A NUMERICAL AND ANALYTICAL WAY FOR DOUBLE-STEPPED PLANING HULL IN REGULAR WAVE

RASUL NIAZMAND BILANDI*, †, SIMONE MANCINI‡, ‡, ABBAS DASHTIMANESH*, SASAN TAVAKOLI‡ AND MARIA DE CARLINI‡

* Faculty of Engineering, Persian Gulf University, Bushehr, Iran; Post Code: 7516913817, rasool.niazmand@mehr.pgu.ac.ir
† Department of Industrial Engineering (DII), University of Naples “Federico II”, Naples, Italy, Post code 80125, simone.mancini@unina.it
‡ Department of Infrastructure Engineering, University of Melbourne, Parkville, Melbourne, Victoria, Australia stavakoli@student.unimelb.edu.au
♀ Eurisco Consulting Srls - R&D Company, Torre del Greco, Italy, Post code 80059; maria.decarlini@euriscoconsulting.com

Keywords: Seakeeping, 2D+T theory, Head sea, CFD simulation, Stepped hull design, Nonlinear mathematical model.

Abstract. The paper presents a comparison analysis between numerical method and nonlinear mathematical model for the prediction of the vertical motions of a double stepped planing hull in regular wave. The numerical method is to Unsteady Reynolds Averaged Navier-Stokes (URANS) equations solution via moving mesh techniques (overset/chimera), performed at different model speeds, wavelengths, and wave heights using the commercial software Siemens PLM Star-CCM+. Instead, the analytical solution is obtained using nonlinear mathematical model. The presented non-linear mathematical model is developed using a combined approach based on 2D+T theory, momentum theory, and linear wake profile. Under such assumption, the double stepped planing hull is divided into three planing surfaces, and hydrodynamic forces acting on each planing surface is found by extension of simulation of symmetric water entry of two-dimensional wedge section bodies. Then, each sub-problem is solved by extending the mathematical simulation of wedge penetrating water. Final vertical force and pitching moment are found and substituted in motion equation. The mathematical model is able to compute heave and pitch motion in calm water and regular waves. Results of numerical method and novel 2D+T analytical method are compared against each other.

1 INTRODUCTION

In recent year, research on motion of stepped planing hulls in wave is important for military, recreational, racing, and transportation applications. The design of stepped planing hull reduces the friction drag creating air cavity and new hydrodynamic forces. For this
reason this kind of hull is characterized by a complex dynamic, in particular in the wave condition. Hence, understanding the behavior of a stepped planing hull in a real seaway is critical for determining its performance. Accordingly, a numerical setup and mathematical tool are directly needed to predict motion of stepped planing hull in wave.

First work on prismatic planing hull in wave has been done with Fridsma [1, 2] experimental work and regression formulas. Fridsma's work considered sixteen model with different L/B, deadrise angle and sea state to calculated added resistance, hull motions, and the vertical impact accelerations. Fridsma found accelerations at the CG and bow and added resistance to have a nonlinear relationship with wave length. Based on the Fridsma works, Savitsky and Brown [3] developed an empirical equation for the measure of vertical acceleration and added resistance of planing hull in wave. Martin [4, 5] developed a linear frequency-domain model for prismatic planing hull in regular waves. Later, Zarnick [6], following the work of Martin, developed a nonlinear mathematical model based on time domain and 2D+T theory for planing hull in head sea. Keuning [7] further extended the basic model of Zarnick [6] to combine a formulation for the sinkage and trim of the ship at wider speed range. He calculated nonlinear added mass for each two dimensional section and wave exciting force in both (ir)regular waves. After Keuning [7], many of researchers similar to Hicks [8] and Alker [9] decided to modify the Zarnick model. Nevertheless, Alker [9] was able to develop power sea software. Then, the researchers were trying to develop the 2D+T theory method for other planing hull motions.

Later, Von Deyzen [10] extended mathematical models of Zarnick [6] and Keuning [7] to three degree of freedom of surge, heave and pitch motion in (ir)regular head sea. On the other hand, Sebastiani et al. [11] presented a mathematical model for roll, heave and pitch motions of planing hull. Then, Ghadimi et al. [12, 13] extended the Sebastiani et al. [11] model to four and six degree of freedom. Finally, Tavakoli et al. [14, 15] developed several mathematical models in order to computed coupled heave, pitch, roll motions, and planar mechanism motion of planing hulls. Then, the researchers were trying to develop mathematical model to reduce the drag-to-lift ratio for planing hulls in calm water and wave. As pointed out by Savitsky and Morabito [16], the stepped hull is characterized by a low hydrodynamic drag-to-lift ratio at high speeds (see Dashtimanesh et al. [17, 18], Niazmand Bilandi et al. [19-21], Di Caterino et al. [22] and De Marco et al. [23].

In this paper, numerical and analytical method are applied in order to investigated double stepped planing hull in regular head waves. The analytical method has been based on 2D+T theory, added mass theory, momentum theory, and linear wake theory in order to compute vertical motion (heave and pitch) of double-stepped planing hulls. In stepped planing hulls, the flow creates a dry area over the bottom body after passing through the steps. The correct prediction of the profile created by the steps will have a great effect on the accuracy of the existing equations. Hull of a double-stepped planing hull has been divided into three parts, and for each part a water entry problem has been simulated. To determine the forces for each planing surface, added mass is considered and it computed by using the momentum variation. To determine the wetted half-beam for each planing surface, two phases are considered. The analytical approach used here follows the works of Niazmand Bilandi et al. [21]. For numerical way, URANS methods has been used. Afterward, the two methods are compared with each other.
2 HULL GEOMETRY AND CONDITION

The seakeeping analysis using the two different approaches was done on a double stepped hull. In Figure 1 the double stepped hull is shown and in Table 1 are summarized the main non-dimensional data of the hull.

![Figure 1: Double stepped hull - side view.](image)

<table>
<thead>
<tr>
<th>Table 1: Main non-dimensional data of the hull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_B$</td>
</tr>
<tr>
<td>$L/B$</td>
</tr>
<tr>
<td>$B/T$</td>
</tr>
<tr>
<td>$L/\sqrt[3]{\Delta}$</td>
</tr>
<tr>
<td>LCG (% from transom)</td>
</tr>
<tr>
<td>$K_{xx}/B$, $K_{yy}/L_W$, $K_{zz}/L_W$</td>
</tr>
</tbody>
</table>

The seakeeping analysis was performed at a one speed, as listed in Table 2. The encountered frequency ($f_e$) has been evaluated according with the following equation:

\[ f_e = \frac{\sqrt{g}}{2\pi \lambda} \frac{V}{\lambda} \]

(1)

<table>
<thead>
<tr>
<th>Table 2: Tests summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed (Kn)</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>30</td>
</tr>
</tbody>
</table>

3 MATHEMATICAL MODEL

In this sub-section, motion in vertical plane of double-stepped planing hull has been considered. It is assumed that the boats under the trim angle $\tau$ is moving forward with velocity $u$ in regular head wave. Two coordinate systems of CG$\xi\eta\zeta$ and Oxyz are considered for the mathematical model (see Figure 1). For this motion, the governing equation for each planing surface is given as

\[ M_{\xi\zeta}(t) = F_{\eta}(t) = \sum_{j=1}^{3} F_{\eta_{j}}(t) + F_{\zeta_{j}}(t) \]

(2)
where, \( z_{CG} \) and \( \theta \) represent vertical (heave) and pitch motions respectively, \( M \) is mass of boat, \( I_\zeta \) is the mass moment of inertia about \( \eta \)-axis. Subscripts \( R_i \), \( H_i \) and \( W_i \) refer to restoring, hydrodynamic, and wave induced forces for each planing surface, respectively. The main problem is to find the forces acting to the double stepped planing hull using a previous mathematical model (Niazmand Bilandi [21]). To this end, similar to previous mathematical models, 2D+T method is used. So, three observation planes for each planing surface in the path of boat as shown in Figure 1. Using 2D+T method, three dimensional problem changes to 2D water entry problem and can be theoretically (added mass variation) solved. To calculate flow separation from the steps, linear wake profile has been considered (see Dashtimanesh et al. [17, 18], and Niazmand Bilandi et al. [19-21]).

\[
I_\zeta \ddot{\zeta}(t) = F_\zeta(t) = \sum_{i=1}^{3} F_{G_i}(t) + F_{H_i}(t) + F_{W_i}(t)
\]  

(3)

Figure 2: View of the coordinate system and 2D+T method for modeling of double stepped planing hull in regular head waves.

In the 2D+T theory, the forces acting on the wedge section entry water for each planing section can be found using momentum variation as in

\[
f_M(i) = -\{f_{w_i} + f_{ch_i} + f_{HS_i}\}; \quad (i = 1, 2, 3)
\]

(4)

where, \( f_{sh} \) is the hydrodynamic lift force due to momentum changes in fluid for each planing surface, \( f_{CD_i} \) is the viscous lift force due to cross-flow drag forces, and \( f_{HS_i} \) is the hydrostatic force for each planing surface. It should be noted that, forces and moments acting on the vessel are found using the added mass theory (see Equations 5 and 6). Formula for calculation mentioned forces can be found in previous work (Niazmand Bilandi [21]). So, equations for determination of heaving force and pitching moment of double stepped planing hull are obtained as shown in Equation 5 and 6, respectively.
By substituting the 3D force obtained in equations 5 and 6 into the motion equation (Equations 2 and 3), it is found to be:

\[ F(t) = \sum_{i=1}^{n} \left[ f_{x(i)} + f_{c(i)} \sin(\theta + \tau) d \xi + f_{m(i)} \cos(\theta + \tau) d \xi \right] = \sum_{i=1}^{n} Q_{a(i)} \cos(\theta + \tau) \sin(\theta + \tau) + \sum_{i=1}^{n} M_{a(i)} \cos^2(\theta + \tau) \sin(\theta + \tau) + \sum_{i=1}^{n} I_{a(i)} \sin(\theta + \tau) \sin(\theta + \tau) \]

By substituting the 3D force obtained in equations 5 and 6 into the motion equation (Equations 2 and 3), it is found to be:

\[ F(t) = \sum_{i=1}^{n} \left[ f_{x(i)} + f_{c(i)} \sin(\theta + \tau) d \xi + f_{m(i)} \cos(\theta + \tau) d \xi \right] = \sum_{i=1}^{n} Q_{a(i)} \cos(\theta + \tau) \sin(\theta + \tau) + \sum_{i=1}^{n} M_{a(i)} \cos^2(\theta + \tau) \sin(\theta + \tau) + \sum_{i=1}^{n} I_{a(i)} \sin(\theta + \tau) \sin(\theta + \tau) \]

where, \( M \) is mass, \( W \) is weight, \( T \) is thrust force, \( D \) is drag force for each planing surface, \( x_p \) and \( x_d \) are distance from CG to the center of action, and \( F_z' \) and \( F_p' \) signifies the total force without considering the defined parameters \( M_{a(i)}, Q_{a(i)} \) and \( I_{a(i)} \). These equations (7 and 8) are second order nonlinear differential. Hence, to solve the equations 6 and 7, the Runge–Kutta–Merson method has been applied.
4 NUMERICAL SIMULATION

A Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) to conjugate pressure and velocity field has been used to find the field of all hydrodynamic unknown quantities, and an Algebraic Multi-Grid (AMG) solver was used to accelerate the convergence of the solution. A segregated flow solver approach has been used for all simulations. The free surface has been modeled with the two phase VOF approach with a High-Resolution Interface Capturing (HRIC) scheme based on the Compressive Interface Capturing Scheme for Arbitrary Meshes (CICSAM). The wall function approach was used for the near wall treatment, in particular, the All wall $y^+$ model. It is a hybrid approach, that attempts to emulate the high $y^+$ wall treatment for coarse meshes, and the low $y^+$ wall treatment for fine meshes. For turbulent flows, $y^+$ values in the range of 30 - 300 are generally acceptable (CD-Adapco, [24]). The values of wall $y^+$ on the hull surface are shown in Figure 2. The Reynolds stress is solved by means of the $k$-$\omega$ SST turbulence model.

![Figure 3: View of the wall $y^+$ on hull in calm water test at 30 Kn.](image)

The URANS simulations were carried out using the technique of the Overset/Chimera grid mesh in order to take into account the hull motion in wave. This technique requires the definition of two different regions: a static region defined background region and a moving region, that incorporates the hull, defined Overset region (Figure 4). More details about this approach in the simulation of hull motion in wave and for calm water resistance simulation are available in Tezdogan et al. [25] and De Luca et al. [26] respectively. The computational domain dimensions are shown in Figure 4 and these dimensions are in compliance with the ITTC [27] recommendations and are close to similar studies, such as Tezdogan et al. [25].

About the time-step value for resistance computations in calm water, the time step size is determined by the following formula:

$$\Delta t = 0.005 - 0.01 \frac{L}{V}$$

where $L$ is the length between perpendiculares, in accordance with the related procedures and guidelines of ITTC [27]. For the prediction of vessel responses to incident regular waves, at least 100 time steps per encounter period were used, as recommended by ITTC [27].
5 RESULTS AND DISCUSSION

In this sub-section results analytical and simulation method for motion of double stepped planing hull in calm water and regular wave compared each other. The main aim of this stage of analysis is to verify the accuracy of the analytical method predicting the heave and pitch motions, CG acceleration, and added resistance of the double stepped planing hull in regular head wave. It should be noted that there is a lack in the literature because there are not available experimental data regarding the motions of double stepped planing hull in head sea.

5.1 Motions in regular wave

The amplitude ratio of heave, pitch and CG acceleration obtained by numerical simulation and analytical method using FFT analysis are shown in table 3. Comparing the time history plots, the amplitude ratio from the analytical and numerical results follows the same trend (see table 3). Comparison of analytical method against numerical study shows error in prediction of pitch, heave, and CG acceleration around 1%, 14%, and 16%, respectively.

### Table 3: results of pitch, heave, and CG acceleration amplitude by FFT analysis.

<table>
<thead>
<tr>
<th>z/A (analytical)</th>
<th>z/A (simulation)</th>
<th>θ/KA (a)</th>
<th>θ/KA (s)</th>
<th>a&lt;sub&gt;CG&lt;/sub&gt;/g (a)</th>
<th>a&lt;sub&gt;CG&lt;/sub&gt;/g (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.093</td>
<td>0.094</td>
<td>3.2</td>
<td>2.8</td>
<td>0.5</td>
<td>0.43</td>
</tr>
</tbody>
</table>
5.2 Added resistance

The resultant added resistance of the double stepped planing hull using the different methods are tabulated in Figure 6 and Table 4. The added resistance is defined as the difference between the steady and $0^{th}$-order harmonic component of resistance. Added resistance calculation is based on the following equation.

$$R_{AW} = R_{AW}^{\infty} - R_T$$  \hspace{1cm} (10)
$\bar{R}_{AW}$ is the mean resistance in waves and $R_T$ is the calm water resistance. Hence, the effective power due to the added resistance in wave has calculated as follow.

$$
\Delta P\% = \frac{P_{calm} - P_{wave}}{P_{calm}} \times 100 \tag{11}
$$

It is observed that the results of both methods (analytical and numerical methods) are relatively accurate (similar manner) to the ship motion predictions in regular wave. Verification of analytical method against numerical study for prediction added resistance in regular wave is around 3%. So, this analytical method was applied, in the preliminary design stage, in order to evaluate the performance in regular wave of several concurrent hull geometries. Hence, this method represents a very fast tool for the designers, in particular in the early design stage, when several configurations (hull shapes, different displacement conditions, different position of CG, etc.) have to be evaluated and compared quickly.

**Figure 6:** comparison of the added resistance – analytical method vs URANS simulations.

**Table 4:** Added resistance analysis of wave with H=0.75 (m) using different methods.

<table>
<thead>
<tr>
<th></th>
<th>Encountered Freq. (Hz)</th>
<th>Encountered Freq. by FFT (Hz)</th>
<th>Error (%)</th>
<th>$\Delta P_{E}$ (kW)</th>
<th>$\Delta P_{E}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star CCM+</td>
<td>1.780</td>
<td>1.753</td>
<td>1.53%</td>
<td>71.978</td>
<td>37.29%</td>
</tr>
<tr>
<td>Analytical method</td>
<td>1.780</td>
<td>1.753</td>
<td>1.53%</td>
<td>64.466</td>
<td>28.81%</td>
</tr>
</tbody>
</table>

6 CONCLUSIONS

In the current paper, a mathematical model and numerical simulation for vertical motion (heave and pitch) of a double stepped planing hull in head sea has been developed. The analytical method represents a very fast way compared with the URANS simulations. A verification of analytical method has been performed against the current numerical simulation study, and error in prediction of added resistance, pitch, heave, and CG acceleration are around 3%, 1%, 14%, and 16%, respectively.

This combined solution demonstrates a very fast way for the designers in order to analyze and compare a high number of concurrent hulls for calculation resonance region in head sea.
In the near future, will be attempt to solve other motions such as roll motion, sway motion, couple heave, pitch, and roll of double stepped planing hull in calm water and regular wave.

REFERENCES


[17] Dashtimanesh, A., Tavakoli, S., Sahoo, P., Development of a simple mathematical model for calculation of trim and resistance of two stepped planing hulls with transverse steps,


