

UNIVERSITAT POLITÈCNICA DE CATALUNYA

ENGINYERIA EN TECNOLOGIES AEROESPACIALS (GRETA)

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Study of buckling phenomena for *SMART* structural applications: Appendices

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Appendices

A MATLAB code

A.1 Free Undamped

```
1 %% Time Integration (Runge-Kutta Order 4)
2 clear;
3 m=4; %kg
4 k=100; %N/m
5 c=0;
6 omega_n=sqrt(k/m); %rad/s
7 beta=0.25;
8 gamma=0.5;
9
10 x_0=2; %m
11 v_0=0; %m/s
12
13 A=sqrt(((x_0*omega_n)^2+v_0^2)/(4*omega_n^2));
14
15 h=0.00001;
16 t2=0:h:3;
17 n=length(t2);
18
19 t3=0:0.05:3
20
21 x=2*A*cos(omega_n*t3);
22 xmodal=2*A*cos(omega_n*t2);
23
24 x2=zeros(n,1);
25 v2=zeros(n,1);
26 x2(1)=x_0;
27 v2(1)=v_0;
28
29 f1 = @(t,x,v) v;
30 f2 = @(t,x,v) -k/m*x;
31
32 for i=1:n-1
33     dx1=f1(t2(i),x2(i),v2(i));
34     dv1=f2(t2(i),x2(i),v2(i));
35     dx2=f1(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
36     dv2=f2(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
37     dx3=f1(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
38     dv3=f2(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
39     dx4=f1(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
40     dv4=f2(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
41     dv=h/6*(dv1+2*dv2+2*dv3+dv4);
42     dx=h/6*(dx1+2*dx2+2*dx3+dx4);
```

```

43     x2(i+1)=x2(i)+dx;
44     v2(i+1)=v2(i)+dv;
45 end
46
47 x3=zeros(n,1);
48 v3=zeros(n,1);
49 a3=zeros(n,1);
50 x3(1)=x_0;
51 v3(1)=v_0;
52 a3(1)=1/m*(-k*x_0);
53
54
55 for i=1:n-1
56     aguess=1;
57     loop=0;
58     while loop==0
59         x3(i+1)=x3(i)+v3(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
60         v3(i+1)=v3(i)+(1-gamma)*aguess*h+gamma*aguess*h;
61         a3(i+1)=(1/m)*(-k*x3(i+1));
62         if abs(a3(i+1)-aguess)<10^-3
63             loop=1;
64         else
65             aguess=a3(i+1);
66         end
67     end
68 end
69
70 eark=abs(xmodal(end)-x2(end));
71 ean=abs(xmodal(end)-x3(end));
72 errk=abs((xmodal(end)-x2(end))/xmodal(end))*100;
73 ern=abs((xmodal(end)-x3(end))/xmodal(end))*100;

```

A.2 Free Underdamped

```
1 %% Time Integration
2 clear;
3 m=1; %kg
4 k=4; %N/m
5 omega_n=sqrt(k/m);
6 c=0.5;
7 dr=c/(2*sqrt(k*m));
8 omega_d=omega_n*sqrt(1-dr^2);
9 beta=0.25;
10 gamma=0.5;
11
12 x_0=2;
13 v_0=0;
14
15 A=sqrt((x_0*omega_d)^2+(dr*omega_n*x_0)^2)/(4*omega_d^2);
16 phase=atan(-dr*omega_n/omega_d);
17
18 h=0.00001;
19 t2=0:h:15;
20 n=length(t2);
21
22 t3=0:0.5:15;
23
24 for i=1:length(t3)
25     xmodal(i)=2*A*exp(-dr*omega_n*t3(i))*cos(omega_d*t3(i)+phase);
26 end
27
28
29 x2=zeros(n,1);
30 v2=zeros(n,1);
31 x2(1)=x_0;
32 v2(1)=v_0;
33
34
35 f1 = @(t,x,v) v;
36 f2 = @(t,x,v) -k/m*x-c/m*v;
37
38 for i=1:n-1
39     dx1=f1(t2(i),x2(i),v2(i));
40     dv1=f2(t2(i),x2(i),v2(i));
41     dx2=f1(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
42     dv2=f2(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
43     dx3=f1(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
44     dv3=f2(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
45     dx4=f1(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
46     dv4=f2(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
47     dv=h/6*(dv1+2*dv2+2*dv3+dv4);
48     dx=h/6*(dx1+2*dx2+2*dx3+dx4);
49     x2(i+1)=x2(i)+dx;
```

```

50     v2(i+1)=v2(i)+dv;
51 end
52
53
54 x3=zeros(n,1);
55 v3=zeros(n,1);
56 a3=zeros(n,1);
57 x3(1)=x_0;
58 v3(1)=v_0;
59 a3(1)=1/m*(-k*x_0-c*v_0);
60
61 cont=0;
62
63 for i=1:n-1
64     aguess=1;
65     loop=0;
66     while loop==0
67         x3(i+1)=x3(i)+v3(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
68         v3(i+1)=v3(i)+(1-gamma)*aguess*h+gamma*aguess*h;
69         a3(i+1)=(1/m)*(-k*x3(i+1)-c*v3(i+1));
70         cont=cont+1;
71         if abs(a3(i+1)-aguess)<10^-3
72             loop=1;
73         else
74             aguess=a3(i+1);
75         end
76     end
77 end
78
79 eark=abs(xmodal(end)-x2(end));
80 ean=abs(xmodal(end)-x3(end));
81 errk=abs((xmodal(end)-x2(end))/xmodal(end))*100;
82 ern=abs((xmodal(end)-x3(end))/xmodal(end))*100;

```

A.3 Free Critically Damped

```
1 %% Time Integration
2 clear;
3 m=4; %kg
4 k=4; %N/m
5 c=8; %Ns/m
6 omega_n=sqrt(k/m);
7 dx=c/(2*sqrt(k*m));
8 beta=0.25;
9 gamma=0.5;
10
11 x_0=2;
12 v_0=0;
13
14 A=x_0;
15 B=v_0+x_0*omega_n;
16
17 t3=0:0.2:10;
18
19 for i=1:length(t3)
20     xmodal(i)=(A+B*t3(i))*exp(-omega_n*t3(i));
21 end
22
23 h=0.00001;
24 t2=0:h:10;
25 n=length(t2);
26
27 x2=zeros(n,1);
28 v2=zeros(n,1);
29 x2(1)=x_0;
30 v2(1)=v_0;
31
32
33 f1=@(t,x,v) v;
34 f2=@(t,x,v) -k/m*x-c/m*v;
35
36 for i=1:n-1
37     dx1=f1(t2(i),x2(i),v2(i));
38     dv1=f2(t2(i),x2(i),v2(i));
39     dx2=f1(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
40     dv2=f2(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
41     dx3=f1(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
42     dv3=f2(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
43     dx4=f1(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
44     dv4=f2(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
45     dv=h/6*(dv1+2*dv2+2*dv3+dv4);
46     dx=h/6*(dx1+2*dx2+2*dx3+dx4);
47     x2(i+1)=x2(i)+dx;
48     v2(i+1)=v2(i)+dv;
49 end
```

```

50
51
52 x3=zeros(n,1);
53 v3=zeros(n,1);
54 a3=zeros(n,1);
55 x3(1)=x_0;
56 v3(1)=v_0;
57 a3(1)=1/m*(-k*x_0-c*v_0);
58
59 cont=0;
60
61 for i=1:n-1
62     aguess=1;
63     loop=0;
64     while loop==0
65         x3(i+1)=x3(i)+v3(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
66         v3(i+1)=v3(i)+(1-gamma)*aguess*h+gamma*aguess*h;
67         a3(i+1)=(1/m)*(-k*x3(i+1)-c*v3(i+1));
68         cont=cont+1;
69         if abs(a3(i+1)-aguess)<10^-3
70             loop=1;
71         else
72             aguess=a3(i+1);
73         end
74     end
75 end
76
77 eark=abs(xmodal(end)-x2(end));
78 ean=abs(xmodal(end)-x3(end));
79 errk=abs((xmodal(end)-x2(end))/xmodal(end))*100;
80 ern=abs((xmodal(end)-x3(end))/xmodal(end))*100;

```

A.4 Free Overdamped

```
1 %% Time Integration
2 clear;
3 m=4; %kg
4 k=4; %N/m
5 c=10; %Ns/m
6 dr=c/(2*sqrt(k*m));
7 omega_n=sqrt(k/m);
8 beta=0.25;
9 gamma=0.5;
10
11 x_0=2;
12 v_0=0;
13
14 X1=(dr+sqrt(dr^2-1))*x_0/(2*sqrt(dr^2-1));
15 X2=(-dr+sqrt(dr^2-1))*x_0/(2*sqrt(dr^2-1));
16
17 t3=0:0.3:15;
18
19 for i=1:length(t3)
20     xmodal(i)=exp(-dr*omega_n*t3(i))*(X1*exp(omega_n*t3(i)*sqrt(dr^2-1))...
21         +X2*exp(-omega_n*t3(i)*sqrt(dr^2-1)));
22 end
23
24 h=0.00001;
25 t2=0:h:15;
26 n=length(t2);
27
28 x2=zeros(n,1);
29 v2=zeros(n,1);
30 x2(1)=x_0;
31 v2(1)=v_0;
32
33
34 f1 = @(t,x,v) v;
35 f2 = @(t,x,v) -k/m*x-c/m*v;
36
37 for i=1:n-1
38     dx1=f1(t2(i),x2(i),v2(i));
39     dv1=f2(t2(i),x2(i),v2(i));
40     dx2=f1(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
41     dv2=f2(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
42     dx3=f1(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
43     dv3=f2(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
44     dx4=f1(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
45     dv4=f2(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
46     dv=h/6*(dv1+2*dv2+2*dv3+dv4);
47     dx=h/6*(dx1+2*dx2+2*dx3+dx4);
48     x2(i+1)=x2(i)+dx;
49     v2(i+1)=v2(i)+dv;
```

```

50 end
51
52
53 x3=zeros(n,1);
54 v3=zeros(n,1);
55 a3=zeros(n,1);
56 x3(1)=x_0;
57 v3(1)=v_0;
58 a3(1)=1/m*(-k*x_0-c*v_0);
59
60 cont=0;
61
62 for i=1:n-1
63     aguess=1;
64     loop=0;
65     while loop==0
66         x3(i+1)=x3(i)+v3(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
67         v3(i+1)=v3(i)+(1-gamma)*aguess*h+gamma*aguess*h;
68         a3(i+1)=(1/m)*(-k*x3(i+1)-c*v3(i+1));
69         cont=cont+1;
70         if abs(a3(i+1)-aguess)<10^-3
71             loop=1;
72         else
73             aguess=a3(i+1);
74         end
75     end
76 end
77
78 eark=abs(xmodal(end)-x2(end));
79 ean=abs(xmodal(end)-x3(end));
80 errk=abs((xmodal(end)-x2(end))/xmodal(end))*100;
81 ern=abs((xmodal(end)-x3(end))/xmodal(end))*100;

```

A.5 Forced Underdamped

```
1 %% Time Integration
2 clear;
3 m=1;
4 k=4;
5 c=0.5;
6 x_0=2;
7 v_0=0;
8 dr=c/(2*sqrt(k*m));
9 F=2;
10 omega_n=sqrt(k/m);
11 omega_d=omega_n*sqrt(1-dr^2);
12 omega_f=1.5;
13 beta=0.25;
14 gamma=0.5;
15
16 X=F/m/(sqrt((omega_n^2-omega_f^2)^2+4*dr^2*omega_n^2*omega_f^2));
17 phase1=atan(-2*omega_f*omega_n*dr/(omega_n^2-omega_f^2));
18 phase2=atan((1/omega_d)*(omega_f*X*sin(phase1)...
19 / (X*cos(phase1)-x_0)-dr*omega_n));
20 B=(x_0-X*cos(phase1))/(cos(phase2));
21
22 t3=0:0.5:20;
23
24 for i=1:length(t3)
25     xmodal(i)=X*cos(omega_f*t3(i)+phase1)+...
26         B*exp(-dr*omega_n*t3(i))*cos(omega_d*t3(i)+phase2);
27 end
28
29 h=0.00001;
30 t2=0:h:20;
31 n=length(t2);
32
33 x2=zeros(n,1);
34 v2=zeros(n,1);
35 x2(1)=x_0;
36 v2(1)=v_0;
37
38 f1 = @(t,x,v) v;
39 f2 = @(t,x,v) -k/m*x-c/m*v+F/m*cos(omega_f*t);
40
41 for i=1:n-1
42     dx1=f1(t2(i),x2(i),v2(i));
43     dv1=f2(t2(i),x2(i),v2(i));
44     dx2=f1(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
45     dv2=f2(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
46     dx3=f1(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
47     dv3=f2(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
48     dx4=f1(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
49     dv4=f2(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
```

```

50   dv=h/6*(dv1+2*dv2+2*dv3+dv4);
51   dx=h/6*(dx1+2*dx2+2*dx3+dx4);
52   x2(i+1)=x2(i)+dx;
53   v2(i+1)=v2(i)+dv;
54   end
55
56   x3=zeros(n,1);
57   v3=zeros(n,1);
58   a3=zeros(n,1);
59   x3(1)=x_0;
60   v3(1)=v_0;
61   a3(1)=1/m*(-k-c-F*cos(omega_f*t2(1)));
62
63   cont=0;
64   for i=1:n-1
65       aguess=1;
66       loop=0;
67       while loop==0
68           x3(i+1)=x3(i)+v3(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
69           v3(i+1)=v3(i)+(1-gamma)*aguess*h+gamma*aguess*h;
70           a3(i+1)=(1/m)*(-k*x3(i+1)-c*v3(i+1)+F*cos(omega_f*t2(i)));
71           cont=cont+1;
72           if abs(a3(i+1)-aguess)<10^-3
73               loop=1;
74           else
75               aguess=a3(i+1);
76           end
77       end
78   end
79
80   eark=abs(xmodal(end)-x2(end));
81   ean=abs(xmodal(end)-x3(end));
82   errk=abs((xmodal(end)-x2(end))/xmodal(end))*100;
83   ern=abs((xmodal(end)-x3(end))/xmodal(end))*100;

```

A.6 Exponential K Damped

```
1 %% Time Integration
2 clear;
3 m=1;
4 k=4;
5 c=0.5;
6 x_0=2;
7 v_0=0;
8 f_0=4;
9 beta=0.25;
10 gamma=0.5;
11
12 h=0.00001;
13 t2=0:h:20;
14 n=length(t2);
15
16 %%Runge-Kutta
17
18 x2=zeros(n,1);
19 v2=zeros(n,1);
20 x2(1)=x_0;
21 v2(1)=v_0;
22
23 f1 = @(t,x,v) v;
24 f2 = @(t,x,v) -f_0/m*(1-exp(-k/f_0*x))-c/m*v;
25
26 for i=1:n-1
27     dx1=f1(t2(i),x2(i),v2(i));
28     dv1=f2(t2(i),x2(i),v2(i));
29     dx2=f1(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
30     dv2=f2(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
31     dx3=f1(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
32     dv3=f2(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
33     dx4=f1(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
34     dv4=f2(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
35     dv=h/6*(dv1+2*dv2+2*dv3+dv4);
36     dx=h/6*(dx1+2*dx2+2*dx3+dx4);
37     x2(i+1)=x2(i)+dx;
38     v2(i+1)=v2(i)+dv;
39 end
40
41 %%Newmark
42
43 x3=zeros(n,1);
44 v3=zeros(n,1);
45 a3=zeros(n,1);
46 x3(1)=x_0;
47 v3(1)=v_0;
48 a3(1)=1/m*(-c-f_0*(1-exp(-k/f_0*x_0)));
49
```

```

50 cont=0;
51
52 for i=1:n-1
53     aguess=1;
54     loop=0;
55     while loop==0
56         x3(i+1)=x3(i)+v3(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
57         v3(i+1)=v3(i)+(1-gamma)*aguess*h+gamma*aguess*h;
58         a3(i+1)=(1/m)*(-c*v3(i+1)-f_0*(1-exp(-k/f_0*x3(i+1))));
59         cont=cont+1;
60         if abs(a3(i+1)-aguess)<0.01
61             loop=1;
62         else
63             aguess=a3(i+1);
64         end
65     end
66 end

```

A.7 Exponential K Forced

```
1 %% Time Integration
2 clear;
3 m=1;
4 k=4;
5 c=0.5;
6 x_0=0;
7 v_0=0;
8 F=2;
9 omega_f=1.5;
10 f_0=4;
11 beta=0.25;
12 gamma=0.5;
13
14 h=0.00001;
15 t2=0:h:40;
16 n=length(t2);
17
18 x2=zeros(n,1);
19 v2=zeros(n,1);
20 x2(1)=x_0;
21 v2(1)=v_0;
22
23 f1 = @(t,x,v) v;
24 f2 = @(t,x,v) -f_0/m*(1-exp(-k/f_0*x))-c/m*v+F/m*cos(omega_f*t);
25
26 for i=1:n-1
27     dx1=f1(t2(i),x2(i),v2(i));
28     dv1=f2(t2(i),x2(i),v2(i));
29     dx2=f1(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
30     dv2=f2(t2(i)+h/2,x2(i)+h*dx1/2,v2(i)+h*dv1/2);
31     dx3=f1(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
32     dv3=f2(t2(i)+h/2,x2(i)+h*dx2/2,v2(i)+h*dv2/2);
33     dx4=f1(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
34     dv4=f2(t2(i)+h,x2(i)+h*dx3,v2(i)+h*dv3);
35     dv=h/6*(dv1+2*dv2+2*dv3+dv4);
36     dx=h/6*(dx1+2*dx2+2*dx3+dx4);
37     x2(i+1)=x2(i)+dx;
38     v2(i+1)=v2(i)+dv;
39 end
40
41 x3=zeros(n,1);
42 v3=zeros(n,1);
43 a3=zeros(n,1);
44 x3(1)=x_0;
45 v3(1)=v_0;
46 a3(1)=1/m*(-c*v_0-f_0*(1-exp(-k/f_0*x_0))+F*cos(omega_f*t2(i)));
47
48 cont=0;
49 for i=1:n-1
```

```
50  aguess=1;
51  loop=0;
52  while loop==0
53      x3(i+1)=x3(i)+v3(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
54      v3(i+1)=v3(i)+(1-gamma)*aguess*h+gamma*aguess*h;
55      a3(i+1)=(1/m)*(-c*v3(i+1)-f_0*(1-exp(-k/f_0*x3(i+1))))+F*cos(omega_f*t2(i+1));
56      cont=cont+1;
57      if abs(a3(i+1)-aguess)<0.01
58          loop=1;
59      else
60          aguess=a3(i+1);
61      end
62  end
63 end
```

A.8 Piecewise linear

```
1 % Time Integration
2 clear;
3 m=1;
4 k_0=4;
5 k_1=2;
6 c=2*sqrt(m*k_1);
7 x_0=0;
8 v_0=0;
9 Fmax=10; %N
10 Fmin=2; %N
11 xmax=Fmax/k_0; %m
12 xmin=xmax+(Fmax-Fmin)/k_0; %m
13 beta=0.25;
14 gamma=0.5;
15 T=sqrt(m/k_0);
16
17 h=0.001;
18 t=0:h:5000;
19 n=length(t);
20
21 %Runge Kutta
22
23 x1=zeros(n,1);
24 v1=zeros(n,1);
25 f_k1=zeros(n,1);
26 f_ext=zeros(n,1);
27 f_a=zeros(n,1);
28 x1(1)=x_0;
29 v1(1)=v_0;
30
31 k=k_0;
32
33 f1 = @(t,x,v) v;
34 f2 = @(t,x,v) T/100*t/m-c/m*v-k_0/m*x;
35 f3 = @(t,x,v) T/100*t/m-c/m*v-(Fmax-k_0*(x-xmax))/m;
36 f4 = @(t,x,v) T/100*t/m-c/m*v-(Fmin+k_1*(x-xmin))/m;
37 f_k1(1)=k_0*x1(1);
38
39 aux1=0;
40 aux2=0;
41
42 for i=1:n-1
43     if(aux1==0)
44         dx1=f1(t(i),x1(i),v1(i));
45         dv1=f2(t(i),x1(i),v1(i));
46         dx2=f1(t(i)+h/2,x1(i)+h*dx1/2,v1(i)+h*dv1/2);
47         dv2=f2(t(i)+h/2,x1(i)+h*dx1/2,v1(i)+h*dv1/2);
48         dx3=f1(t(i)+h/2,x1(i)+h*dx2/2,v1(i)+h*dv2/2);
49         dv3=f2(t(i)+h/2,x1(i)+h*dx2/2,v1(i)+h*dv2/2);
```

```

50     dx4=f1(t(i)+h,x1(i)+h*dx3,v1(i)+h*dv3);
51     dv4=f2(t(i)+h,x1(i)+h*dx3,v1(i)+h*dv3);
52     dv=h/6*(dv1+2*dv2+2*dv3+dv4);
53     dx=h/6*(dx1+2*dx2+2*dx3+dx4);
54     x1(i+1)=x1(i)+dx;
55     v1(i+1)=v1(i)+dv;
56     f_k1(i+1)=k_0*x1(i+1);
57 end
58
59 if (f_k1(i) > Fmax) && (aux2==0)
60     aux1=1;
61     aux2=1;
62     save(aux2)=i;
63     k=-k_0;
64 end
65
66 if (f_k1(i) < Fmin) && (aux2==1)
67     aux1=2;
68     aux2=2;
69     save(aux2)=i;
70     k=k_1;
71 end
72
73 if(aux1==1)
74     dx1=f1(t(i),x1(i),v1(i));
75     dv1=f3(t(i),x1(i),v1(i));
76     dx2=f1(t(i)+h/2,x1(i)+h*dx1/2,v1(i)+h*dv1/2);
77     dv2=f3(t(i)+h/2,x1(i)+h*dx1/2,v1(i)+h*dv1/2);
78     dx3=f1(t(i)+h/2,x1(i)+h*dx2/2,v1(i)+h*dv2/2);
79     dv3=f3(t(i)+h/2,x1(i)+h*dx2/2,v1(i)+h*dv2/2);
80     dx4=f1(t(i)+h,x1(i)+h*dx3,v1(i)+h*dv3);
81     dv4=f3(t(i)+h,x1(i)+h*dx3,v1(i)+h*dv3);
82     dv=h/6*(dv1+2*dv2+2*dv3+dv4);
83     dx=h/6*(dx1+2*dx2+2*dx3+dx4);
84     x1(i+1)=x1(i)+dx;
85     v1(i+1)=v1(i)+dv;
86     f_k1(i+1)=Fmax-k_0*(x1(i+1)-xmax);
87 end
88
89 if(aux1==2)
90     dx1=f1(t(i),x1(i),v1(i));
91     dv1=f4(t(i),x1(i),v1(i));
92     dx2=f1(t(i)+h/2,x1(i)+h*dx1/2,v1(i)+h*dv1/2);
93     dv2=f4(t(i)+h/2,x1(i)+h*dx1/2,v1(i)+h*dv1/2);
94     dx3=f1(t(i)+h/2,x1(i)+h*dx2/2,v1(i)+h*dv2/2);
95     dv3=f4(t(i)+h/2,x1(i)+h*dx2/2,v1(i)+h*dv2/2);
96     dx4=f1(t(i)+h,x1(i)+h*dx3,v1(i)+h*dv3);
97     dv4=f4(t(i)+h,x1(i)+h*dx3,v1(i)+h*dv3);
98     dv=h/6*(dv1+2*dv2+2*dv3+dv4);
99     dx=h/6*(dx1+2*dx2+2*dx3+dx4);
100    x1(i+1)=x1(i)+dx;
101    v1(i+1)=v1(i)+dv;

```

```

102     f_k1(i+1)=Fmin+k_1*(x1(i+1)-xmin);
103     end
104     f_ext(i+1)=T/100*t(i+1);
105 end
106
107 % Newmark
108
109 x2=zeros(n,1);
110 v2=zeros(n,1);
111 a2=zeros(n,1);
112 f_k2=zeros(n,1);
113
114 x2(1)=x_0;
115 v2(1)=v_0;
116 a2(1)=1/m*(-c*v_0-k_0*x_0);
117 aux1=0;
118 aux2=0;
119
120 Ep=0;
121 Eext=0;
122 Ed=0;
123
124 for i=1:n-1
125     aguess=1;
126     loop=0;
127     k=k_0;
128
129     if (f_k2(i)≥Fmax) && (aux2==0)
130         aux1=1;
131         aux2=1;
132         k=-k_0;
133     end
134
135     if (f_k2(i)≤Fmin) && (aux2==1)
136         aux1=2;
137         aux2=2;
138         k=k_1;
139     end
140
141     if aux1==0
142         while loop==0
143             x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
144             v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
145             f_k2(i+1)=k_0*x2(i+1);
146             a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
147             if abs(a2(i+1)-aguess)<0.001
148                 loop=1;
149             else
150                 aguess=a2(i+1);
151             end
152         end
153     end

```

```

154
155
156   if aux1==1
157       while loop==0
158           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
159           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
160           f_k2(i+1)=Fmax-k_0*(x2(i+1)-xmax);
161           a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
162           if abs(a2(i+1)-aguess)<0.001
163               loop=1;
164           else
165               aguess=a2(i+1);
166           end
167       end
168   end
169
170   if aux1==2
171       while loop==0
172           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
173           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
174           f_k2(i+1)=Fmin+k_1*(x2(i+1)-xmin);
175           a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
176           if abs(a2(i+1)-aguess)<0.001
177               loop=1;
178           else
179               aguess=a2(i+1);
180           end
181       end
182   end
183
184   Pc(i+1)=v2(i+1)*m*a2(i+1);
185   Pp(i+1)=f_k2(i+1)*v2(i+1);
186   Pd(i+1)=c*v2(i+1)^2;
187   Pext(i+1)=f_ext(i+1)*v2(i+1);
188   Ep=Ep+f_k2(i+1)*v2(i+1)*h;
189   Eext=Eext+f_ext(i+1)*v2(i+1)*h;
190   Ed=Ed+c*v2(i+1)^2*h;
191
192   end

```

A.9 C value sweep (Piecewise Spring)

```
1 %% Time Integration
2 clear;
3 m=1;
4 k_0=4;
5 k_1=2;
6 C=[0.5 1.5 2*sqrt(m*k_1) 4 9];
7 x_0=0;
8 v_0=0;
9 Fmax=10; %N
10 Fmin=2; %N
11 xmax=Fmax/k_0; %m
12 xmin=xmax+(Fmax-Fmin)/k_0; %m
13 beta=0.25;
14 gamma=0.5;
15 T=sqrt(m/k_0);
16
17 h=0.001;
18 t=0:h:5000;
19 n=length(t);
20
21 %Newmark
22
23 x2=zeros(length(C),1);
24 v2=zeros(length(C),1);
25 a2=zeros(length(C),1);
26 f_k2=zeros(length(C),1);
27
28 for p=1:length(C)
29 c=C(p)
30 x2(p,1)=x_0;
31 v2(p,1)=v_0;
32 a2(p,1)=1/m*(-c*v_0-k_0*x_0);
33 aux1=0;
34 aux2=0;
35
36
37 for i=1:n-1
38     aguess=1;
39     loop=0;
40     k=k_0;
41
42     if (f_k2(p,i) >= Fmax) && (aux2==0)
43         aux1=1;
44         aux2=1;
45         k=-k_0;
46     end
47
48     if (f_k2(p,i) <= Fmin) && (aux2==1)
49         aux1=2;
```

```

50     aux2=2;
51     k=k_1;
52 end
53
54 if aux1==0
55     while loop==0
56         x2(p,i+1)=x2(p,i)+v2(p,i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
57         v2(p,i+1)=v2(p,i)+(1-gamma)*aguess*h+gamma*aguess*h;
58         f_k2(p,i+1)=k_0*x2(p,i+1);
59         a2(p,i+1)=(1/m)*(-c*v2(p,i+1)-f_k2(p,i+1)+T/100*t(i+1));
60         if abs(a2(p,i+1)-aguess)<0.001
61             loop=1;
62         else
63             aguess=a2(p,i+1);
64         end
65     end
66 end
67
68
69 if aux1==1
70     while loop==0
71         x2(p,i+1)=x2(p,i)+v2(p,i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
72         v2(p,i+1)=v2(p,i)+(1-gamma)*aguess*h+gamma*aguess*h;
73         f_k2(p,i+1)=Fmax-k_0*(x2(p,i+1)-xmax);
74         a2(p,i+1)=(1/m)*(-c*v2(p,i+1)-f_k2(p,i+1)+T/100*t(i+1));
75         if abs(a2(p,i+1)-aguess)<0.001
76             loop=1;
77         else
78             aguess=a2(p,i+1);
79         end
80     end
81 end
82
83 if aux1==2
84     while loop==0
85         x2(p,i+1)=x2(p,i)+v2(p,i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
86         v2(p,i+1)=v2(p,i)+(1-gamma)*aguess*h+gamma*aguess*h;
87         f_k2(p,i+1)=Fmin+k_1*(x2(p,i+1)-xmin);
88         a2(p,i+1)=(1/m)*(-c*v2(p,i+1)-f_k2(p,i+1)+T/100*t(i+1));
89         if abs(a2(p,i+1)-aguess)<0.001
90             loop=1;
91         else
92             aguess=a2(p,i+1);
93         end
94     end
95 end
96 end
97 end

```

A.10 Load-Unload (Metastable)

```
1  % Time Integration
2  clear;
3  m=1;
4  k_0=4;
5  k_1=2;
6  c=2*sqrt(m*k_1);
7  x_0=0;
8  v_0=0;
9  Fmax=10; %N
10 Fmin=2; %N
11 FMAX=12;
12 xmax=Fmax/k_0; %m
13 xmin=xmax+(Fmax-Fmin)/k_0; %m
14 beta=0.25;
15 gamma=0.5;
16 T=sqrt(m/k_0);
17
18 h=0.0001;
19 t=0:h:4800;
20 n=length(t);
21
22 % Newmark
23
24 x2=zeros(n,1);
25 v2=zeros(n,1);
26 a2=zeros(n,1);
27 f_k2=zeros(n,1);
28
29 x2(1)=x_0;
30 v2(1)=v_0;
31 a2(1)=1/m*(-c*v_0-k_0*x_0);
32 aux1=0;
33 aux2=0;
34
35 Eploding=0;
36 Eextloading=0;
37 Edloading=0;
38 Ecloding=0;
39
40 Epunloading=0;
41 Eextunloading=0;
42 Edunloading=0;
43 Ecunloading=0;
44
45 for i=1:n-1
46     aguess=1;
47     loop=0;
48     k=k_0;
49
```

```

50  if (f_k2(i) ≥ Fmax) && (aux2==0)
51      aux1=1;
52      aux2=1;
53      k=-k_0;
54      save2(aux2)=i;
55  end
56
57  if (f_k2(i) ≤ Fmin) && (aux2==1)
58      aux1=2;
59      aux2=2;
60      k=k_1;
61      save2(aux2)=i;
62  end
63
64  if (f_k2(i) > FMAX) && (aux2==2)
65      c=2*sqrt(m*k_0);
66      aux1=3;
67      aux2=3;
68      k=k_1;
69      save2(aux2)=i;
70      tmax=t(i);
71      ffinal=T/100*tmax;
72  end
73
74  if (f_k2(i) ≤ Fmin) && (aux2==3)
75      aux1=4;
76      aux2=4;
77      k=-k_0;
78      save2(aux2)=i;
79  end
80
81  if (f_k2(i) ≥ Fmax) && (aux2==4)
82      aux1=5;
83      aux2=5;
84      k=k_0;
85      save2(aux2)=i;
86  end
87
88  if aux1==0
89      while loop==0
90          x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
91          v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
92          f_k2(i+1)=k_0*x2(i+1);
93          f_ext(i+1)=T/100*t(i+1);
94          a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
95          if abs(a2(i+1)-aguess)<0.001
96              loop=1;
97          else
98              aguess=a2(i+1);
99          end
100      end
101  end

```

```

102
103
104   if aux1==1
105       while loop==0
106           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
107           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
108           f_k2(i+1)=Fmax-k_0*(x2(i+1)-xmax);
109           f_ext(i+1)=T/100*t(i+1);
110           a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
111           if abs(a2(i+1)-aguess)<0.001
112               loop=1;
113           else
114               aguess=a2(i+1);
115           end
116       end
117   end
118
119   if aux1==2
120       while loop==0
121           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
122           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
123           f_k2(i+1)=Fmin+k_1*(x2(i+1)-xmin);
124           f_ext(i+1)=T/100*t(i+1);
125           a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
126           if abs(a2(i+1)-aguess)<0.001
127               loop=1;
128           else
129               aguess=a2(i+1);
130           end
131       end
132   end
133
134   if aux1==3
135       while loop==0
136           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
137           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
138           f_k2(i+1)=Fmin+k_1*(x2(i+1)-xmin);
139           f_ext(i+1)=ffinal-T/100*(t(i+1)-tmax);
140           a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+(ffinal-T/100*(t(i+1)-tmax)));
141           if abs(a2(i+1)-aguess)<0.001
142               loop=1;
143           else
144               aguess=a2(i+1);
145           end
146       end
147   end
148
149   if aux1==4
150       while loop==0
151           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
152           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
153           f_k2(i+1)=Fmax-k_0*(x2(i+1)-xmax);

```

```

154     f_ext(i+1)=ffinal-T/100*(t(i+1)-tmax);
155     a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+(ffinal-T/100*(t(i+1)-tmax)));
156     if abs(a2(i+1)-aguess)<0.001
157         loop=1;
158     else
159         aguess=a2(i+1);
160     end
161 end
162 end
163
164 if aux1==5
165     while loop==0
166         x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
167         v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
168         f_k2(i+1)=k_0*x2(i+1);
169         f_ext(i+1)=ffinal-T/100*(t(i+1)-tmax);
170         a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+(ffinal-T/100*(t(i+1)-tmax)));
171         if abs(a2(i+1)-aguess)<0.001
172             loop=1;
173         else
174             aguess=a2(i+1);
175         end
176     end
177 end
178
179 Pc(i+1)=v2(i+1)*m*a2(i+1);
180 Pp(i+1)=f_k2(i+1)*v2(i+1);
181 Pd(i+1)=c*v2(i+1)^2;
182 Pext(i+1)=f_ext(i+1)*v2(i+1);
183
184 if aux2==0 || aux2==1 || aux2==2
185     Eploding=Eploding+f_k2(i+1)*v2(i+1)*h;
186     Eextloading=Eextloading+f_ext(i+1)*v2(i+1)*h;
187     Edloading=Edloading+c*v2(i+1)^2*h;
188     Ecloding=Ecloding+Pc(i+1)*h;
189 else
190     Epunloading=Epunloading+f_k2(i+1)*v2(i+1)*h;
191     Eextunloading=Eextunloading+f_ext(i+1)*v2(i+1)*h;
192     Edunloading=Edunloading+c*v2(i+1)^2*h;
193     Ecunloading=Ecunloading+Pc(i+1)*h;
194 end
195 end

```

A.11 Load-Stop (Metastable)

```
1 % Time Integration
2 clear;
3 m=1;
4 k_0=4;
5 k_1=2;
6 c=2*sqrt(m*k_1);
7 x_0=0;
8 v_0=0;
9 Fmax=10; %N
10 Fmin=2; %N
11 FMAX=12;
12 xmax=Fmax/k_0; %m
13 xmin=xmax+(Fmax-Fmin)/k_0; %m
14 beta=0.25;
15 gamma=0.5;
16 T=sqrt(m/k_0);
17
18 h=0.0001;
19 t=0:h:4800;
20 n=length(t);
21
22 % Newmark
23
24 x2=zeros(n,1);
25 v2=zeros(n,1);
26 a2=zeros(n,1);
27 f_k2=zeros(n,1);
28
29 x2(1)=x_0;
30 v2(1)=v_0;
31 a2(1)=1/m*(-c*v_0-k_0*x_0);
32 aux1=0;
33 aux2=0;
34
35 Eploding=0;
36 Eextloading=0;
37 Edloading=0;
38 Ecloding=0;
39
40 Epunloading=0;
41 Eextunloading=0;
42 Edunloading=0;
43 Ecunloading=0;
44
45 for i=1:n-1
46     aguess=1;
47     loop=0;
48     k=k_0;
49
```

```

50  if (f_k2(i) ≥ Fmax) && (aux2==0)
51      aux1=1;
52      aux2=1;
53      k=-k_0;
54      save2(aux2)=i;
55  end
56
57  if (f_k2(i) ≤ Fmin) && (aux2==1)
58      aux1=2;
59      aux2=2;
60      k=k_1;
61      save2(aux2)=i;
62  end
63
64  if (f_k2(i) > FMAX) && (aux2==2)
65      c=2*sqrt(m*k_0);
66      aux1=3;
67      aux2=3;
68      k=k_1;
69      save2(aux2)=i;
70      tmax=t(i);
71      ffinal=T/100*tmax;
72  end
73
74  if (f_k2(i) ≤ Fmin) && (aux2==3)
75      aux1=4;
76      aux2=4;
77      k=-k_0;
78      save2(aux2)=i;
79  end
80
81  if (f_k2(i) ≥ Fmax) && (aux2==4)
82      aux1=5;
83      aux2=5;
84      k=k_0;
85      save2(aux2)=i;
86  end
87
88  if aux1==0
89      while loop==0
90          x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
91          v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
92          f_k2(i+1)=k_0*x2(i+1);
93          f_ext(i+1)=T/100*t(i+1);
94          a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
95          if abs(a2(i+1)-aguess)<0.001
96              loop=1;
97          else
98              aguess=a2(i+1);
99          end
100      end
101  end

```

```

102
103
104   if aux1==1
105       while loop==0
106           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
107           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
108           f_k2(i+1)=Fmax-k_0*(x2(i+1)-xmax);
109           f_ext(i+1)=T/100*t(i+1);
110           a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
111           if abs(a2(i+1)-aguess)<0.001
112               loop=1;
113           else
114               aguess=a2(i+1);
115           end
116       end
117   end
118
119   if aux1==2
120       while loop==0
121           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
122           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
123           f_k2(i+1)=Fmin+k_1*(x2(i+1)-xmin);
124           f_ext(i+1)=T/100*t(i+1);
125           a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
126           if abs(a2(i+1)-aguess)<0.001
127               loop=1;
128           else
129               aguess=a2(i+1);
130           end
131       end
132   end
133
134   if aux1==3
135       while loop==0
136           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
137           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
138           f_k2(i+1)=Fmin+k_1*(x2(i+1)-xmin);
139           f_ext(i+1)=0;
140           a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1));
141           if abs(a2(i+1)-aguess)<0.001
142               loop=1;
143           else
144               aguess=a2(i+1);
145           end
146       end
147   end
148
149   if aux1==4
150       while loop==0
151           x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
152           v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
153           f_k2(i+1)=Fmax-k_0*(x2(i+1)-xmax);

```

```

154         f_ext(i+1)=0;
155         a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1));
156         if abs(a2(i+1)-aguess)<0.001
157             loop=1;
158         else
159             aguess=a2(i+1);
160         end
161     end
162 end
163
164 if aux1==5
165     while loop==0
166         x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
167         v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
168         f_k2(i+1)=k_0*x2(i+1);
169         f_ext(i+1)=0;
170         a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1));
171         if abs(a2(i+1)-aguess)<0.001
172             loop=1;
173         else
174             aguess=a2(i+1);
175         end
176     end
177 end
178
179 Pc(i+1)=v2(i+1)*m*a2(i+1);
180 Pp(i+1)=f_k2(i+1)*v2(i+1);
181 Pd(i+1)=c*v2(i+1)^2;
182 Pext(i+1)=f_ext(i+1)*v2(i+1);
183
184 if aux2==0 || aux2==1 || aux2==2
185     Eploding=Eploding+f_k2(i+1)*v2(i+1)*h;
186     Eextloading=Eextloading+f_ext(i+1)*v2(i+1)*h;
187     Edloading=Edloading+c*v2(i+1)^2*h;
188     Ecloding=Ecloding+Pc(i+1)*h;
189 else
190     Epunloading=Epunloading+f_k2(i+1)*v2(i+1)*h;
191     Eextunloading=Eextunloading+f_ext(i+1)*v2(i+1)*h;
192     Edunloading=Edunloading+c*v2(i+1)^2*h;
193     Ecunloading=Ecunloading+Pc(i+1)*h;
194 end
195
196 end

```

A.12 Load-Stop (Bistable)

```
1  % Time Integration
2  clear;
3  m=1;
4  k_0=4;
5  k_1=2;
6  c=2*sqrt(m*k_1);
7  x_0=0;
8  v_0=0;
9  Fmax=10; %N
10 Fmin=-2; %N
11 FMAX=12;
12 xmax=Fmax/k_0; %m
13 xmin=xmax+(Fmax-Fmin)/k_0; %m
14 beta=0.25;
15 gamma=0.5;
16 T=sqrt(m/k_0);
17
18 h=0.0001;
19 t=0:h:4800;
20 n=length(t);
21
22 % Newmark
23
24 x2=zeros(n,1);
25 v2=zeros(n,1);
26 a2=zeros(n,1);
27 f_k2=zeros(n,1);
28
29 x2(1)=x_0;
30 v2(1)=v_0;
31 a2(1)=1/m*(-c*v_0-k_0*x_0);
32 aux1=0;
33 aux2=0;
34
35
36 for i=1:n-1
37     aguess=1;
38     loop=0;
39     k=k_0;
40
41     if (f_k2(i) >= Fmax) && (aux2==0)
42         aux1=1;
43         aux2=1;
44         k=-k_0;
45         save2(aux2)=i;
46     end
47
48     if (f_k2(i) <= Fmin) && (aux2==1)
49         aux1=2;
```

```

50     aux2=2;
51     k=k_1;
52     save2(aux2)=i;
53 end
54
55 if (f_k2(i)>FMAX) && (aux2==2)
56     c=2*sqrt(m*k_0);
57     aux1=3;
58     aux2=3;
59     k=k_1;
60     save2(aux2)=i;
61     tmax=t(i);
62     ffinal=T/100*tmax;
63 end
64
65
66 if aux1==0
67     while loop==0
68         x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
69         v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
70         f_k2(i+1)=k_0*x2(i+1);
71         f_ext(i+1)=T/100*t(i+1);
72         a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
73         if abs(a2(i+1)-aguess)<0.001
74             loop=1;
75         else
76             aguess=a2(i+1);
77         end
78     end
79 end
80
81
82 if aux1==1
83     while loop==0
84         x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
85         v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
86         f_k2(i+1)=Fmax-k_0*(x2(i+1)-xmax);
87         f_ext(i+1)=T/100*t(i+1);
88         a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
89         if abs(a2(i+1)-aguess)<0.001
90             loop=1;
91         else
92             aguess=a2(i+1);
93         end
94     end
95 end
96
97 if aux1==2
98     while loop==0
99         x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
100        v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
101        f_k2(i+1)=Fmin+k_1*(x2(i+1)-xmin);

```

```

102     f_ext(i+1)=T/100*t(i+1);
103     a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+T/100*t(i+1));
104     if abs(a2(i+1)-aguess)<0.001
105         loop=1;
106     else
107         aguess=a2(i+1);
108     end
109 end
110 end
111
112 if aux1==3
113     while loop==0
114         x2(i+1)=x2(i)+v2(i)*h+(0.5-beta)*aguess*h^2+beta*aguess*h^2;
115         v2(i+1)=v2(i)+(1-gamma)*aguess*h+gamma*aguess*h;
116         f_k2(i+1)=Fmin+k_1*(x2(i+1)-xmin);
117         f_ext(i+1)=0;
118         a2(i+1)=(1/m)*(-c*v2(i+1)-f_k2(i+1)+0);
119         if abs(a2(i+1)-aguess)<0.001
120             loop=1;
121         else
122             aguess=a2(i+1);
123         end
124     end
125 end
126
127 end

```

A.13 2 Mass System

```
1 clear;
2 m2=1;
3 m3=m2;
4 k_0=4;
5 K1=[1 10 100];
6 p=length(K1);
7 k_1=2;
8 c1=2;
9 c2=c1;
10
11 Fmax=10;
12 Fmin=-2;
13 FMAX=12;
14 xmax=Fmax/k_0;
15 xmin=xmax+(Fmax-Fmin)/k_0;
16 beta=0.25;
17 gamma=0.5;
18
19
20 h=0.001;
21 t=0:h:2000;
22 n=length(t);
23
24 v=0.01;
25
26 d2inicial=0;
27 d3inicial=0;
28
29 d2=zeros(p,n);
30 v2=zeros(p,n);
31 v3=zeros(p,n);
32 a2=zeros(p,n);
33 a3=zeros(p,n);
34 f_1=zeros(p,n);
35 f_2=zeros(p,n);
36
37 for l=1:p
38     k_2=K1(l)
39     v3(l,1)=v;
40     d2(l,1)=d2inicial;
41     d3(l,1)=d3inicial;
42     f_1(l,1)=k_0*(d2(l,1));
43     f_2(l,1)=k_2*(d3(l,1)-d2(l,1));
44     a2(l,1)=(1/m2)*(c2*(v3(l,1)-v2(l,1))-c1*v2(l,1)+f_2(l,1)-f_1(l,1));
45
46     aux1=0;
47     aux2=0;
48     for i=1:n-1
49         d3(l,i+1)=d3(l,1)+v*t(i+1);
```

```

50     loop=0;
51     a2guess=1;
52     v3(l,i+1)=v;
53     while loop==0
54         d2(l,i+1)=d2(l,i)+v2(l,i)*h+(0.5-beta)*a2(l,i)*h^2+beta*a2guess*h^2;
55         v2(l,i+1)=v2(l,i)+(1-gamma)*a2(l,i)*h+gamma*a2guess*h;
56         [f_1(l,i+1),aux1]=piecewisespring(k_0,k_1,0,d2(l,i+1),...
57             xmax,xmin,Fmax,Fmin,aux1,f_1(l,i));
58         f_2(l,i+1)=k_2*(d3(l,i+1)-d2(l,i+1));
59         a2(l,i+1)=(1/m2)*(c2*(v3(l,i+1)-v2(l,i+1))-c1*v2(l,i+1)+f_2(l,i+1)-f_1(l,i+1));
60         if abs(a2(l,i+1)-a2guess)<0.001
61             loop=1;
62         else
63             a2guess=a2(l,i+1);
64         end
65     end
66
67     R1(l,i+1)=-f_1(l,i+1);
68     R3(l,i+1)=c2*(v3(l,i+1))+f_2(l,i+1);
69 end
70 end

```

A.14 Piecewise function

```
1 function [F,aux] = piecewisespring(k_1,k_2,x_1,x_2,xmax,xmin,Fmax,Fmin,aux,f)
2 if f > Fmax && aux ==0
3     aux = 1;
4 end
5
6 if f<Fmin && aux == 1
7     aux = 2;
8 end
9
10 if aux ==0
11     F = k_1*(x_2-x_1);
12 elseif aux==1
13     F = Fmax-k_1*(x_2-x_1-xmax);
14 elseif aux == 2
15     F = Fmin + k_2*(x_2-x_1-xmin);
16 end
17 end
```

A.15 N Spring System

```
1 clear;
2 tic
3 N=2+1; %% number of piecewise springs plus 1 linear spring
4
5 m=1; %% mass [kg]
6 k_0=4; %% spring constant [N/m]
7 k_1=2; %% spring constant [N/m]
8 k_2=4; %% spring constant [N/m]
9 c=2; %% damping constant [Ns/m]
10
11
12
13 Fmax=10; %% maximum force of the piecewise spring [N]
14 Fmin=2; %% minimum force of the piecewise spring [N]
15 dmax=50; %% maximum value the last mass reaches before going back
16 xmax=Fmax/k_0; %% elongation of the maximum force [m]
17 xmin=xmax+(Fmax-Fmin)/k_0; %% elongation of the minimum force [m]
18 beta=0.25; %% parameter for the newmark time integration
19 gamma=0.5; %% parameter for the newmark time integration
20
21 velocity=0.01; %%velocity of the last mass [m/s]
22
23 h=0.01; %% time step
24 t=transpose(0:h:(dmax)*2/velocity); %% time domain
25 n=length(t);
26
27 f=zeros(N,n); %% internal force of each spring
28 R = zeros(N,1); %% reaction at the last node
29
30 d=zeros(N+1,n); %% displacement of each mass
31 v=zeros(N+1,n); %% velocity of each mass
32 a=zeros(N+1,n); %% acceleration of each mass
33
34 aguess=zeros(N+1,n); %% guess value for the newmark integration
35
36 v(N+1,1)=velocity;
37
38 %Matrices for the energy analysis
39 M = sparse(N+1,N+1);
40 C = sparse(N+1,N+1);
41 F = zeros(N+1,1);
42 for i=1:N+1
43     for j=1:N+1
44         if j==i
45             M(i,j)=m;
46             if i==1 || i==N+1
47                 C(i,j)=c;
48             else
49                 C(i,j)=2*c;
```

```

50         end
51     end
52     if j == i+1 || j == i-1
53         C(i,j)=-c;
54     end
55 end
56 end
57
58 Pint=zeros(n,1);
59 Pc=zeros(n,1);
60 Pext=zeros(n,1);
61 Pd=zeros(n,1);
62
63 aux = 0;
64
65 for i=1:n-1
66     if d(N+1,i) ≥ dmax && aux == 0
67         aux = 1;
68         maximum = i;
69     elseif d(N+1,i)<0 && aux == 1
70         aux = 2;
71     end
72     if aux == 0
73         d(N+1,i+1)=velocity*t(i+1);
74         v(N+1,i+1)=velocity;
75     elseif aux == 1
76         d(N+1,i+1)=dmax-velocity*(t(i+1)-t(maximum));
77         v(N+1,i+1)=-velocity;
78     elseif aux == 2
79         d(N+1,i+1)=0;
80     end
81     loop=0;
82     while loop == 0
83         for j=2:N
84             d(j,i+1)=d(j,i)+v(j,i)*h+(0.5-beta)*a(j,i)*h^2+beta*aguess(j,i+1)*h^2;
85             v(j,i+1)=v(j,i)+(1-gamma)*a(j,i)*h+gamma*aguess(j,i+1)*h;
86         end
87
88         for j=1:N-1
89             [f(j,i+1)]=piecewispring(k_0,k_1,d(j,i+1),d(j+1,i+1),...
90 xmax,xmin,Fmax,Fmin);
91         end
92         f(N,i+1)=k_2*(d(N+1,i+1)-d(N,i+1));
93
94         for j=2:N
95             a(j,i+1)=(1/m)*(f(j,i+1)-f(j-1,i+1)+c*(v(j+1,i+1)-v(j,i+1))-c*(v(j,i+1)-v(j-1,i+1)));
96         end
97
98         cont = 0;
99
100        for k=1:N+1
101            if abs(aguess(k,i+1)-a(k,i+1))<0.001

```

```

102         cont = cont + 1;
103         else
104             aguess(k,i+1)=a(k,i+1);
105         end
106     end
107
108     if cont == N+1
109         loop=1;
110     end
111 end
112
113 for p=1:N+1
114     if p==1
115         F(p)=-f(p,i+1);
116     elseif p==N+1
117         F(p)=f(p-1,i+1);
118     else
119         F(p)=f(p-1,i+1)-f(p,i+1);
120     end
121 end
122
123 R(i+1)=c*(v(N+1,i+1)-v(N,i+1))+f(N,i+1);
124
125 Pc(i+1)=(v(:,i+1)')*M*a(:,i+1);
126 Pd(i+1)=(v(:,i+1)')*C*v(:,i+1);
127 Pint(i+1)=(v(:,i+1)')*F;
128 Pext(i+1)=v(N+1,i+1)*R(i+1);
129 end
130
131 Ecloding=sum(Pc(1:maximum-1)*h);
132 Ecnloading=sum(Pc(maximum:end)*h);
133 Eintloading=sum(Pint(1:maximum-1)*h);
134 Eintunloading=sum(Pint(maximum:end)*h);
135 Edloading=sum(Pd(1:maximum-1)*h);
136 Edunloading=sum(Pd(maximum:end)*h);
137 Eextloading=sum(Pext(1:maximum-1)*h);
138 Eextunloading=sum(Pext(maximum:end)*h);
139 Ed=trapz(d(N+1,1:maximum-1),R(1:maximum-1))+trapz(d(N+1,maximum:end),R(maximum:end));

```