Factor (blue) of transverse sections



Figure 6: Longitudinal Basic Curves: Deadrise and Flare Second Point of NURBS polygon

This parametric model, as opposed to the others developed in previous works [5] is not based on the Sectional Area Curve, so each parametric variation corresponds in general to a different SAC and hence implies a variation of the LCB and the displaced volume  $\nabla$ . If, as in usual design cases, these two hull properties are to be kept fixed to the design value with a certain tolerance, then the optimization need to consider the values of these two parameters as constrained to lie inside a given range centered around their design value.

### 3.2 Free Form Deformation Method

In the Free Form Deformation (FFD) method the shape of the “static” hull surface is modified by a general transformation applied through a BSpline control volume. The hull surface is not parametric modelled and it represents a generic reference geometry to which the shape transformation (FFD) is applied. This time, the BSpline control volume is defined in a parametric fashion such that the coordinates of its control points become the free design variables of optimization problem.

The method is better explained by a practical example. Let us consider a  $(U, V, W)$  Cartesian reference framework in which each point has  $(u, v, w)$  coordinates; consider also a sphere and a prismatic BSpline control polygon,  $P_{ijk}$ , in which the shape is embedded, shown in Figure 7. By the control polygon  $P_{ijk}$  the BSpline control volume,  $Q(u, v, w)$ , is generated (Figure 7) according to:

$$Q(u, v, w) = \sum_{i=0}^{du} \sum_{j=0}^{dv} \sum_{k=0}^{dw} P_{ijk} B_i(u) B_j(v) B_k(w) \quad (1)$$

where the  $B$  matrixes are the Basis Functions of the BSpline formulation. The coordinates of the surface points  $(u, v, w)$  in the transformation reference system, i.e.  $(U, V, W)$  or the fixed reference system with the BSpline volume, are computed according to:

$$u = \frac{V \times W * (X - X_0)}{V \times W * U} \quad v = \frac{U \times W * (X - X_0)}{U \times W * V} \quad w = \frac{U \times V * (X - X_0)}{U \times V * W} \quad (2)$$

In Figure 8, the iso-parameter outer planes are highlighted with the same color (the same color of the points means the same  $u$ ,  $v$  or  $w$  value). The distance of the origin of the volume reference system with respect to the global Cartesian one is shown with black arrows.

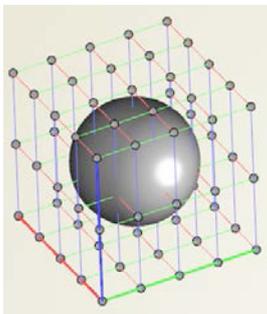


Figure 7: Initial sphere shape and BSpline control polygon

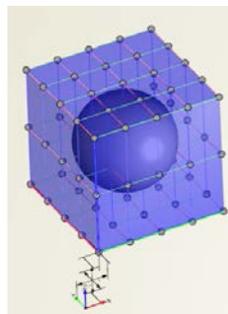


Figure 8: BSpline control volume built on the control polygon

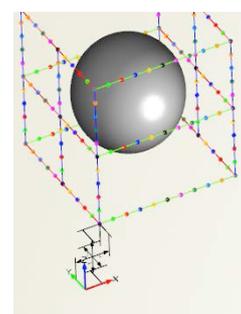


Figure 9: Iso-parameter outer planes of the BSpline volume reference system

Once this initial structure of the FFD control volume is defined, changing the position of its control points result in a modification of the shape of the embedded surface (the sphere). The control polygon can be modified by single points translations or by translations or rotations of group of points, depending on the amount of the initial surface that wants to be varied. Once the transformation is applied to the control points, the BSpline volume is recomputed in the global Cartesian reference system as well as the  $(u, v, w)$  parametrization of the deformed local reference system  $(U_1, V_1, W_1)$ . Then the position vector,  $X_1$ , of the points of the modified shape

in the global Cartesian reference system  $(X, Y, Z)$  is found by:

$$X_1(\mathbf{u}, \mathbf{v}, \mathbf{w}) = X + \mathbf{u}U_1 + \mathbf{v}V_1 + \mathbf{w}W_1 \quad (3)$$

where  $X$  is the new position vector defining the origin of the local reference system in the Cartesian global one.

Applying for example a single translation to a plane of points of the defined control polygon (see Figure 10) the new parametrization of the BSpline volume is shown in Figure 11 and the initial sphere shape is modified accordingly (see Figure 12).

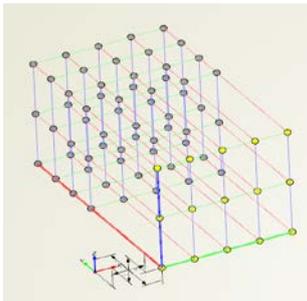


Figure 10: Modification of BSpline control polygon by a single translation of its points

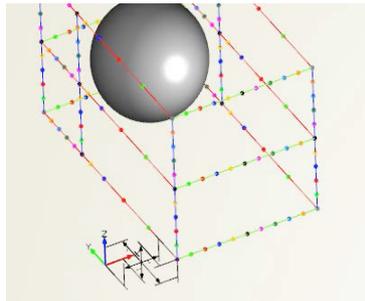


Figure 11: New positions of the BSpline control polygon points

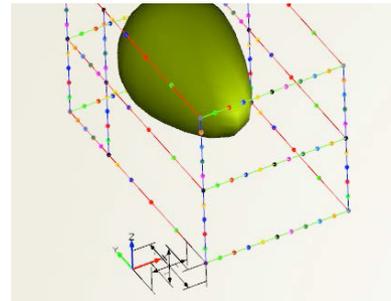


Figure 12: Deformed sphere after the application of the FFD

An example of double translation of the same plane of points of the initial control polygon is shown in Figure 13 to Figure 15.

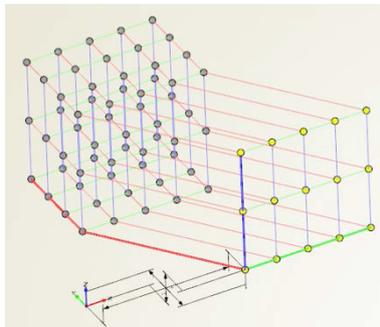


Figure 13: Modification of the BSpline control polygon by a double translation of its points

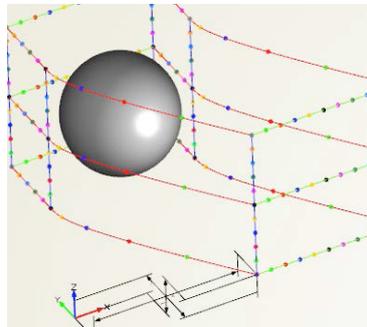


Figure 14: New positions of the BSpline control polygon points

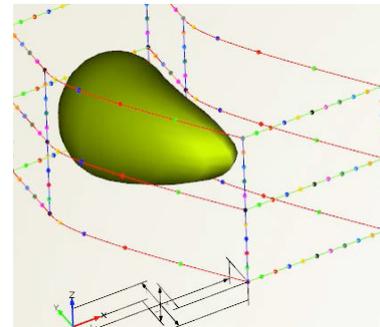


Figure 15: Deformed sphere shape after the application of the FFD

When the practical example of the reference hull is considered, the application of the FFD method results less trivial than the previous example. The reason is that in this particular case only a portion of the hull surface needs to be modified, namely the surface portion comprised between the keel profile and the chine. Hence a simple quadrangular box used for example in [6] cannot be adopted in this case, but more complex BSpline volume shapes are generated to respect the topological constraints of the hull, that is to preserve the chine shape during the optimization process. In order to accomplish this task, six BSpline volumes are created, each

one performing a different transformation over a portion of the hull static surface. The six FFD volumes are shown in Figure 16 to Figure 21; the control volumes are highlighted in red; the points of the control polygon by which the transformation is applied are highlighted in yellow. Due to the complexity given by the 3D chine-line constraint, most of the control points are used to force BSpline volumes to fit this 3D chine-line edge and to impose at least the tangent constraint and curvature continuity at the volume boundaries.

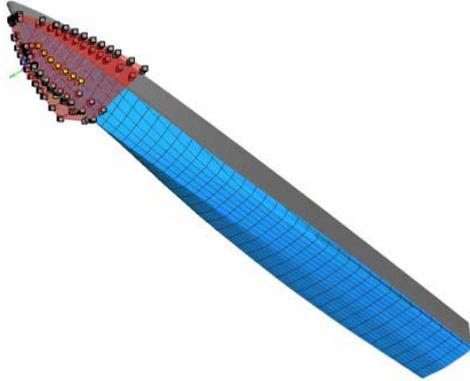


Figure 16: 1<sup>st</sup> BSpline Volume for extreme bow transformation (y translation of control points)

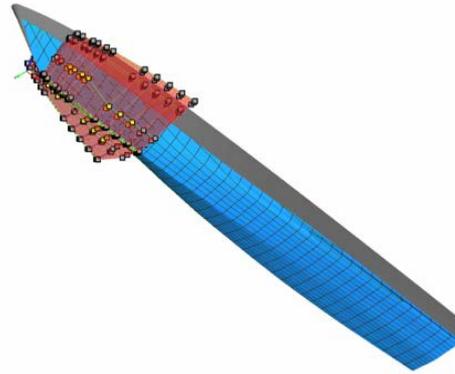


Figure 17: 2<sup>nd</sup> BSpline Volume for bow transformation (y translation of control points)

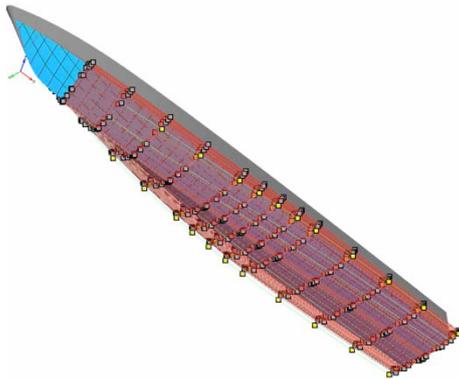


Figure 18: 3<sup>rd</sup> BSpline Volume for mid- and aft-body transformation (y translation of control points)

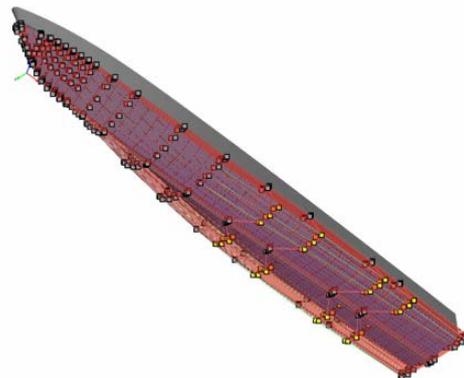


Figure 19: 4<sup>th</sup> BSpline Volume for longitudinal shift of the mid- and aft- body sections (x-translation)

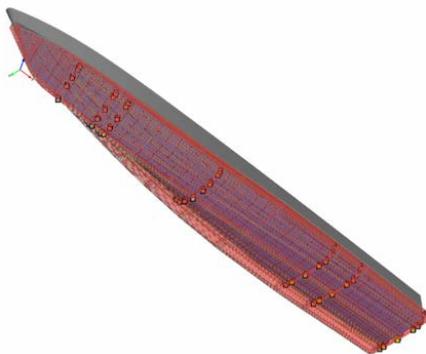


Figure 20: 5<sup>th</sup> BSpline Volume for mid- and aft-body transformation (z translation of control points)

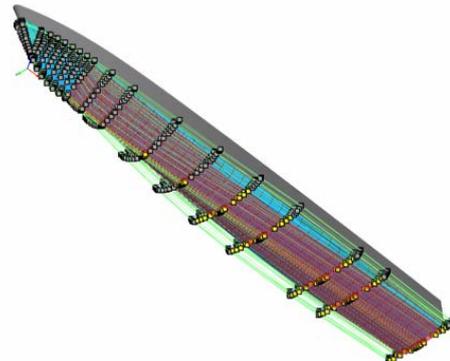


Figure 21: 6<sup>th</sup> BSpline Volume for mid- and aft-body transformation (z translation of control points)

#### 4 HYDRODYNAMIC SHAPE OPTIMIZATION STUDY

A hydrodynamic shape optimization automatic process is performed by means of each shape modification technique, namely the FPA and the FFD method, briefly described in the previous section. Non-dominant Sorting Genetic Algorithm, NSGA-II [7] drive both runs, ensuring the global convergence within the complex domain defined by the free parameters (in the order of 20) subject to design constraints. This evolutionary algorithm has been recently used with success for hydrodynamic multi-objective shape optimization of more complex hull forms [13] and it is surely less fast to converge than other greedier stochastic algorithms, such as differential evolution [8] or particle swarm [9]. But, again, main stress here is to find the global optimum with the two different geometry modification methods without being trapped in relative minima. *ModeFrontier* optimization framework [10] is used to setup the two optimization procedures. The wave resistance relative to displacement,  $R_w/D$ , at the reference design speed  $Fn_L=0.59$  is selected as objective function for the present analysis. This form of the objective function allows for a linear compensation of the small tolerance considered for the given displacement constraint. A low order Boundary Element Method (BEM) is used for the wave resistance computation; this numerical method was been extensively validated on different types of fast mono and multihulls [11], and other unconventional types of fast multihulls, such as fast SWATHs [8] [12] and semi-SWATH [13].

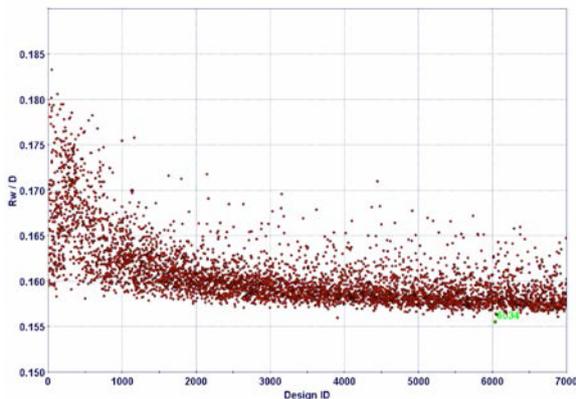


Figure 22: Optimization history of the Full parametric Model approach computation

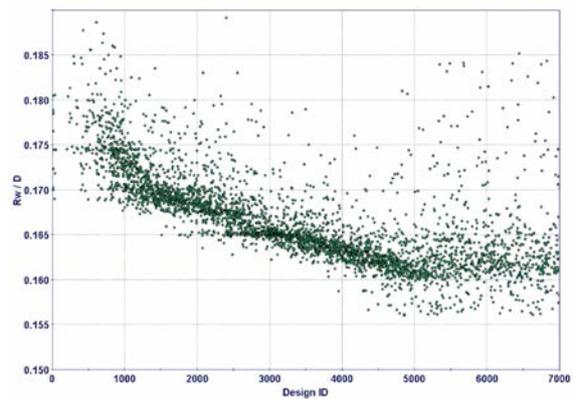


Figure 23: Optimization history of the Free Form Deformation approach computation

A tolerance of 5% has been used to allow a relative difference on the volume with respect to its reference (initial) value while for the Longitudinal Centre of Buoyancy,  $LCB$ , the variation relative to its reference value has been allowed within the 3%. An even keel draft of 2.05 meters has been imposed to create the mesh of the hull and run the calculation with the panel method. This draft is the maximum even keel draft before intersecting the spray rail at the chine at midship and it corresponds to the reference displacement. At higher drafts the waterline starts to intersect the horizontal spray rail at the chine introducing a new type of physics, the jet spray flow, which is not adequately reproduced by the Rankine source panel method used in this study.

The history of the FPA and FFD based optimizations are shown in Figures 22 and Figure 23 respectively. The computation based on the full parametric model (FPA) reaches a stable

convergence after about 7000 design evaluations. After the same number of numerical evaluations, the alternative procedure based on FFD does not achieve a distinct asymptotic value, although the initial trend is very good and promising. Moreover, as the run goes on, after about 5000 cases (Figure 23), the genetic algorithm starts to explore a portion of the designs with unrealistic shapes, compromising the convergence of the optimization populating the individuals of each generation with false optimal solutions (these false optimal designs have been removed from the optimization history).

The perspective view of the optimized hulls from the two processes are shown in Figure 22 (FPA) and in Figure 23 (FFD), while the comparison of the stations of both hulls with the reference hull are shown in Figure 24 (FPA) and Figure 25 (FFD). The free wave pattern generated by the optimum hull found by FPA is shown in Figure 26, while in Figure 27 there we present the same comparison but for the FFD optimum hull. Both wave patterns are compared with the free wave pattern generated by the reference hull form. The two optimization processes had worked mainly on the mid- and aft-body of the hull. Both methods achieve quite the same relative reduction of the objective function, namely about 8.5% for the FPA run and 8.4% for the FFD run. Although the two approaches lead to the same numeric reduction, the two optimum hulls that achieve it result very different. The hull found by the FPA, due to the higher intrinsic fairness of the modeling philosophy, preserves faired lines corresponding to a feasible (from a wider design perspective). On the other side, the FFD results in a quite weird shape, showing a tumblehome in the sections aft of midship: tis probably to compensate for the volume loss in rising the transom and considerably reducing the deadrise aft of midship. Both hull respect the given design constraints, in fact.

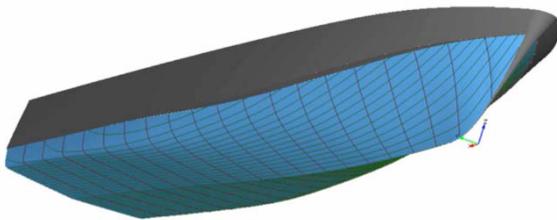


Figure 24: Perspective view of the optimized hull with the Full parametric Model approach

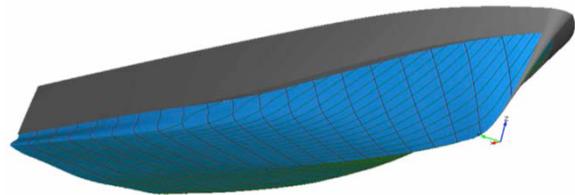


Figure 25: Perspective view of the optimized hull with the Free Form Deformation approach

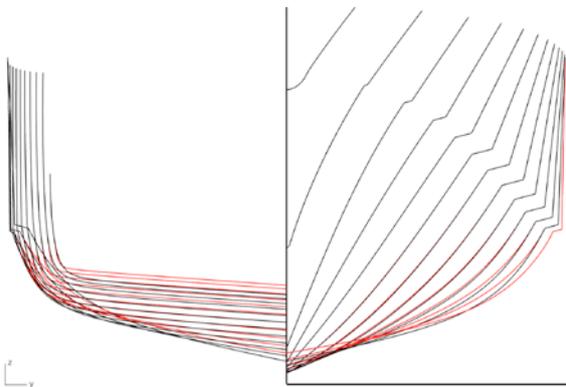


Figure 26: Stations of the FPA optimized hull (red) and the reference hull (black)

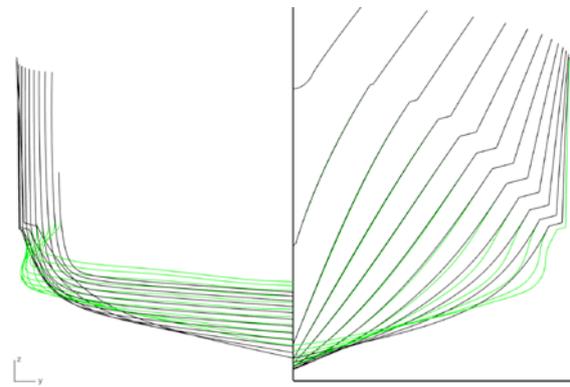


Figure 27: Stations of the FFD optimized hull (green) and the reference hull (black)

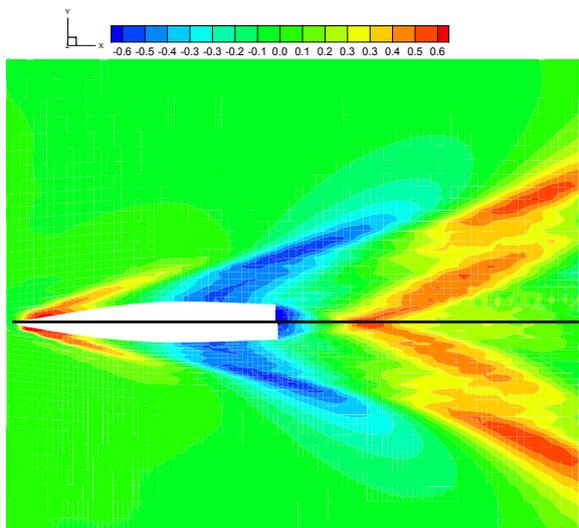


Figure 28: Wave patterns generated by the reference hull (top) and the optimum FPA hull (bottom)

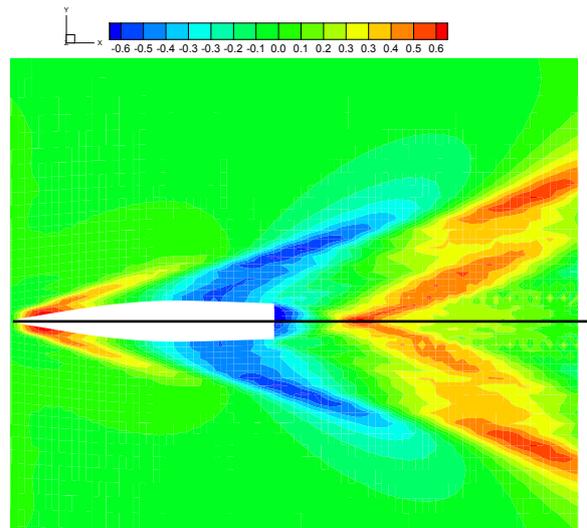


Figure 29: Wave patterns generated by the reference hull (top) and the optimum FFD hull (bottom)

## 12 CONCLUSIONS

The comparison of two relevant and modern hull shape modification techniques has been performed. The Full parametric Approach (FPA) has been compared with the Free Form Deformation (FFD) technique. Both methods have been applied to the same initial hull shape and have been used in two separate hydrodynamic shape optimization processes. A classic Boundary Element Method has been used to compute wave resistance during the automatic optimization, driven by a modern genetic algorithm.

Both techniques lead to the same relative reduction of the wave resistance over displacement ratio. However, while the FPA results in a feasible and realistic (faired) hull shape the FFD method ends with a feasible but unrealistic design.

This comparison is not exhaustive and more systematic variation of the FFD transformation used here should be considered as well as different sets of parameters should be used for both models. If anything else, this study demonstrates how difficult and hardly practical it might be in practice to define an effective FFD transformation that can be at least equivalent to the full parametric approach proposed in this paper. The FPA, in fact, relies on geometric entities well known and easily adaptable by naval architects to different hull forms. As expected, in this example at least, the FPA is able to achieve better performances both in terms of stability/convergence of the optimization procedure and in terms of design viability of the attained optimum shape.

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