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# A model that assesses proposals for infrastructure improvement and capacity expansion on a mixed railway network 

Francisca Rosell ${ }^{\text {a,* }}$, Esteve Codina ${ }^{\text {a }}$<br>${ }^{a}$ Universitat Politècnica de Catalunya, Statistics and Operations Research Depart. Jordi Girona 1, Campus Nord, C5, 08034 Barcelona, Spain.


#### Abstract

This paper presents a mathematical programming-based model for evaluating the impact on a mixed railway network from proposals for infrastructure improvement and capacity expansion that are oriented mainly toward increasing freight transportation. The authors have applied this model to extend elements of an existing mixed railway network, perform relatively less costly actions on the network, and to enhance capacity by adding new blocking/control systems at specific locations. These aspects are usually not taken into account in models for regional planning. Rather than a model whose sole focus is on railway capacity expansion, our approach combines capacity-expansion with network design. Because the way investments generate returns to the freight transportation system is of utmost relevance for these types of problems, this model is based on the efficient frontier between investment and operating costs. The model is tested on a regional network in the Mediterranean Corridor.


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## 1. Introduction

As the traffic of goods increases locally, nationally and internationally, challenges arise for reducing pollution and congestion on the roads. To this end, new focus is being placed on finding the most adequate means of transportation while making railway use more accessible. This work originates from the increasingly urgent need to improve the current situation for rail freight transport, which it pursues with a focus on upgrading infrastructures to adapt capacity expansion and have an adjusted rolling stock.

Railway infrastructure design is a well-known topic in operations research literature. Many research studies are oriented toward strategic designs of large-scale multiproduct networks that adopt important simplifications, as in Crainic and Laporte (1997) and Fernández et al. (2004). Other approaches are more tactical and evaluate different alternatives in disjointed scenarios (Crainic et al. (1990), Arnold et al. (2004)). In all of them, capacity conditions usually appear indirectly through penalizations to the cost functions or directly through a previously calculated external parameter.

[^0]Estimating railway capacity constitutes key element to planning at any level, not only for tracks but also for classifying terminals. A remarkable review of capacity concepts and evaluation methodologies is reported in Abril et al. (2008), Pachl (2015), and more recently in Bešinović and Goverde (2018). Different proposals for calculating capacity have been put forth, among which the standardization proposed by UIC (2013) is worth highlighting, as it presents a method for estimating capacity while following common international definitions, criteria and methodologies. It is worth mentioning the effect on capacity from signalling systems shown by UIC (2008), which is a study supported by the UIC to analyze the influence on capacity consumption from the European Train Control System (ETCS). Particularly in Burdett (2016), the author develops a model for expanding capacity using track subdivisions and optimal locations of blocking systems. In addition, it is essential to take into account how rolling stock is used, especially regarding freight trains, railway freight shunting, empty railcars, and the process of converting arriving freight trains into departing trains.

This paper presents a design model for improving railway networks by increasing the efficiency of goods transportation by rail in order to more greatly encourage its use. The main innovation in the study presented here is its approach, which seeks to determine an optimal trade-off between infrastructure investments and the operating costs from using rail freight transport. Classical decision variables for enhancing or not infrastructure are mainly conditioned by the prospective volume of merchandise to be transported by rail freight. This demand is reflected in the total number of trains and railcars that are needed to transport it, including empty railcar flow. Moreover, we apply hard capacity limits to tracks as a function of blocking time in order to reflect more realistically the shared use of the railway networks.

The remainder of the article is organized as follows. Section 2 presents a brief introduction to the main elements of the model. Then, Section 3 explains in detail the constraints and components of the objective function. Section 4 provides a case study to show the applicability of the model. Finally, Section 5 summarizes the main conclusions of the work presented in this paper.

## 2. Model introduction

The railway network will be represented using a directed graph $G=(N, A)$. Each track on the network will be represented by two links of the set of arcs $A, a=(i, j)$ and its opposite $-a=(j, i)$, and yards and diverting/crossing points will be represented by nodes $i \in N$. The subset of links $A^{-}=\{a \in A ; a=(i, j), i<j\}$ should also be considered. Each arc $a$ is assumed to have similar physical characteristics along its length, i.e., we can assume that trains run on each of them at a constant speed.

Yards are considered relevant elements at which railcars may be re-classified to assemble outgoing trains using incoming ones. The compatibility of track and loading gauges determines the configuration of these facilities. Terminals are considered a particular type of yard in which a net amount of products enters or leaves from the "external world" and, thus, new trains must be formed or, on the contrary, railcars are left empty and available to form new trains. The set of yards will be denoted by $Y$, while the set of terminals by $T \subseteq Y$. Diverting/crossing points consist simply of points at which several tracks may merge, where two opposite trains can cross or where overtaking between trains may occur. The set of paths between two terminals, or t-paths, is denoted by $R$, and $\Gamma$ is the set of paths between two yards, or y-paths. Given a t-path, it contains at least one y-path. Paths between yards are assumed to have homogeneous characteristics in terms of types of rolling stock allowed on the y-path, while t-paths do not necessarily preserve this condition. Given a yard $i \in Y, \Gamma_{i}^{+}$and $\Gamma_{i}^{-}$will be denoted as the sets of $i$-emergent and $i$-incident y-paths, respectively, while $\Gamma_{a}$ corresponds to the set of all y-paths that contain the arc $a \in A$. The (inelastic) demand for products is known in advance and will be associated with the set of possible origin-destination terminal pairs (OD pairs) on the network. The set of all OD pairs will be denoted by $W$. For a given OD pair $\omega \in W$, the set of $t$-paths joining its origin and destination terminal will be denoted by $R_{\omega}$. The amount of demand for products of type $q \in P$ corresponding to OD-pair $\omega \in W$ will be denoted by $\chi^{\omega q}$. Also considered will be special types of products that require a shorter delivery time, named "priority products" and denoted by $P^{H}$.

Let $\mathcal{V}$ be the set of railcar types. Each type of railcar $v$ can transport only a given subset of product types, named $P(v) \subseteq P$. Also, $P^{H}(v)$ denotes the set of priority products that can be transported for $v$-class railcars. $\Gamma(v)$ denotes the set of $y$-paths on which railcars in $v$-class may circulate, and which reflect that different gauges may possibly be operating on the network. Reciprocally, $\mathcal{V}_{\rho}(q)$ is the set of railcars that are compatible with y-path $\rho$ and which can
transport product $q$. Following the previous notation, $\Gamma_{i}^{+}(v)$ and $\Gamma_{i}^{-}(v)$ will represent, respectively, the sets of y-path emergent from $i \in Y$ and incident to $i \in Y$, which are compatible with $v$-class railcars. Trains are classified into different types in order to reflect the variety of trains that run on the network, and $K$ is the set of train types. Each type $\varsigma \in K$ is characterized by its average speed $\left(\sigma^{\varsigma}\right)$ and its usage (goods or passengers). $K_{M}$ is the subset of freight train types, while $K_{P}$ is the subset of trains from external and traversal flows in the area under study (passenger or other freight trains). Then we have $K=K_{M} \cup K_{P}$. Given that at a strategic or tactical level a detailed train schedule is not known in advance, a simplification must be stated. Here, the approach is as follows: trains are supposed to run on each track in groups that are assumed to follow a predetermined order during the running period. Let $\Theta(K)$ be the set of train groups. Each group $k \in \Theta(K)$ contains only one type of train, $\varsigma(k) \in K$. Different groups $k, k^{\prime}$ may be made up by trains of the same type. $k \rightarrow k^{\prime}$ represents that group $k$ precedes $k^{\prime}$ group. Variables and parameters used in the model will be introduced when required.

## 3. Model description

The primary goal of this work is to evaluate the impact on mixed railway transportation from proposals for infrastructure improvement and capacity expansion, with a focus mainly on increasing rail freight transport. In this situation, three elements are critical: first, the proper movement of goods; second, capacity limits on a rail network; and third, rolling stock requirements, which are basically defined by the use of railcars. Different components of the life-cycle costs are taken into account by a multi-objective analysis in order to analyze the impact of investments.

Goods transportation constraints. The way how goods are transported throughout the railway network defines the first group of constraints. Thus, (1) states that flows $\chi^{\omega, q}$, must be allocated on the corresponding OD-pair t-paths, where variable $h_{r}^{q}$ represents the total tons of $q$ transported using $r$. (2) establishes the relationship between paths and railcars, with variables $f_{\rho}^{v, q}$ being the total $v$-class railcars that runs on $\rho$ transporting product $q$, and $\alpha_{v q}$ is a parameter for the total tons of $q$ when transported on a $v$-class railcar. Ideally, railcars run along the entire network, and yards do not have spare units in stock. This is guaranteed by (3), with $f_{\rho}^{v, \emptyset}$ variables being the total empty $v$-class railcars that run on $\rho$. Variables $m_{\rho}^{\varsigma}$ denote the total $\varsigma$-type trains that run on $\rho$. Then, (4) reflects the maximum length of trains allowed on tracks, while (5) limits maximum weight on tracks, which is defined by the power of engines and track slopes. The parameters used are as follows: $\ell_{v}$ is $v$-class railcar length; $\ell_{\rho}$ is the maximum train length permitted on y-path $\rho$; $\alpha_{v}$ represents the $v$-class railcar tare; and $\alpha_{\rho \varsigma}$ is the maximum weight for $\varsigma$-type trains on y-path $\rho$. Let $h_{\rho}^{L}$ be the variable for the total tons of priority products transported by non-fast trains on the y-path $\rho$. Then (6) is applied jointly with a penalty factor for these variables included in the objective function, thus ensuring that capacity limitations in trains will be considered and prioritized products will be the first option for filling faster trains.

$$
\begin{array}{lr}
\chi^{\omega q}=\sum_{r \in R_{\omega}} h_{r}^{q} & \forall \omega \in W, \forall q \in P . \\
\sum_{r \in R(\rho)} h_{r}^{q} \leq \sum_{v \in \mathcal{V}_{\rho}(q)} \alpha_{v q} f_{\rho}^{v, q}, & \forall \rho \in \Gamma, \forall q \in P . \\
\sum_{\rho \in \Gamma_{i}^{-}(v)}\left(f_{\rho}^{\nu \emptyset}+\sum_{q \in P(v)} f_{\rho}^{v, q}\right)=\sum_{\rho \in \Gamma_{i}^{+}(v)}\left(f_{\rho}^{\nu \emptyset}+\sum_{q \in P(v)} f_{\rho}^{v, q}\right) & \forall i \in N, \forall v \in \mathcal{V} . \\
\sum_{v \in \mathcal{V}} \ell_{v}\left(f_{\rho}^{v, \emptyset}+\sum_{q \in P(v)} f_{\rho}^{v, q}\right) \leq \ell_{\rho} \sum_{\varsigma \in K_{M}} m_{\rho}^{\varsigma} & \forall \rho \in \Gamma . \\
\sum_{v \in \mathcal{V}}\left(\alpha_{v} f_{\rho}^{v, \emptyset}+\sum_{q \in P(v)} \alpha_{v q} f_{\rho}^{v, q}\right) \leq \sum_{\varsigma \in K_{M}} \alpha_{\rho \varsigma} m_{\rho}^{S} & \forall \rho \in \Gamma . \\
\sum_{v \in \mathcal{V}} \sum_{q \in P^{H}(v)} \alpha_{v q} f_{\rho}^{v, q}-\sum_{\varsigma \in K_{M}} \alpha_{\rho \varsigma} m_{\rho}^{S} \leq h_{\rho}^{L} & \forall \rho \in \Gamma . \\
h_{r}^{q}, h_{\rho}^{L} \in \mathbb{R}^{+}, \quad f_{\rho}^{v, q}, f_{\rho}^{v, \emptyset}, m_{\rho}^{S} \in \mathbb{Z}^{+} &
\end{array}
$$

Capacity constraints. In the present work, the capacity analysis is based on the concept of compression from UIC (2013), and our adjustment for fixed blocking systems on some sections to increase their capacity follows the approach in Burdett (2016). First, we introduce the elements to define minimum headway time between two consecutive trains as a function of blocking distance between two crossing points, as reflected in Figure 1. Figure 2 shows the components of the blocking time, following UIC definitions. Switching and reaction time are previously known values $\left(t_{s}\right.$ and $t_{r}$, respectively). Physical occupancy time $\left(t_{o t}\right)$ is equivalent to the blocking signal distance divided by the average speed


Figure 1. Blocking time between two trains. Source: Pachl (2015)


Figure 2. Components of blocking time. Source: UIC (2013)
of the train. Approaching time $\left(t_{a t}\right)$ is equivalent to the approaching distance divided by the average speed of the train. We can assume that approaching distance is proportional to physical occupancy distance, and let $\gamma$ be the factor of proportionality. Clearing time is equivalent to the length of the train divided by its average speed. We suppose that the length of the train is the maximum length allowed on the track.

As (Burdett, 2016, Lemma 2) shows, a section (a link in this work) of rail should be split into equidistant subsections (here blocking distance) in order to increase the maximum theoretical capacity. So, let $d_{a}$ and $b_{a}$ be the variables that denote, respectively, the length of the blocking distance and the total number of blocking signals on $a$. Also, we define the next previously known parameters: $\ell_{a}$ is the length of link $a ; \ell_{a}^{\max }$ is the maximum train length allowed on link $a$; and $\sigma^{\zeta(k)}$ is the average speed of $\zeta(k)$-type train. The next set of equations defines the minimum headway as a function of train characteristics, track length, and blocking time. Variables $\Delta t_{a}^{k}$ and $\Delta t_{a}^{k, k^{\prime}}$ are, respectively, the minimum headway between two consecutive trains of the same class and different class. (7) establishes the hypothesis that on each track two consecutive blocking signals are equidistant, while (8) and (9) set the equivalence of headways among trains of the same or different characteristics, respectively, in terms of blocking time.
$b_{a} \cdot d_{a}=\ell_{a}$
$\forall a \in A$
$\Delta t_{a}^{k}=t_{s}+t_{r}+\left(\gamma \cdot d_{a}+d_{a}+\ell_{a}^{\max }\right) / \sigma^{s(k)}+t_{s}$
$\Delta t_{a}^{k, k^{\prime}}=t_{s}+t_{r}+\left(\gamma \cdot d_{a}\right) / \sigma^{S^{\left(k^{\prime}\right)}}+\left(d_{a}+\ell_{a}^{\text {max }}\right) / \sigma^{s(k)}+t_{s}$

$$
\begin{equation*}
\forall k, k^{\prime} \in \Theta(K), k \rightarrow k^{\prime}, \forall a \in A . \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
\forall k \in \Theta(K), \forall a \in A . \tag{7}
\end{equation*}
$$

$b_{a} \in \mathbb{Z}^{+}, b_{a} \geq 1, \quad d_{a} \in \mathbb{R}^{+}$

We use the concept of train groups introduced in Section 2 to build capacity constraints. Figure 3 shows how trains run in groups, and the correspondence of the variables used to define the constraints. To simplify, we use the hypothesis that trains run in groups of trains with similar characteristics and, in the case of a single track, all trains run in one direction first, followed by all trains running in the opposite direction. These hypotheses are rather restrictive, and allow us to obtain only an upper bound for the theoretical capacity of tracks. Then, by trying to better adjust to realistic cases, and in following the recommendations of UIC (2013), a factor is applied over the maximum theoretical capacity in order to reduce it. $\mathcal{T}$ represents the reduced period that is allowed. To gather train movements, the following variables are defined: $\tau_{a}^{k}$ and $t_{a}^{k}$ are, respectively, the initial and ending running time for the train group $k \in \Theta(K)$ on the link $a$; while $r_{a}$ is a binary variable to denote whether the link is single or double; and $\delta_{a}^{k}$ is another binary variable for specifying whether or not trains of group $k$ run on link $a$. Parameter $\pi^{k}$ is a percentage that represents the portion of $\varsigma(k)$-train class that run in the $k$-group; $\theta_{a}^{\zeta}$ is the running time on link $a$ for $\varsigma$-train class; and $k_{0}$ and $k_{s}$ represent, respectively, first and last group of trains on link $a$. Then, (10) and (11) ensure that each freight train runs in one group. (12) and (13) link the initial time of a train group with its ending time and the total number of trains that run on it. (14) and (15) define the relationship between two consecutive train groups. Also, (15) ensures that initial and ending times are equal if no trains run on the group; and (16) guarantees that no train runs on the group if there is no difference between the initial and ending times. (17) applies only for single tracks, in order to verify that trains run in one direction first before running in the opposite. Last, (18) allows to include external flows (passenger or other freight trains) that are currently running (or are expected to run).


Figure 3. Time-distance diagram and minimum headway times between train paths on a block signal section


Figure 4. Yard congestion approximation.

$$
\begin{array}{r}
\forall k \in \Theta(K), \varsigma(k) \in K_{M}, \forall a \in A . \\
\forall S \in K_{M}, \forall a \in A . \\
\forall k \in \Theta(K), \forall a \in A . \\
\forall k \in \Theta(K), \forall a \in A . \\
\forall k, k^{\prime} \in \Theta(K), k \rightarrow k^{\prime}, \forall a \in A . \\
\forall k, k^{\prime} \in \Theta(K), k \rightarrow k^{\prime}, \forall a \in A . \\
\forall k \in \Theta(K), \forall a \in A . \\
\forall a \in A^{-} . \\
\forall k \in \Theta(k), \varsigma(k) \in K_{P}, \forall a \in A . \tag{18}
\end{array}
$$

$\delta_{a}^{k}, r_{a} \in\{0,1\}, \quad \tau_{a}^{k}, t_{a}^{k} \in \mathbb{R}^{+}, \tau_{a}^{k} \leq \mathcal{T}, t_{a}^{k} \leq \mathcal{T}$
Rolling stock requirements. Because the rolling stock is expensive, it is important to include an approximation of the total number of railcars required for composing the trains. This approach assumes that railcars run all along the network as if it was a closed circuit, and that all railcars needed for transporting all the demand must be inside the system. For a better approximation, a new graph $\mathcal{G}=(Y, \Gamma)$ is defined in order to properly state the relationship between the required rolling stock and the sojourn time in the cycles in $\mathcal{G}$. Thus, we can presume that railcars run by following cycles inside the network. Let $C$ be the subset of cycles in $(Y, \Gamma)$ that railcars are supposed to follow more likely, and let $C(\rho)$ be the subset of cycles in $C$ containing the y-path $\rho$. Equally, $\mathcal{V}(c)$ will denote the subset of railcar types that are compatible with all y-paths that form $c$. It will be assumed that the subset of cycles in $\mathcal{G}$ is such that $C(\rho) \neq \emptyset, \forall \rho \in \Gamma$. The variables used are as follows: $\lambda_{c}^{\nu}$ represents total $v$-class railcars for each cycle $c$ and for the total period $T ; w_{c}^{v}$ is the minimum number of $v$-type railcars required for covering cycle $c ; \tau_{c}$ represents running time on cycle $c$; and $\mu^{v}$ is the minimum total number of $v$-class railcars required to cover all services. Then, given a railcar class and a y-path, (19) connects railcars that run on cycles with full and empty railcars that run on the y-path. (20) defines a lower bound for the railcars of each class that are needed to cover a cycle. (21) groups all railcars of the same class, and estimates the total number of railcars of each class that are needed to cover all services, including a maintenance factor $\eta^{\nu}$, which is previously known.
$\begin{array}{lr}\sum_{q \in P} f_{\rho}^{v, q}+f_{\rho}^{\nu,( }=\sum_{c \in C(\rho)} \lambda_{c}^{v} & \forall \rho \in \Gamma, \quad \forall v \in \mathcal{V} . \\ w_{c}^{v} \geq \tau_{c} \cdot \lambda_{c}^{v} & \forall c \in C, \forall v \in \mathcal{V} . \\ \sum_{c \in C} w_{c}^{v}=\mu^{v} \cdot \eta^{v} & \forall v \in \mathcal{V} . \\ \lambda_{c}^{v}, \tau_{c}, w_{c}^{v}, \mu^{v} \in \mathbb{R}^{+} & \end{array}$
Aditional constraints. Usual location/allocation constraints should be included in the model. If a yard does not exist, neither can links exist that enter or exit from that yard. Trains run only on existing links. One link exists if and only if its opposite exists, and that the condition of being a single or double track is the same for each link and its opposite.

Replacing non-linearities. Notice that some of the constraints are non-linear. Specifically, capacity constraints (7), (12), (13), and (14), and rolling stock constraint (20). (12) is easy to linearise using common techniques for these type of conditions. (7), (13) and (14) are the product of a bounded integer and a non-negative continuous variable. Here, it is possible to reformulate them as linear constraints using additional binary variables. First, the integer variable must be decomposed as a sum of products of power-of-two terms multiplied by a binary variable. Then, the standard techniques for linearising the product of a binary variable and a continuous variable can be applied. In contrast, (20) needs another approximation. In this case, we can first see that variable $\tau_{c}$, which represents the cycle $c$ run time, depends on a deterministic run time along the tracks and the waiting time on yards, $\tau_{i}$, which, in turn, is depending on yard congestion. From that, $\tau_{i}$ can be approximated by an increasing stepwise function, depending on the total trains being handled and yard capacity. Figure 4 shows an example for a yard $i$, where $\hat{B}_{i}^{0}, \hat{B}_{i}^{1}, \hat{B}_{i}^{2}$ are the total number of handled trains per hour and track, while $\bar{g}_{i}$ is an integer variable that represents the size of the yard in terms of the number of tracks. $b_{i}^{0}, b_{i}^{1}, b_{i}^{2}, b_{i}^{3}$ are time congestion levels. These levels can be established by using, for example, simulation or surveys. Then, $\tau_{i}$ can be decomposed as a sum of products (one for each level of congestion) of one binary variable multiplied by the corresponding level of congestion and thus, for each of the new products resulting, the previous technique can be applied in order to linearise the product of a binary variable and a continuous variable.

Life cycle costs involved. The life-cycle costs associated with building investments, maintenance and operational development are now formulated. Binary variables $g_{i}$ and $z_{a}$ denote respectively, whether or not yard $i$ and track $a$ are to be built, while $\bar{g}_{i}$ is a non-negative integer variable for representing the size of yard $i \in Y$ in terms of the number of tracks. To estimate the monetary cost of each component, the following parameters are defined: $s_{i}$ and $\bar{s}_{i}$ correspond (respectively, fixed and per track) to building or maintenance costs for yard $i ; s_{a}$ and $\tilde{s}_{a}$ are (respectively, fixed and additional - if double) the building or maintenance costs of a track on link $a \in A^{-} ; S_{a}$ is the maintenance cost per blocking signal on link $a \in A ; \bar{c}_{i} / \hat{c}_{i} / \tilde{c}_{i}$ are the inbound/outbound/reclassification costs at yard $i ; c_{\rho}^{S}$ is the travel cost for $\varsigma$-type trains on y-path $\rho ; D_{\rho}$ is a penalty for prioritized products on non-priority trains on y-path $\rho$; and $e_{v}$ is the investment or maintenance cost per railcar. Thus, (22) reflects capital investment in infrastructure (yards, diverting/crossing points, tracks and signals), while (23) gathers the operating costs to yards from handling trains and railcars. Lastly, (24) is related to run time costs and rolling stock maintenance, and it is composed of three elements: a) railroad cost for goods transport, handling and classification (monetary and/or travel time and/or delay costs) on tracks and in terminals/yards; b) penalty cost for tons of prioritized products transported on non-fast trains; and c) initial investment and maintenance costs for rolling stock, depending on the type of railcar.
$\sum_{i \in N} s_{i} \cdot g_{i}+\sum_{i \in Y} \bar{s}_{i} \cdot \bar{g}_{i}+\sum_{a \in A^{-}}\left(s_{a} \cdot z_{a}+\bar{s}_{a} \cdot r_{a}\right)+\sum_{a \in A} S_{a} \cdot b_{a}+$
$\sum_{\varsigma \in K_{M}} \sum_{i \in Y}\left(\bar{c}_{i} \sum_{\rho \in \Gamma_{i}^{-}} m_{\rho}^{\varsigma}+\hat{c}_{i} \sum_{\rho \in \Gamma_{i}^{+}} m_{\rho}^{\varsigma}\right)+\sum_{v \in \mathcal{V}} \sum_{q \in P(v)} \sum_{i \in Y} \tilde{c}_{i}\left(\sum_{\rho \in \Gamma_{i}^{-}(v)} f_{\rho}^{v, q}+\sum_{\rho \in \Gamma_{i}^{+}(v)} f_{\rho}^{v, q}\right)+$
$\sum_{\rho \in \Gamma} \sum_{\varsigma \in K_{M}} c_{\rho}^{S} \cdot m_{\rho}^{\varsigma}+\sum_{\rho \in \Gamma} \sum_{q \in P^{H}} D_{\rho} \cdot h_{\rho}^{L}+\sum_{v \in \mathcal{V}} e_{v} \cdot \mu_{v}$.

Pareto efficiency analysis. A good way to find out how investments generate returns in the form of freight transportation costs is to analyze points on the efficient frontier of investment and operational costs. In our model, a bi-objective function $f=\left(f_{0}, f_{1}\right)$ is considered, where $f_{0}$ comprises the infrastructure investments given in (22), and $f_{1}$ comprises the operating costs in yards that are expressed in (23), and the travelling costs and rolling stock investments that are defined by (24). Then, the efficient points will be found by solving the following bi-objective problem, with $0 \leq \beta \leq 1$ :

$$
\begin{equation*}
\min _{x \in X} \beta \frac{f_{0}(x)}{f_{0}\left(x_{1}^{*}\right)-f_{0}\left(x_{0}^{*}\right)}+(1-\beta) \frac{f_{1}(x)}{f_{1}\left(x_{0}^{*}\right)-f_{1}\left(x_{1}^{*}\right)} \tag{25}
\end{equation*}
$$

where $x_{0}^{*}$ is a solution of $\min _{x \in X} \beta \cdot f_{0}(x)+(1-\beta) \cdot f_{1}(x)$ with $\beta \approx 1$, and $x_{1}^{*}$ is a solution of $\min _{x \in X} \beta \cdot f_{0}(x)+(1-\beta) \cdot f_{1}(x)$ with $\beta \approx 0$. In these problems, $X$ represents the feasible set defined by the sets of constraints (1) - (21), $x$ is the vector of the variables in our model, and $f_{0}\left(x_{0}^{*}\right), f_{0}\left(x_{1}^{*}\right), f_{1}\left(x_{0}^{*}\right), f_{1}\left(x_{1}^{*}\right)$ are used to rescale functions $f_{0}, f_{1}$. Obtaining the complete set of efficient points can be very difficult because we must solve problem (25), which is a mixed-linear
integer problem. Instead, we choose a limited subset of values for $\beta$ in our analysis of the case study in Section 4. Graphically, the Pareto efficient frontier will be represented approximately by both values of $f_{0}$ and $f_{1}$, from the chosen set of values for $\beta$, as can be seen in Figure 5.

## 4. Results and discussion

This case study is based on the Catalan region of the Mediterranean Corridor, which is one of the nine railway corridors defined by the EU for promoting the use of trains to sustainable transport passengers and goods. The covered area extends from the Spanish freight terminals in Valencia and Zaragoza to those on the French border in Le Boulou (Perpignan) and Portbou. The actual dimensions of the tested rail network are: 55 tracks, of which 23 are to be built; 4,387 kilometers, of which 2,572 are to be built; and 23 classification yards, of which 14 are to be built. The time period is 18 hours. The amount of products transported is 20,300 tons and 1,850 urban cars, calculated from the total goods transported by train in the analyzed area during 2016, and divided by the number of work days per year.

The model was implemented using Python 3 and CPLEX V12.7. Tests were carried out on a R5500 workstation using Intel ${ }^{\circledR}$ Xeon ${ }^{\circledR}$ CPU 5645 with 2.40 GHz and 48 Gb RAM. Two different groups of tests were executed to check the model and its efficiency. The first considers that no infrastructure exists and that the railway network must be created ex-novo; while the second tests an existing infrastructure with new elements or modifications added. In each of these groups, runs were the result of applying different values for $\beta$.

Table 1 summarizes the results for some experiments. All costs are expressed in thousands of monetary units. Each row in the table corresponds to a test, while the meaning of the columns is as follows. First, the case identification and the $\beta$ value used appear on the left. The next three columns are: the GAP percentage for the best feasible solution found; the total CPU time required in seconds; and the total life-cycle costs for the best feasible solution. Then, the next six columns show this total cost divided into different concepts: infrastructure cost investments and maintenance (yards - Infr. Y, tracks - Infr.T. and signals - Infr.S.); operating costs (costs related to the operation of trains) (Op.Tr.); the penalty cost for transporting priority products using non-fast trains (Prior); and rolling stock investment and maintenance (Wag). The five last columns show some indicators of how costs are distributed: column " T " shows the percentage of kilometers of new tracks built of the total kilometers of candidate tracks to be built; column "DT" shows the percentage of new double tracks of the total track kilometers built; column "C" shows the percentage of tracks of the total number of tracks built and with usage greater than $55 \%$; column " P " shows the total tons, in thousands, of priority products transported by non-fast trains; and, finally, column "W" shows the total number of railcars needed to cover all transportation.

Table 1. Some results of test examples

| Case | $\beta$ | GAP | CPU | Total | Infr.Y. | Infr.T. | Infr S | Op.Tr. | Prior | Wag. | T | DT | C | P | W |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NMV2 | 0.2 | 3.7 | 5,030 | $1,007,324$ | 227,799 | 551,136 | 7,387 | 205,964 | 2,011 | 13,026 | 60 | 23 | 75 | 2.0 | 416 |
| NMV6 | 0.6 | 2.4 | 5,019 | $1,001,082$ | 217,481 | 551,136 | 7,363 | 206,104 | 1,860 | 17,139 | 59 | 23 | 47 | 0.9 | 547 |
| NMV9 | 0.9 | 1.9 | 5,000 | $1,000,054$ | 217,481 | 547,079 | 8,408 | 206,514 | 3,097 | 17,475 | 57 | 20 | 67 | 1.1 | 557 |
| UMV2 | 0.2 | 5.0 | 5,000 | 551,498 | 120,584 | 203,404 | 7,071 | 206,748 | 1,353 | 12,338 | 43 | 83 | 75 | 3.6 | 393 |
| UMV6 | 0.6 | 3.6 | 5,018 | 542,188 | 108,413 | 203,404 | 6,804 | 204,973 | 2,378 | 16,215 | 43 | 83 | 78 | 4.1 | 520 |
| UMV9 | 0.9 | 2.5 | 5,006 | 543,413 | 107,884 | 202,345 | 6,804 | 206,629 | 2,738 | 17,013 | 43 | 83 | 76 | 3.3 | 542 |

Figure 5 graphically represents the relationship between infrastructure and operating costs for both test groups. The left vertical axis shows the infrastructure costs for the experiments in which rail infrastructure is intended to be upgraded ("UM" tests), while the right vertical axis displays the infrastructure costs for the ex-novo tests ("NM" tests). Only a limited number of points has been obtained, and this does not allow detecting probable unsupported points or discontinuities in the frontier. Tests "UM" and "NM" reveal points that indicate a "reasonable value" for the minimum investment in infrastructure, whereas any investments above this reasonable value barely improve rail operating costs in any significant way. As Table 1 shows for these tests, infrastructure costs and operating costs vary within a relatively narrow interval, no matter which value $\beta$ takes. In contrast, the costs that are more sensitive to variations in the value of $\beta$ are rolling stock costs (directly related to the number of railcars needed for transportation) and penalty costs for priority products transported on non-fast trains. This fact reflects that even though investments appear to be sufficient, operating costs may depend on which elements receive the investments. Furthermore, it is notable that neither the


Figure 5. Approximate Pareto frontier UM and NM tests
operating costs, nor the number of railcars needed for rail transportation, nor the number of high usage tracks depend on the infrastructure being built from scratch or being upgraded from an existing network. This result may mean that the base infrastructure in the scenario of the upgrade is a good starting point. With some enhancements, the operating costs that result from using the improved network are similar to those costs obtained in case of having an ad-hoc infrastructure.

## 5. Conclusion and future research

A mathematical programming-based model has been developed and tested with the aim of evaluating the impact of infrastructure improvements and capacity expansion, specifically when applied to a mixed railway network. The problem presented is a multi-objective minimization problem, where each of the objectives corresponds to: a) cost related to infrastructure investments and maintenance; and b) operating costs. An approximate Pareto efficiency analysis was applied to determine a trade-off solution between infrastructure costs and operating costs. A set of experiments allowed us to validate the model, using as an example the Catalan region of the Mediterranean Corridor.

In this model it is assumed that traffic flows obey centralized decisions and that a single operator establishes which services will be appropriate for transporting specific freight flows; considering the effect of several operators competing under some possibly regulated conditions could be a potential future contribution. In addition, the effects on the passenger network is taken into account here only in the constraints by fixing some variable values. A better approximation to passenger flows and the impact that different train speeds have on the rail network may be another line of future research.

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[^0]:    * Corresponding author.

    E-mail address: francisca.rosell@upc.edu

