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Marangoni Ducts for Energy Harvesting

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The possibility for harnessing energy from thermal or concentration gradients by the use of ducts lined with hydrophobic microcavities is discussed. Utilizing a physical simplified model an expression for the attainable pumping power from this approach was derived.

Keywords. Thermal energy harvesting; Marangoni mass transfer

I. INTRODUCTION

The urgency of taking better advantage of all the forms of energy has resulted into a growing interest for harvesting energy from thermal sources. Latent heat, [1], thermomagnetic and magnetic materials, [2], new pyrolytic reactors, [3], thermal storage from nuclear reactors, [4], thermal shocks, [5], or industrial wastes, [6] are just some of the myriad of sources being currently investigated. Current thermal converters are based mostly in the thermoelectric effect or the induction of convection currents. At the same time and in other topic of research, Marangoni convection induced by thermal or concentration gradients has been researched in several aspects in the last decade, as for example, as mechanism for heat transfer enhancement form condensations [7]-[15]; reverse osmosis, [16]; distribution of suspended radioactive particles, [17]; or microfluidic application [18], just to name a few. The thermal dependence of surface tension open up the possibility to add Marangoni effect to the portfolio of thermal energy harvesters as will be discussed.

A. Marangoni ducts by using microcavities

Despite that the idea of thermocapillary-driven fluid flow within microchannels for micro pumps by using walls lined with hydrophobic microcavities can be traced back to the early work of Baier et al (2010), [19], however it was only recently (2018), [20], when a proof of concept for a microfluidic channel was presented. The basis behind the idea is conceptually straightforward. By means of microcavities along the wall it is possible to introduce air-fluid interfaces which by applying a thermal gradient across a Marangoni thermocapillary stress appears owing to the thermal dependence of the surface tension with temperature. The object of this work was a first assessment for the use of ducts with walls deliberately lined with air-filled hydrophobic microcavities for harvesting energy from thermal gradients or even from concentration gradients.

II. MATERIALS AND METHODS

Consider two regions with different temperatures or concentrations and separated by an air-filled cavity as depicted in Fig. 1. Because the temperature or concentration gradient along the free interface air-fluid and the thermal or concentration dependence of surface tension, a Marangoni stress appears which pulls the fluid from the region of low surface tension towards the region of high surface tension. It is desired to know what is the pumping power resulting from this simple configuration.

To begin with, we can divide the cavity region into a fluid and air region and separated by the interface and dynamically connected by the boundary conditions. In the air region (left region in Fig. 1) the air is trapped; and the fluid region (right region in Fig. 1) a fluid flows parallel to the interface and the wall.
For the air region, if it is allowable to assume that the interface normal velocity component $u_x$ of the velocity $u$ can be neglected in comparison with the velocity parallel to the interface $u_z$, i.e., $u \approx u_z$ and that the effect of the cavity walls returning backward the air in the convective loop is causing a kind of Couette flow with the appearance of a constant pressure drop $\frac{dp_a}{dz}$ along the $z$-axis, then the component Navier-Stokes momentum equations for an incompressible (for our regime of velocities) and viscous gas in steady state motion written in Cartesian co-ordinates is given by, \[ dp_a \frac{dz}{dz} = \mu_a \frac{d^2 u}{dx^2} \] (1)

where the subscripts $a$ stands for air, $p$ is pressure, $\mu$ dynamic viscosity, $u$ velocity, $z$ the co-ordinate parallel to the interface and $x$ the normal co-ordinate (See Fig. 1). By solving Eq.(1) and considering the pressure gradient constant, we obtain the velocity profile as

$$u = c_1 + c_2 x + \frac{1}{2 \mu_a} \frac{dp_a}{dz} x^2$$ (2)

where $c_1$ and $c_2$ are constants to be determined by the boundary conditions. For the fluid region, the velocity $v$ is parallel to the interface and wall and then $v \approx v_z$, and proceeding as before we obtain a similar solution

$$v = c_3 + c_4 x + \frac{dp_f}{dz} x^2$$ (3)

where, the subscript $f$ stands for fluid and is referring the central or main fluid. The constants $c_3$ and $c_4$ are to be determined also by the boundary conditions.

For a Cartesian system centered at the interface air-liquid as is shown in Fig. 1, we have the following set of boundary conditions:

- Zero slip conditions

$$u(x = -h) = 0$$ (4)

$$v(x = b) = 0$$ (5)

- Mass continuity inside the cavity (convective loop)

$$\int_{-h}^{0} u \, dx = 0$$ (6)

- Continuity at the interface

$$u(x = 0) = v(x = 0)$$ (7)

- Continuity of the tangential stress at the interface which implies that viscous tensor stress equates the Marangoni stress

$$\mu_f \partial_x v|_{x=0} + \mu_a \partial_x u|_{x=0} = -\partial_z \sigma$$ (8)

where $\sigma$ is the liquid-gas surface tension. By applying systematically the boundary conditions, Eq.(4)-Eq.(8) into Eq.(2) and Eq.(3) we obtain for the velocity profile of the liquid

$$v = \partial_z \sigma \frac{(b - x)}{\mu_f} \left[ 1 + 4 \frac{\mu_a}{\mu_f} \frac{b}{h} \right] - \frac{1}{2 \mu_f} \frac{dp_f}{dz} (bx - x^2)$$ (9)

If the size of the duct is similar than the size of the cavity, i.e., $h \approx b$ and with $\mu_a \ll \mu_f$ for an air filled cavity, Eq.(9) simplifies as

$$v = \frac{\partial_z \sigma}{\mu_f} (b - x) - \frac{1}{2 \mu_f} \frac{dp_f}{dz} (bx - x^2)$$ (10)

and the mean fluid velocity defined as

$$\bar{v} = \frac{1}{b} \int_{0}^{b} v \, dx$$

is

$$\bar{v} = \frac{b \partial_z \sigma}{2 \mu_f} - \frac{1}{12} \frac{\mu_f}{\mu_f} \frac{dp_f}{dz}$$ (11)
If the length of the duct is similar than its width, i.e., then \( \frac{\Delta p_f}{\Delta z} \approx \Delta \sigma \), and \( \partial z \sigma \approx \Delta \sigma \) yielding

\[
\bar{v} = \frac{\Delta \sigma}{2 \mu_f} - \frac{b \Delta p_f}{12 \mu_f}
\]

and then Eq.(11) becomes

\[
\Delta p_f = \frac{6 \Delta \sigma}{b} - \frac{12 \mu_f \bar{v}}{b}
\]

**A. Pumping power**

Finally, the power attained per unit of cross section area of the duct is calculated as

\[
W = \Delta p_f \cdot \bar{v}
\]

which taking into account Eq.(13) yields

\[
W = \frac{6 \Delta \sigma \bar{v}}{b} - \frac{12 \mu_f \bar{v}^2}{b}
\]

and the maximum power occurs when its derivative with respect to \( \bar{v} \) is maximized:

\[
\frac{d(W)}{d\bar{v}} \bigg|_{\bar{v}=\bar{v}^*} = 0
\]

and then from Eq.(15) one obtains

\[
\bar{v}^* = \frac{1}{4} \frac{\Delta \sigma}{\mu_f}
\]

and

\[
W^* = W(\bar{v}^*)
\]

\[
W^* = \frac{3 \Delta \sigma^2}{4 b \mu_f}
\]

If the surface tension comes from a concentration or thermal gradient, Eq.(18) becomes

\[
W^* = \frac{3 \sigma_c^2}{4 b \mu_f} \Delta c^2
\]

and

\[
W^* = \frac{3 \sigma_T^2}{4 b \mu_f} \Delta T^2
\]

where \( \sigma_c \) and \( \sigma_T \) are the surface tension coefficient for concentration and temperature, respectively.

**III. RESULTS AND CONCLUSIONS**

In order to obtain some idea of the density power by a Marangoni duct, we assume some typical values of the parameters: \( \sigma_T = 10^{-4} \) N/(m)(K); \( \sigma_c = 10^{-3} \) N/(mM) -where \( M \) stands for molarity; \( \mu_f = 10^{-3} \) Pa s, and a micrometric duct \( d = 100 \mu m \). The resulting curves are shown in Fig. 2 and Fig. 3 for the power density as function of the mean velocity and as function of the width of the duct for a Marangoni duct driven by thermal gradient, respectively. Fig. 4 and Fig. 5 are similar figures than Fig. 2 and Fig. 3 but for a Marangoni driven by concentration gradients. It is seen that for reasonable small duct with 0.5 mm to 1mm width the power density
FIG. 5: Power density as function of the duct width for Marangoni driven by concentration gradients.

could be around 1 W per m² of cross section area of the duct considering temperatures differences around 10 K or concentrations differences up to 1 M. It is interesting to see, that for very small ducts with widths around 100μm or less the attainable power density can be arise up to a 10-fold i.e., around 10 W/m² for the same thermal and concentration gradients.

NOMENCLATURE

\( b \) = width of duct, (mm)
\( c \) = concentration, (mol/m³)
\( h \) = width of cavity, (mm)

Greek symbols

\( \mu \) = dynamic viscosity, (Pas)
\( \sigma \) = surface tension, (N/m)
\( \sigma_T \) = surface tension coefficient of temperature, (N/mK)
\( \sigma_c \) = surface tension coefficient of concentration, (N/mM)

subscripts

\( a \) = air
\( f \) = fluid

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IV. REFERENCES


