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THERMOELASTIC STRESS ANALYSIS FOR A TUBE UNDER GENERAL MECHANOCHEMICAL CORROSION CONDITIONS

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Abstract. This work deals with the uniform surface mechanochemical corrosion of an elastic thick-walled long cylindrical tube subjected to internal and external pressure of environments at different temperatures. The rate of uniform corrosion is supposed to be linear with the stress and inversely as the exponent of time. The problem is then reduced to the first-order ordinary differential equation in a tube cross-sectional size. Analytical solutions of this equation are found. To determine the tube lifetime and the cause of its failure, the method based on various estimating functions is proposed. The algorithm for environmental contamination by corroded material is developed.

1 INTRODUCTION

Most structures are exploited being subjected to both mechanical loads and operating environments. This often causes the process of so called mechanochemical wear which is more intensive in comparison with the simple superposition of damages induced by mechanical stresses and electrochemical corrosion taken separately. The rate of corrosion depends on many factors. It is to be noted that mechanical stress does not affect the corrosiveness in neutral and weakly acid media. According to most experimental results [1, 2, 3], in other media the rate of uniform corrosion often is linear with the stress when traction increases beyond a given threshold. This depends on the stress sign and properties of the "material–medium" system. Furthermore, the corrosion rate is inversely as the exponent of time if closed oxide layer leads to the inhibition of corrosion.

This paper is concerned with equal-rate mechanochemical corrosion often observed in practice. The theoretical research in this area has been conducted by numerous authors. A comprehensive review of models and calculations for structures taking into account corrosive wear was given e.g. in [3, 4]. Among the first works in the field there were some articles by V.M.Dolinskii, concerned with mechanochemical corrosion of a thin-walled structural members [1, 5]. Elegant mathematical apparatus was presented in the work [6] where the lifetime of a loaded pipe had been assessed under the assumption of the exponential dependence of the corrosion rate on the mean stress. Using the linear relation between the corrosion rate and the stress, the corrosive wear of a nonlinearly elastic cylinder subjected to pressure and temperature was simulated by numerical methods in the book [3]. In the articles [7, 8] the equal-rate mechanochemical corrosion of a linearly elastic thick-walled cylindrical tube subjected to a longitudinal force, internal and external pressure was discussed without taking temperature stresses into consideration. The problem was then reduced to the first-order ordinary differential equation in either tube cross-sectional sizes or the maximum principal stress as the situation requires. The purpose of this work is to study the uniform mechanochemical corrosion of a hollow cylinder under the joint action of pressure and temperature.

2 PROBLEM STATEMENT

The uniform surface corrosion of an elastic long thick-walled cylindrical tube is investigated. The tube is subjected to constant internal pressure p_r of a corrosive medium at temperature T_r and external pressure p_R of another corrosive medium at temperature T_R. The inner and outer tube radii at the initial time $t = 0$ are denoted by r_0 and R_0 ($r_0 < R_0$). The action of the ends of the cylinder is not taken into account. Changes of the tube radii are assumed to be quasi-static. The corrosion rates at the internal ($\rho = r$) and external ($\rho = R$) boundaries are given correspondingly by the expressions [2, 3]:

$$v_r = \frac{dr}{dt} = [a_r + m_r \sigma_1(r)] \exp(-bt) \exp(\beta_r [T_r - T_r^0]) \quad \text{at} \quad |\sigma_1(r)| \geq |\sigma_r^{th}|, \tag{1}$$

$$v_R = -\frac{dR}{dt} = [a_R + m_R \sigma_1(R)] \exp(-bt) \exp(\beta_R [T_R - T_R^0]) \quad \text{at} \quad |\sigma_1(R)| \geq |\sigma_R^{th}|. \tag{2}$$

Here, $a_r, a_R, m_r, m_R, b, \beta_r, \beta_R, T_r^0$ and T_R^0 are observable quantities; $a_r = v_r^0 - m_r \sigma_r^{th}$ and $a_R = v_R^0 - m_R \sigma_R^{th}$; σ_r^{th} and σ_R^{th} are the threshold stresses (as a matter of fact, which are different for traction and compression); v_r^0 and v_R^0 are the initial corrosion rates at $|\sigma_1(r)| < |\sigma_r^{th}|$ and $|\sigma_1(R)| < |\sigma_R^{th}|$, respectively; σ_1 is the maximum principal stress.

It is necessary to trace the change of the thermoelastic stresses and amount of corroded material with time t and to estimate the tube lifetime.

3 BASIC EQUATIONS

The problem of a tube under pressure and temperature has been discussed by numerous writers including G. Lame and R. Lorenz [9, 10]. At $r = 0$, $p_R = p$ or $p_r = p_R = p$ and $T_r = T_R$, there is a homogeneous stress $\sigma_{\theta\theta} \equiv \sigma_{\rho\rho} \equiv -p$ irrespective of corrosion. In other cases, the stress-components are expressed, by reference to cylindrical coordinates ρ, θ, z, by the equations

$$\sigma_{\theta\theta}(\rho) = \frac{(p_r + T_\sigma/2)r^2 - p_R R^2}{R^2 - r^2} + \frac{p_r - p_R + T_\sigma/2}{R^2 - r^2}\,\frac{r^2 R^2}{\rho^2} + T_\sigma\,\frac{\ln(R/\rho) - 1}{2\ln(R/r)}, \tag{3}$$

$$\sigma_{\rho\rho}(\rho) = \frac{(p_r + T_\sigma/2)r^2 - p_R R^2}{R^2 - r^2} - \frac{p_r - p_R + T_\sigma/2}{R^2 - r^2}\,\frac{r^2 R^2}{\rho^2} + T_\sigma\,\frac{\ln(R/\rho)}{2\ln(R/r)},$$

$$\sigma_{zz}(\rho) = \frac{p_r r^2 + (T_\sigma - p_R)R^2}{R^2 - r^2} - T_\sigma\,\frac{1 + 2\ln(\rho/r)}{2\ln(R/r)}, \tag{4}$$

where

$$T_\sigma = \frac{\alpha E}{1-\nu}\,(T_R - T_r),$$

α is thermal expansion coefficient, E is Young's modulus, ν is Poisson's ratio. Formula (4) holds true for a closed cylindrical vessel with allowance for the total pressure to its bottoms.

If stress does not affect the corrosion rate, the stress-components at any time can be easily calculated by the above equations (3)–(4) for given laws of change of radii $r(t)$ and $R(t)$. This is valid for neutral and alkalescent media and when load is lower than the threshold value σ^{th}. We now study mechanochemical corrosion when the conditions (1) and (2) hold true. The solution to the problem for $T_r = T_R$ and $p_r \neq p_R$ has been given in [7, 8]. We now consider other situations when $p_r \neq p_R$ and $T_r \neq T_R$.

It is evident that the maximum principal stress is the circumferential one: $\sigma_1(\rho) = \sigma_{\theta\theta}(\rho)$. This stress is then to be used in formulae (1) and (2). If $T_r = T_R$, then the greatest tension is at the inner surface: $\sigma_{\theta\theta}(r) \geq \sigma_{\theta\theta}(R)$. So we must follow the amount of $\sigma_{\theta\theta}(r)$ to determine the time t^* when $\sigma_{\theta\theta}(r)$ reaches the limiting stress. But sometimes, heat stress can compensate mechanical one. For example, $\sigma_{\theta\theta}(r) < \sigma_{\theta\theta}(R)$ when $0 < p_r - p_R < -T_\sigma$. When this is the case, we have to watch the $\sigma_{\theta\theta}(R)$. Let the greatest stress be the stress $\sigma_1(r) = \sigma_{\theta\theta}(r)$. The equation (3) gives

$$\sigma_{\theta\theta}(r) = \frac{(p_r - 2p_R + T_\sigma)\eta^2 + p_r}{\eta^2 - 1} - \frac{T_\sigma}{2\ln\eta}, \tag{5}$$

where

$$\eta = \frac{R}{r} = \frac{R_0 - \delta_R}{r_0 + \delta_r}. \tag{6}$$

For the relation $\eta = R/r$ can not be expressed from (5), it is impossible to derive the differential equation in $\sigma_{\theta\theta}(r)$ or $\sigma_{\theta\theta}(R)$, as it was made in [7]. So let us deduce an equation in η.

If we eliminate σ_1 from the formulae (1) and (2) using (3), we can obtain the relationship

$$RM_r + rM_R = M_R\left(r_0 - \frac{B_r}{b}\left[\exp(-bt) - 1\right]\right) + M_r\left(R_0 + \frac{B_R}{b}\left[\exp(-bt) - 1\right]\right), \qquad (7)$$

where

$$B_r = a_r \exp(\beta_r[T_r - T_r^0]), \qquad M_r = m_r \exp(\beta_r[T_r - T_r^0]),$$

$$B_R = (a_R - m_R[p_r - p_R + T_\sigma]) \exp(\beta_R[T_R - T_R^0]), \quad M_R = m_R \exp(\beta_R[T_R - T_R^0]). \qquad (8)$$

On differentiating the expression (6) with respect to t and then using (5) and (7)–(8), we can deduce the ordinary differential equation for changing η

$$\frac{d\eta}{dt} = -\frac{M_R + M_r\eta}{\exp(bt)} \frac{B_R + B_r\eta + (M_R + M_r\eta)\left[\dfrac{(p_r - 2p_R + T_\sigma)\eta^2 + p_r}{\eta^2 - 1} - \dfrac{T_\sigma}{\ln \eta^2}\right]}{M_R r_0 + M_r R_0 - \left(M_R\dfrac{B_r}{b} - M_r\dfrac{B_R}{b}\right)\left[\exp(-bt) - 1\right]}. \qquad (9)$$

The initial condition to be satisfied at $t = 0$ is

$$\eta_0 = \frac{R_0}{r_0}. \qquad (10)$$

4 SOLUTIONS OF THE BASIC DIFFERENTIAL EQUATION

The integral of the equation (9), satisfying the condition (10), is

$$t = -\frac{1}{b}\ln\left\{1 - b\frac{M_R r_0 + M_r R_0}{M_R B_r - M_r B_R}\Big(\exp\left[(M_R B_r - M_r B_R)\, J(\eta)\right] - 1\Big)\right\},$$

where

$$J(\eta) = \int_\eta^{\eta_0} \frac{(\eta^2 - 1)\ \ln \eta^2}{M_R + M_r\eta}\Big\{[B_R + B_r\eta](\eta^2 - 1)\ln\eta^2 + $$
$$+ [M_R + M_r\eta]\{[(p_r - 2p_R + T_\sigma)\eta^2 + p_r]\ln\eta^2 - T_\sigma(\eta^2 - 1)\}\Big\}^{-1} d\eta.$$

For one-sided corrosion, the analytical solution can be simplified. For example, at $M_r = 0$ and $B_r = 0$ (external corrosion), the result may be written in the form

$$t = -\frac{1}{b}\ln\{1 - bJ(\eta)\} \quad \text{at} \ \ b \neq 0, \qquad (11)$$
$$t = J(\eta) \quad \text{at} \ \ b = 0,$$

where

$$J(\eta) = r_0 \int_{\eta}^{\eta_0} \frac{(\eta^2 - 1)\ \ln \eta^2 \quad d\eta}{\ln \eta^2 \{B_R\,(\eta^2 - 1) + M_R[(p_r - 2p_R + T_\sigma)\eta^2 + p_r]\} - M_R T_\sigma (\eta^2 - 1)}.$$

For internal corrosion, at $M_R = 0$ and $B_R = 0$, the solution is of the form (11), where

$$J(\eta) = R_0 \int_{\eta}^{\eta_0} \frac{(1 - 1/\eta^2)\ \ln \eta^2 \quad d\eta}{\ln \eta^2 \{B_r\,(\eta^2 - 1) + M_r[(p_r - 2p_R + T_\sigma)\eta^2 + p_r]\} - M_r T_\sigma (\eta^2 - 1)}.$$

The solutions of the basic differential equation give us the t-to-η corresponding. Using the equation (5), we can then calculate the stress $\sigma_{\theta\theta}(r)$ (and other stress-components) at any time t.

5 ASSESSMENT OF ENVIRONMENTAL CONTAMINATION

Using above relations, it is also possible to assess the amount of the corroded material at the inner and outer surfaces. This material is solved (evaporated) in the environments or precipitated into corrosion products (oxide film) as the case may be. The volume of the inside and the outside corroded material per unit length at any t are evaluated correspondingly by the formulae

$$V_r^p = \pi(r^2 - r_0^2), \qquad V_R^p = \pi(R_0^2 - R^2),$$

where

$$r = r_0 + \delta_r = \frac{M_R r_0 + M_r R_0 - \left(M_R \dfrac{B_r}{b} - M_r \dfrac{B_R}{b}\right)[\exp(-bt) - 1]}{\eta\, M_r + M_R},$$

$$R = R_0 - \delta_R = \eta r,$$

η is uniquely determined by the integral curve of the differential equation (9) for the t involved.

If the internal and external environments are closed, then it is easy to calculate the impurity concentration in it at any t using above relations and chemical equations.

When the volume $V^l = \pi r_1^2$ of internal environment flows in a pipe for a known period from t_1 to t_2, the volume of corroded material V_r^p per the environment volume (e.g., concentration of pipe material solved in the environment volume) can be estimated by the formulae

$$C = \frac{V_r^p}{V^l}\, 100\% = \frac{1}{2}\left(\frac{r_2^2}{r_1^2} - \frac{r_1^2}{r_2^2}\right) 100\% \qquad \text{or} \qquad C = 4\,\frac{r_2 - r_1}{r_1 + r_2}\, 100\%,$$

valid for $r_2 - r_1 << r_1$ and $t_2 - t_1 << t^*$, where $r_1 = r|_{t=t_1}$, $r_2 = r|_{t=t_2}$; t^* is the predicted lifetime of a pipe.

6 LIFETIME PREDICTION

Taking into account interference of general corrosion and thermomechanical stress, lifetime of a tube may be estimated. It is obvious that pipe failure can be initiated by many causes. The life of the body is determined by the minimum time it takes for a limiting state to be achieved owing to any case: when the ultimate tensile strength is reached, upon loss of stability, upon damage accumulation (e.g., during quasi-static cyclic loading or aging) and so on. To determine the rupture source and the life of a tube it is reasonable to introduce scalar estimating functions. Following the L. Kachanov approach [11], various kinds of damage are represented by dimensionless functions varying from zero to unity (or from $-\infty$ to 1) and mounting to unity in the moments of fault related to a concrete criteria.

To assess the durability functions of the type $\Pi_s(t) = \dfrac{f(\sigma,\epsilon)}{f_s} \leq 1$ may be used. For the maximum normal stress criterion, we can write $\Pi_s(t_s^*) = 1$, where $\Pi_s(t) = \dfrac{\sigma(t)}{\sigma_s(t)}$, $\sigma_s(t)$ is the ultimate tensile strength of material that may change in time.

Another approach to determine the rupture life consists in damage accumulation assessment. For instance, according to Bailey's principle, the time to destruction t_d^* is determined by the equation $\Pi_d(t_d^*) = \displaystyle\int_0^{t_d^*} \frac{dt}{\tau[\sigma(t),T]} = 1$, where $\tau[\sigma,T]$ is the working life of the material under the stress σ and temperature T.

Functions to assess the stability factor may be of the form $\Pi_{cr}(t) = \dfrac{\sigma_{zz}(t)}{\sigma_{zz}^{cr}(t)} + \dfrac{\sigma_{\theta\theta}(t)}{\sigma_{\theta\theta}^{cr}(t)} \leq 1$, where σ_{zz}^{cr} is buckling stress for the tube under only longitudinal force (under no pressure), $\sigma_{\theta\theta}^{cr}$ is the upper critical stress for the tube under pressure (in the absence of axial force), expressed through the current tube sizes and mechanical quantities. It is obvious that the cylindrical shape is always stable at $\Pi_{cr}(t) \leq 0$. Stability of thin-walled shells under conditions of corrosive action have been investigated by many scientists, e.g. [12, 13].

Furthermore, failure may be determined apparently by accidental circumstances. For such assessment we can introduce estimating function as being equal to the probability, i.e. accident risk $\Pi_p(t) = P(x_1, ...x_n, t/y_1, ...y_n)$. It is to be emphasized that the unreliability function depends on the other estimating functions.

The graphs of all the estimating functions are plotted on one figure and compared with each other. The curve being the first to reach unity indicates the most probable cause of breakdown and the lifetime of an item

$$t^* = \min\{t_i^* : \ \Pi_i(t_i^*) = 1\}.$$

Computation results have shown that an increase in corrosion inhibition index b considerably increases the service life of the tube. At sufficiently high values of b, corrosion can practically finish without reaching the critical state. In this case, the life is controlled by damage accumulation (decrease in the ultimate strength) or random factors.

7 CONCLUSIONS

- The problem under study has been reduced to the first-order ordinary differential equation. The analytical solution of this equation has been obtained.
- The algorithm for environmental contamination has been developed.
- To predict the life of a tube various estimating functions have been proposed.

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REFERENCES

[1] Dolinskii, V.M. Calculation of stressed pipes subjected to corrosion. *Khim. Neft. Mashinostr.* 1967, N 2: 18–21.

[2] Pavlov, P.A., Kadyrbekov, B.A. and Kolesnikov, V.A. *Strength of steels in corrosive environments.* Nauka, Alma-Ata, (1987). [in Russian].

[3] Naumova, G.A. and Ovchinnikov, I.G. *Strength calculations of compound frameworks and pipeline structures with taking into account corrosion damages.* Saratov State University, Saratov, (2000). [in Russian].

[4] Ovchinnikov, I.G. and Pochtman, Yu.M. Calculation and rational design of structures subjected to corrosive wear (review). *Materials Science.* (1992) **27**, N 2: 105–116.

[5] Dolinskii, V.M. and Sirotenko, V.A. Calculation of heat exchange systems of reduced design subjected to the effect of a corrosive medium. *Khim. Mashinostr.* 1970, N 11: 4–11.

[6] Gutman, E.M., Zainullin, R.S., Shatalov, A.T. et al. *Strength of gas industry pipes under corrosive wear conditions.* Nedra, Moscow, (1984). [in Russian].

[7] Pronina, Yu.G. Analytical study of general mechanochemical corrosion of the pipe under the axial force and pressure. *7th Euromech Solid Mechanics Conference (Lisbon, Portugal, September 7-11, 2009): CD-Rom Proceedings.* European Mechanics Society, APMTAC. CD-Rom\papers\Gs-sm\pap_0486_GS-SM.pdf.

[8] Pronina, Yu.G. Estimation of the life of an elastic tube under the action of a longitudinal force and pressure under uniform surface corrosion conditions. *Russian Metallurgy (Metally).* (2010) **2010**, N 4: 361–364.

[9] Lame, G. *Lecons sur la theorie mathematique de l'elasticite des corps solides.* Paris, (1852).

[10] Lorenz, R. Temperaturspannungen in hohlzylindern. *Zeitschrift des vereines Deutscher ingenieure.* (1907) **51**: 743–747.

[11] Kachanov, L.M. *Introduction to continuum damage mechanics.* Martinus Nijhoff Publishers, Dordrecht, Netherlands, (1986).

[12] Karpunin, V.G., Kleshchev, S.I. and Kornishin, M.S. Endurance of plates and shells under conditions of the corrosive action of a medium. *Strength and Endurance of Structures.* Naukova Dumka, Kiev, (1980), 35–44. [in Russian].

[13] Gutman, E.M., Haddad, J. and Bergman, R. Stability of thin-walled high-pressure vessels subjected to uniform corrosion. *Thin-Walled Structures.* (2000) **38**: 43–52.