Comparative performance of linear and nonlinear control techniques applied to a permanent-magnet synchronous motor

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Permanent-magnet synchronous motors (PMSM) are widely used in industrial appliances, wind and aerospace applications and also in electrical vehicles. This motor presents better efficiency, higher power density, less weight and volume than other machines for the same power. The electrical characteristics of the motor are nonlinear and require the application of nonlinear control techniques (or linear ones by linearizing the dynamics) to regulate the speed, the position or the torque of the machine. Moreover, the encoder used to measure the position or the speed is quite expensive and observers are usually needed. In this project, we study different control techniques applied to a PMSM for regulating the speed of the machine. Matlab simulations are used to compare the behaviors obtained with the different controllers.

I. INTRODUCTION

The main problem we face when designing and studying different control techniques of the PMSM system [1] is its nonlinear behavior. The aim is to stabilize the system with the proper control so we consider two potential approaches, either use the Lyapunov direct method [2], a non-linear technique, or linearize the system via feedback so that we can design a control for a linear system [3].

The remainder of this paper is organized as follows. In section II, the mathematical model of the motor is described. In section III the tests that will be performed are introduced. In sections IV to VI, various control strategies are presented and the results obtained with each of them are discussed. In section VII, the different controllers are compared, in terms of their performance. Finally, in section VIII some conclusions are presented.

II. MATHEMATICAL MODEL OF THE PMSM

We will work in the rotor reference (dq) frame. The electrical and mechanical equations of the PMSM [4] are the following:

$$L_d \frac{di_d}{dt} = -R_S i_d + pL_q i_q \omega + u_d \quad (1)$$

$$L_q \frac{di_q}{dt} = -R_S i_q - pL_d i_d \omega - p\phi_v \omega + u_q \quad (2)$$

$$J\frac{d\omega}{dt} = \frac{3}{2}p\phi_v i_q + \frac{3}{2}p(L_d - L_q)i_d i_q - B\omega - T_L \quad (3)$$

where the inputs, variables and parameters are:

u_d and u_q	dq frame stator voltages
i_d and i_q	dq frame stator currents
L_d and L_q	dq axes inductances
p	number of pole pairs
ϕ_v	rotor flux
R_s	stator resistance
ω	speed
T_L	load torque
J	inertia
B	viscous friction coefficient

In this paper we will assume that the state variables i_d, i_q and w are available, whereas u_d and u_q are used as control variables. For the controller design, the load torque will be considered to be an unknown disturbance and the parameter values are shown in table I.

TABLE I. Parameters of the PMSM

$R_s = 1.2\ \Omega$	$\phi_v = 0.18\ V.s/rad$	$J = 0.006\ kg.m^2$
$p = 3$	$L_d = L_q = 0.011\ H$	$B = 0.0001\ N.m.s/rad$

III. TESTING METHODOLOGY

By searching for the equilibrium points of the system we can discern two degrees of freedom, allowing us to set the value of two state variables. We use $i_d^e = 0$, which is a typical current value for this kind of motor, and $w^d = 300 rad/s$ as our target speed in equilibrium.

Three tests will be carried out for each of the control designs. The first one will focus on the start-up performance of the system, assuming known invariant system parameters and that no torque is applied. The second one will assume a change in the B and J parameters no larger than one order of magnitude, most probable in a practical environment, once the system has stabilized. The third and final one will once again assume known invariant parameters but will introduce a constant torque value once the system has reached the steady state.

IV. LYAPUNOV-BASED CONTROL

The first approach we use to design the control is finding a Lyapunov function such that guarantees stability of the system in terms of the error with no need of solving any equation. If we consider that $L_d = L_q = L$, one can prove that the function

$$V = \frac{1}{2}\left(L\overline{i_d}^2 + L\overline{i_q}^2 + \frac{2}{3}J\overline{\omega}^2\right)$$

where $\overline{x} = x - x^*$, being x^* the equilibrium point, using the following proper control

$$\bar{u}_q = pLi_d^e\bar{\omega}, \quad \bar{u}_d = -\frac{2}{3}\frac{LBw^d}{\phi_v}\bar{\omega}$$

fulfils the condition. Of course, if it proves global stability it also demonstrates local stability of the system around the equilibrium point.

The test results are shown in figure 1. For the first test we see that the system takes around 5 seconds to reach the target speed, with no error in the steady-state. When running the second test we observe no error when changing J, but a discrepancy can be seen for a change in B. Lastly, an important error appears in the third test results when applying a torque of value 1 Nm.

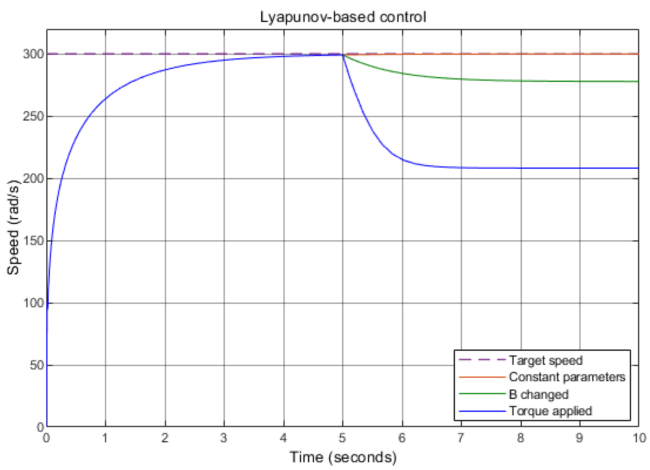

FIG. 1. Lyapunov-based control performance

V. FULL STATE CONTROL USING FEEDBACK LINEARIZATION

The other approach is to linearize the system using feedback linearization. We perform the following change of control variables

$$u_d = v_d - pL_q w i_q \tag{4}$$
$$u_q = v_q + pL_d w i_d + pw\phi_v \tag{5}$$

that allows us to decouple the electric dynamic equations (1) and (2) and make the nonlinear terms disappear so that we can design v_n using linear methods. To do that, one can assume that the electrical variables' dynamics are much faster than the mechanical one, so the control algorithm may be split into to parts: current control loop and mechanical control loop. For the fast dynamics, ω changes very slowly, while for the slow one the intensities rapidly achieve their stationary value so that we use $i_q = r_q$ as the input control. The equation (3) can be considered linear as well.

A. Control without integral action

Both electrical and mechanical loop have a decoupled and linear dynamics. The three controls v_d, v_q and r_q will have a typical feedback controller structure with a reference gain per a reference value plus a feedback gain per the corresponding state variable that will ensure a stable dynamic equation ending up in the desired reference value. In order to do that we have to place the poles p_d, p_q and p_w in a proper location. It is important to note that by construction of the solution, the real part of the slow pole p_w must be at least 5 times smaller in absolute value than the electric poles p_d and p_q so that they become the fast ones.

Setting the pole values to $p_d = p_q = -4000$ and $p_\omega = -40$, the first test shows that the system reaches steady state in close to 0.15s, without any error (see figure 2). Once again, a change in the J parameter does not effect the steady state of the system, but the error when B is changed is practically imperceptible. This change has to be of at least an order of magnitude higher to have a notable effect. On the contrary, when applying a torque of 10 Nm in the third test, the error in speed is still important.

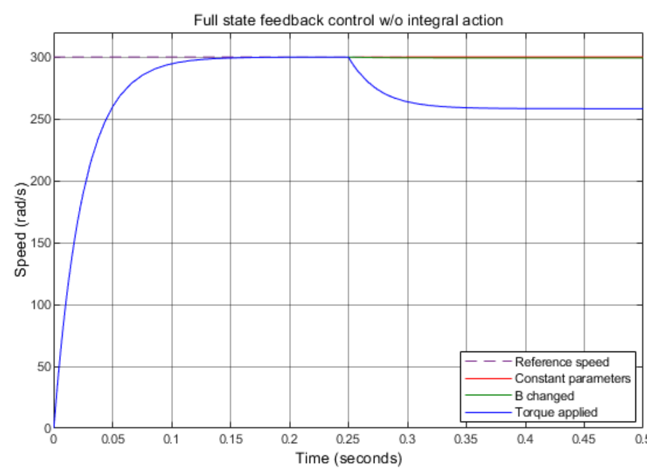

FIG. 2. FSF control w/o integral action performance

B. Control with integral action

The problem encountered using the previous implementation is due to the added disturbance, that is, the torque, as it affects the equilibrium point and consequently the term responsible for the linearization. In order to fix it, one could extend the mechanical loop while leaving the electrical loop the same, since the torque only appears in the mechanical equation. This means adding an integral action to the controller so that the error in ω is integrated over time. Since we have increased the range, we need a new condition, i.e to place a fourth pole to the controller.

This new pole is set to $p_z = -40.8$. Running the tests again (see figure 3), the settling time of the system is now slightly higher, of around 0.2s for the first one. In the second test, the error becomes not negligible, but nonexistent. Additionally, the target speed is reached when applying the third test conditions with a torque of 50 Nm.

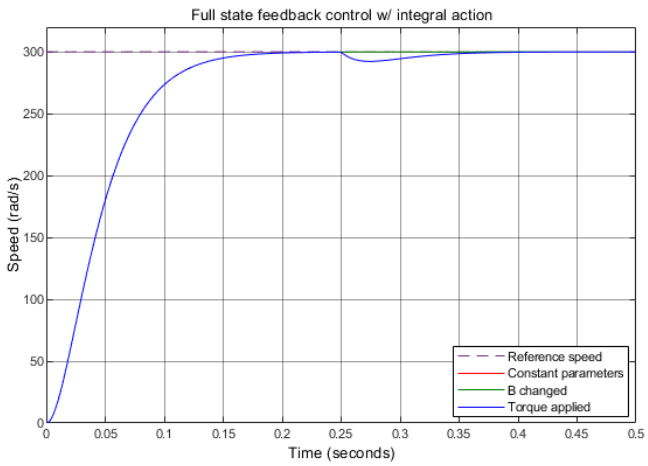

FIG. 3. FSF control w/ integral action performance

VI. OBSERVER DESIGN

Until now we have assumed that the entire state is known. However, it is not always the case, since especially the speed measurement might have a lot of noise or perturbations. Normally, the measurements that are available are the currents' (and sometimes the rotor position, but we are not taking it into consideration).

Starting from the same idea of dividing the system into two loops, the electrical control will be exactly the same as in the previous point, but in the mechanical, we will use the angular speed observed $\hat{w}$ instead of the measured. We will work with the dependence there is between the dynamics of i_q (2) and w (3) so that we can observe the angular velocity from the measurements of the intensity. It's important to mention that with the design of the observer we can not deal with the non-linear term appearing in the third equation due to $L_q \neq L_d$. It has an observability degree of 2 but since we already have i_q measured, we can design a minimum order observer. The observer gain is obtained with the eigenvalue assignment problem of the closed-loop error system characteristic polynomial with the desired one, which involves a single observer pole p_o.

We set $p_o = -100$. Starting with the constant target speed test, we can see in figure 4 that the behavior of the system is the same as with the previous controller, and the observed speed is apparently identical to the real speed. Regarding the second test, the change of both the B and J follows a behavior similar to the one observed when using the feedback controller without integral action, and therefore the error is negligible. However, when running the third test with a torque of 50 Nm, a large error appears in the steady-state speed. This is due to the fact that the observer does not take into account the effect of the torque in the mechanical equation, so the observed speed is not correct. Therefore one can see that the controller makes the observed speed reach the desired value, but not the real speed.

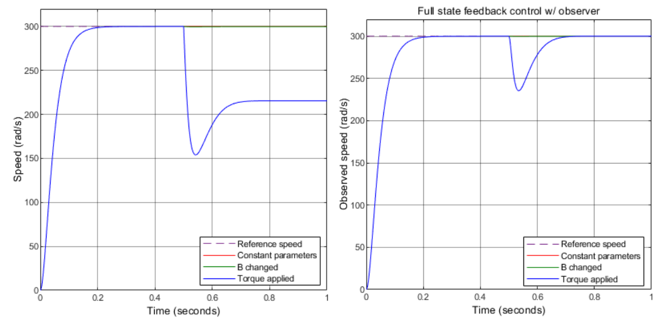

FIG. 4. Feedback control with observer performance

VII. FIELD ORIENTED CONTROL

Field oriented control (FOC), also called vector control, is a variable-frequency drive control method, that is, the AC motor speed is adjusted and controlled by varying motor input frequency and voltage. It helps us with the issue of entering a disturbance (the torque).

We use again the linearizing control (4) and (5) and take the same considerations as in the full state control

To proceed in a much more comfortable and appropriate way we will work with Laplace transformations as we are interested in the frequency domain and with the $\overline{X}$ variables.

The linear control designs are: $V_n(s) = k_P \overline{I}_n(s)$ and $R_q(s) = (\frac{k_I}{s} + k_P)\overline{W}(s)$. Now we want the poles of the closed loop transfer function to be located in such a way that the system is stable and meets the specifications we want. Notice that the integral action is adding a zero that must be placed far enough from the poles for the control to be valid. We impose a pole p_1 on the open left-half plane for the electrical loop controller and a pair of complex poles such as $\lambda_\pm = -\sigma \pm j w_d$ for the mechanical loop controller.

For the simulation, we set the current pole to the same value as the feed-back controller, $p_1 = -4000$. As for the mechanical poles, we choose the poles by taking into account the properties of a second-order system: the settling time $t_s = \frac{4}{\sigma}$ and the overshoot $M_p = e^{-\frac{\pi\sigma}{w_d}}$. On the one hand, we can choose w_d as small as we want, it might as well be zero. On the other hand, in principle, one could expect to get a better response of the system the larger σ is. However, as we approach the zero of the transfer function, located in $\alpha = -\frac{\sigma^2 + w_d^2}{2\sigma J + B}$, these formulas are no longer valid. In practice, a larger σ gives a shorter settling time, but for a certain value of σ (in our case, $\sigma > 400$, approximately) the overshoot starts to increase. We will then arbitrarily use this value to get the minimum possible overshoot.

TABLE II. Comparison of the controllers' performance

	Lyapunov	**Feedback**	**Integral**	**Observer**	**FOC**
Number of sensors	3	3	3	2	3
Global stability	Guaranteed	Guaranteed	Guaranteed	Guaranteed	Guaranteed
Steady-state error	Nonexistent	Nonexistent	Nonexistent	Nonexistent	Nonexistent
Dynamic response	Slow	Fast	Fast	Fast	Very fast
Overshoot	Null	Null	Null	Null	Noteworthy
Robustness to B	Very low	High	Very high	High	Very high
Robustness to torque	Very low	Low	Very high	Low	Very high
$L_d \neq L_q$	No	Yes	Yes	No	Yes

Having said this, we proceed with the tests, whose results are shown in figure 5. We start with the first test, for which we obtain a settling time of around 0.025s and an overshoot slightly below 50 rad/s (16%). There is no error in steady-state. The second test does not show any noticeable effect on the specified parameter variations. It is not until the variations in B are of an order of magnitude 4 times larger than its original value that we begin to see those effects. Lastly, we see that the system is also robust to a large load torque. Applying a torque of 300 Nm, we get a maximum error in the transient stage of 5 rad/s, and an overshoot of 1 rad/s, whereas no error is observed in the steady-state.

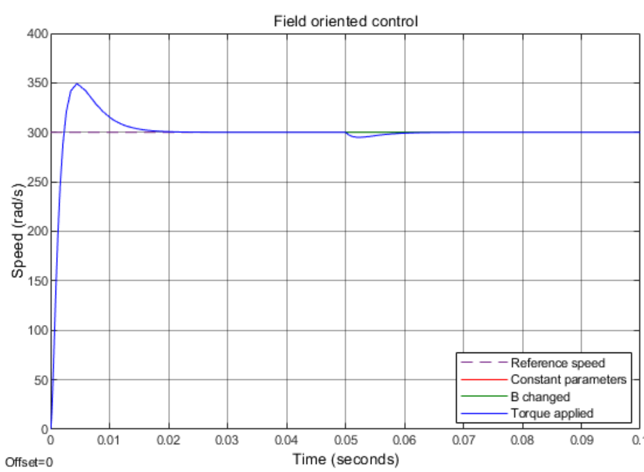

FIG. 5. FOC performance

VIII. CONTROLLER COMPARISON

We have seen that all the controllers provide global stability with no steady-state error when the system parameters are known and no torque is applied. However when this is not the case they present quite different behaviours.

These different behaviours are observed in terms of start-up performance, considering both time needed to reach stationary state and overshoot, robustness to variations of the B parameter (having obtained no visible effects given by the variation of J) and the load torque, and whether their analytical treatment allows for different *dq* axes inductances consideration or not.

Another major property is the number of sensors needed for the implementation of the controllers, which corresponds to the number of state variables that they assume to be known.

A summary of the controllers' properties is shown in table II.

IX. CONCLUSIONS

In this paper we have studied some approaches to the control of a PMSM non-linear system. We have shown that either using a proper Lyapunov function or feedback linearization techniques one can design a controller with global stability. Moreover, with the second approach it is possible to obtain a design that is robust to variations (or uncertainty) of the system parameters, as well as external disturbances such as an applied load torque.

The most convenient designs presented in this paper, assuming that all the state of the system is available, are the full-state feedback linearization with integral action in the mechanical loop and a field-oriented control. The first one has no overshoot, while the second one provides a much faster response.

Moreover, we have shown a possible observer design that can be used to estimate the speed of the motor, in such a way that the speed sensor is no longer needed. However, the results manifest poor robustness to an external disturbance.

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