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Two-temperatures thermo-porous-elasticity with microtemperatures

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Abstract

In this note, we study a linear system of partial differential equations modelling a one-dimensional two-temperatures thermo-porous-elastic problem with microtemperatures. A new system of conditions is proposed to guarantee the existence, uniqueness and exponential decay of solutions. Our arguments are based on the theory of semigroups of linear operators.

Keywords: Two-temperatures heat conduction, thermo-porous-elasticity, microtemperatures, existence, exponential decay, semigroups.

1. Introduction

It is accepted that the thermoelasticity with voids can be seen as the easiest extension of the classical theory of thermoelasticity [6, 7, 20]. It incorporates the existence of voids at the microstructure of the elastic material. This theory has deserved a big interest in the recent years [8, 9, 13, 15, 16, 18, 19, 21]. In fact, over the last years a big deal has been developed to understand the different mechanisms working at the microstructure level. Apart of the voids, we want to highlight the possibility of the microtemperatures at the microstructure level [1, 2, 10, 11, 12, 17, 22, 26, 27]. In this short note, we want to focus

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our attention to porous-thermo-elastic materials with microtemperatures. It is relevant to note the huge quantity of contributions for this kind of materials in the literature. This is because they have been shown as a type of materials with a wide applicability.

The thermoelasticity with two temperatures is a theory proposed by Gurtin and several colleagues [3, 4, 5, 28], where the heat conduction is described by means of two temperatures: the thermodynamic temperature and the inductive temperature. It has also received a big attention in the recent years [23, 24, 25]; however, it has not been combined previously with the microtemperatures. We here propose a one-dimensional linear theory by using both points of view (two temperatures and the microtemperatures) which are incorporated at the same time. In this sense, the present paper has three main objectives: the first one corresponds to propose the two-temperatures thermo-porous-elasticity with microtemperatures in the one-dimensional case. The second one is to give a suitable family of conditions on the constitutive parameters to guarantee the existence and uniqueness of solutions in a suitable Hilbert space. The third one is to give an exponential time decay result for the solutions to this problem.

2. Basic equations

In this section, we propose the basic equations for the one-dimensional two-temperatures thermo-porous-elasticity with microtemperatures for isotropic and homogeneous materials. The length of the body is assumed to be π. According to this theory, the evolution equations are the following:

$$\rho\ddot{u} = t_x + \rho f, \quad J\ddot{\phi} = h_x + g + \rho l,$$
$$\rho T_0\dot{\eta} = q_x + \rho s, \quad \rho\dot{\epsilon} = P_x + q - Q + \rho G,$$

and the constitutive equations:

$$
\begin{aligned}
& t=\mu u_x+\mu_0\phi-\beta_0\theta, \quad h=a_0\phi_x-\mu_2 T, \\
& g=-\mu_0 u_x-\xi\phi+\beta_1\theta, \quad \rho\eta=\beta_0 u_x+\beta_1\phi+a\theta, \\
& \rho\epsilon=-\mu_2\phi_x-bT, \quad q=k\vartheta_x+k_1 S, \\
& P=-k_4 S_x, \quad Q=(k-k_3)\vartheta_x+(k_1-k_2)S.
\end{aligned}
$$

In this system, ρ is the mass density, u is the displacement, t is the stress, h is the equilibrated stress, g is the equilibrated body force, η is the entropy, q is the heat flux, J is the equilibrated inertia, T_0 is the reference temperature at the equilibrium state (assumed uniform and equal to one to simplify the calculations), ϵ is the first moment of the energy, Q is the microheat flux average, P is the first heat flux moment, ϕ is the volume fraction, θ is the thermodynamic temperature, T is the thermodynamic microtemperature, ϑ is the inductive temperature, S is the inductive microtemperature and f, l, s and G are the supply terms. Moreover, $\mu, \mu_0, \beta_0, \beta_1, a_0, \mu_2, \xi, k$ and k_i are the constitutive parameters defining the couplings among the different components of the material. In this paper, we assume that the constitutive coefficients satisfy the conditions:

$$\mu>0, \quad \mu\xi>\mu_0^2, \quad a_0>0, \quad k>0, \quad k_4>0, \quad \rho>0, \tag{1}$$

$$J>0, \quad a>0, \quad b>0, \tag{2}$$

$$4\alpha k(k_4+\alpha k_2)>\alpha^2 k_1^2, \quad 4\alpha k k_2 k_4-\alpha k_4(k_1+k_3)^2-\alpha^2 k_2 k_3^2>0. \tag{3}$$

It is worth noting that condition (3) is more restrictive than the usual one in classical thermoelasticity with microtemperatures. It is needed to guarantee that the dissipation is positive. In fact, for the classical theory we usually assume that $4kk_2-(k_1+k_3)^2$ is positive meanwhile in the present theory we need to impose condition (3).

It is relevant recalling that the temperatures and the microtemperatures satisfy the relations:

$$\theta=\vartheta-\alpha\vartheta_{xx}, \quad T=S-\alpha S_{xx}, \tag{4}$$

where α is a positive constant. If we substitute the constitutive equations into the evolution equations, we obtain the following linear system:

$$\rho\ddot{u} = \mu u_{xx} + \mu_0\phi_x - \beta_0\theta_x + \rho f,$$
$$J\ddot{\phi} = a_0\phi_{xx} - \mu_0 u_x - \mu_2 T_x + \beta_1\theta - \xi\phi + \rho l,$$
$$a\dot{\theta} = -\beta_0\dot{u}_x - \beta_1\dot{\phi} + k\vartheta_{xx} + k_1 S_x + \rho s,$$
$$b\dot{T} = -\mu_2\dot{\phi}_x + k_4 S_{xx} - k_2 S - k_3\vartheta_x - \rho G.$$

To study a well posed problem, we should define initial and boundary conditions. Hence, as initial conditions we impose that, for a.e. $x \in (0,\pi)$,

$$u(x,0) = u^0(x), \quad \dot{u}(x,0) = v^0(x), \quad \phi(x,0) = \phi^0(x), \quad \dot{\phi}(x,0) = \varphi^0(x),$$
$$\theta(x,0) = \theta^0(x), \quad T(x,0) = T^0(x),$$

where u^0, v^0, ϕ^0, φ^0, θ^0 and T^0 are given functions.

Since we assume homogeneous Dirichlet boundary conditions, it follows, for a.e. $t \in [0,\infty)$ and $x = 0,\pi$,

$$u(x,t) = \phi(x,t) = \vartheta(x,t) = S(x,t) = 0.$$

It is worth noting that under the assumption of these boundary conditions we have

$$\int_0^\pi \theta^2 dx = \int_0^\pi (\vartheta^2 + 2\alpha\vartheta_x^2 + \alpha^2\vartheta_{xx}^2)dx, \quad \int_0^\pi T^2 dx = \int_0^\pi (S^2 + 2\alpha S_x^2 + \alpha^2 S_{xx}^2)dx,$$

and therefore,

$$\int_0^\pi \theta^2 dx \approx \int_0^\pi (\vartheta_x^2 + \vartheta_{xx}^2)dx, \ \int_0^\pi T^2 dx \approx \int_0^\pi (S_x^2 + S_{xx}^2)dx.$$

3. Existence and uniqueness

In this section, we transform our problem into an abstract problem for a suitable Hilbert space and we prove the well posedness of the problem.

To this end, we recall that $I - \alpha\delta_{xx} : \vartheta \to \theta = \vartheta - \alpha\vartheta_{xx}$ is an isomorphism from $W^{2,2} \cap W_0^{1,2}$ to L^2 (where $W^{2,2}, W_0^{1,2}$ and L^2 are the usual Sobolev spaces).

We denote by $\Upsilon(\theta) = \vartheta$ its inverse. Obviously, the L^2-norm of θ is equivalent to the $W^{2,2}$-norm of ϑ. We also note that we can do the same comments for the functions T and S. That is, we have $S - \alpha\delta_{xx}(S) = T$ and $\Upsilon(T) = S$, and also that the L^2-norm of T is equivalent to the $W^{2,2}$-norm of S.

We will work in the Hilbert space:

$$\mathcal{H} = W_0^1 \times L^2 \times W_0^1 \times L^2 \times L^2 \times L^2.$$

An element in this space will be denoted by $(u, v, \phi, \varphi, \theta, T)$.

Defining the following matrix operator:

$$\mathcal{A} = \left(\begin{array}{c|c|c|c|c|c} 0 & I & 0 & 0 & 0 & 0 \\ \frac{\mu}{\rho}\delta_{xx} & 0 & \frac{\mu_0}{\rho}\delta_x & 0 & -\frac{\beta_0}{\rho}\delta_x & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ -\frac{\mu_0}{J}\delta_x & 0 & \frac{a_0\delta_{xx}-\xi}{J} & 0 & \frac{\beta_1}{J} & -\frac{\mu_2}{J}\delta_x \\ 0 & -\frac{\beta_0}{a}\delta_x & 0 & -\frac{\beta_1}{a} & \delta_{xx}(\frac{k}{a}\Upsilon) & \delta_x(\frac{k_1}{a}\Upsilon) \\ 0 & 0 & 0 & -\frac{\mu_2}{b}\delta_x & -\delta_x(\frac{k_3}{b}\Upsilon) & \delta_{xx}(\frac{k_4}{b}\Upsilon) - \frac{k_2}{b}\Upsilon \end{array} \right),$$

we can write our problem as

$$\frac{dU}{dt} = \mathcal{A}U + \mathcal{F}, \quad U(0) = U^0, \tag{5}$$

where $\mathcal{F} = (0, f, 0, l, s, -G)$ and $U^0 = (u^0, v^0, \phi^0, \varphi^0, \theta^0, T^0)$.

In this section, we are going to prove that operator $\mathcal{A}$ generates a contractive semigroup. We first note that the domain of the operator, denoted by $\mathcal{D}(\mathcal{A})$, is given by the elements in the Hilbert space $\mathcal{H}$ such that

$$v, \varphi \in W_0^1, \quad \mu u_{xx} - \beta_0\theta_x \in L^2, \quad a_0\phi_{xx} - \mu_2 T_x \in L^2.$$

Given $U = (u, v, \phi, \varphi, \theta, T)$ and $U^* = (u^*, v^*, \phi^*, \varphi^*, \theta^*, T^*)$, we consider the inner product defined as

$$\langle U, U^*\rangle = \frac{1}{2}\int_0^\pi (\rho v\bar{v}^* + J\varphi\bar{\varphi}^* + \mu u_x\bar{u}_x^* + \mu_0(u_x\bar{\phi}^* + \bar{u}_x^*\phi) + \xi\phi\bar{\phi}^* + a_0\phi_x\bar{\phi}_x^* + c\theta\bar{\theta}^* + bT\bar{T}^*)\,dx.$$

Here, and from now on, the bar denotes the conjugated complex. It is clear that this inner product is equivalent to the usual one in the Hilbert space $\mathcal{H}$.

Theorem 1. *Let the conditions (1)-(3) hold. Then, operator $\mathcal{A}$ generates a contractive semigroup.*

Proof. It is straightforward to see that the domain of the operator is dense in our Hilbert space. We can easily show that

$$\mathrm{Re}\langle \mathcal{A}U, U\rangle = -\frac{1}{2}\int_0^\pi (k|\vartheta_x|^2 + (k_1+k_3)\mathrm{Re}\vartheta_x\bar{S} + k_2|S|^2 + k_4|S_x|^2)dx$$
$$-\frac{\alpha}{2}\int_0^\pi (k|\vartheta_{xx}|^2 + k_1\mathrm{Re}\bar{\vartheta}_{xx}S_x + k_2|S_x|^2 + k_3\mathrm{Re}\vartheta_x\bar{S}_{xx} + k_4|S_{xx}|^2)dx \le 0.$$

Last inequality is a consequence of assumptions (1)-(3).

Now, we prove that zero belongs to the resolvent of the operator. Given $(f_1, f_2, f_3, f_4, f_5, f_6) \in \mathcal{H}$, we must show that the system

$$\begin{aligned}
&v = f_1,\\
&\varphi = f_3,\\
&\mu u_{xx} + \mu_0\phi_x - \beta_0\theta_x = \rho f_2,\\
&a_0\phi_{xx} - \mu_0 u_x - \mu_2 T_x + \beta_1\theta - \xi\phi = J f_4,\\
&-\beta_0 v_x - \beta_1\varphi + k\vartheta_{xx} + k_1 S_x = a f_5,\\
&-\mu_2\varphi_x + k_4 S_{xx} - k_2 S - k_3\vartheta_x = b f_6,
\end{aligned}$$

has a solution in the domain of operator $\mathcal{A}$. The solution for v and φ is obtained. If we substitute it in the last two equations, we obtain that

$$\begin{aligned}
k\vartheta_{xx} + k_1 S_x &= a f_5 + \beta_1 f_3 + \beta_0 f_{1,x} = F_3,\\
k_4 S_{xx} - k_2 S - k_3\vartheta_x &= b f_6 + \mu_2 f_{3,x} = F_4.
\end{aligned}$$

To prove the existence of solutions to this system, we note that $(F_3, F_4) \in L^2 \times L^2$. Moreover, if we define the bilinear form:

$$\begin{aligned}
&B((\theta_1, T_1), (\theta_2, T_2))\\
&\quad = -\langle (k(\Upsilon(\theta_1))_{xx} + k_1(\Upsilon(T_1))_x, k_4(\Upsilon(T_1))_{xx} - k_2\Upsilon(T_1) - k_3(\Upsilon(\theta_1))_x), (\theta_2, T_2)\rangle_{L^2\times L^2},
\end{aligned}$$

we can see that it is bounded and coercive. Therefore, in view of the Lax-Milgram lemma, we can prove the existence of θ, $T \in L^2$ satisfying the system.

Thus, we can study now the another system:

$$\mu u_{xx} + \mu_0 \phi_x = \rho f_2 + \beta_0 \theta_x = F_1,$$
$$a_0 \phi_{xx} - \mu_0 u_x - \xi \phi = J f_4 + \mu_2 T_x - \beta_1 \theta = F_2.$$

To prove the existence of solutions to this second system, we note that $(F_1, F_2) \in W^{-1,2} \times W^{-1,2}$. The bilinear form

$$B^*((u_1,\phi_1),(u_2,\phi_2)) = -\langle(\mu u_{1,xx} + \mu_0 \phi_{1,x}, a_0 \phi_{1,xx} - \mu_0 u_{1,x} - \xi \phi_1), (u_2, \phi_2)\rangle_{L^2 \times L^2}$$

is bounded and coercive in $W^{1,2} \times W^{1,2}$. Again, the use of the Lax-Milgram lemma allows us to obtain the solution.

Therefore, in view of the Lumer-Phillips corollary to the Hille-Yosida theorem we find that our operator generates a contractive semigroup.

Now, we can obtain the following existence and uniqueness result.

Theorem 2. *If we assume that conditions (1)-(3) hold, then, for every $U^0 \in \mathcal{D}$, there exists a unique solution to problem (5).*

We note that, since the operator generates a contractive semigroup, the problem is well posed in the sense of Hadamard.

4. Exponential stability

Now, we show the exponential decay of the solutions to our problem when the supply terms vanish and some conditions hold. To this end, we need to assume that $\beta_0 \neq 0$ and $\mu_2 \neq 0$. In order to prove the exponential decay, we recall the characterization stated in the book of Liu and Zheng [14].

Theorem 3. *Let $S(t) = \{e^{\mathcal{A}t}\}_{t \geq 0}$ be a C_0-semigroup of contractions on a Hilbert space. Then $S(t)$ is exponentially stable if and only the imaginary axis is contained in the resolvent of $\mathcal{A}$ and*

$$\overline{\lim_{|\lambda| \to \infty}} \|(i\lambda \mathcal{I} - \mathcal{A})^{-1}\|_{\mathcal{L}(\mathcal{H})} < \infty. \tag{6}$$

Then, we prove the following result which states the exponential decay of the energy system.

Theorem 4. *Let the conditions (1)-(3) still hold. If we also assume that* $\beta_0 \neq 0$ *and* $\mu_2 \neq 0$*, then operator* $\mathcal{A}$ *generates a semigroup exponentially stable. That is, there exist two positive constants* M, ω *such that*

$$||U(t)|| \leq M \exp(-\omega t)||U(0)||$$

for every $U(0) \in \mathcal{D}(\mathcal{A})$.

Proof. We here follow the arguments given in the book of Liu and Zheng ([14], page 25). Let us assume that the intersection of the imaginary axis and the spectrum is non-empty. Therefore, there exist a sequence of real numbers λ_n with $\lambda_n \to \varpi$, $|\lambda_n| < |\varpi|$ and a sequence of vectors $U_n = (u_n, v_n, \phi_n, \varphi_n, \theta_n, T_n)$ in $\mathcal{D}(\mathcal{A})$, and with unit norm, such that

$$\|(i\lambda_n \mathcal{I} - \mathcal{A})U_n\| \to 0.$$

It follows that

$$i\lambda_n u_n - v_n \to 0 \text{ in } W^1, \tag{7}$$

$$i\rho\lambda_n v_n - (\mu u_{n,xx} + \mu_0 \phi_{n,x} - \beta_0 \theta_{n,x}) \to 0 \text{ in } L^2, \tag{8}$$

$$i\lambda_n \phi_n - \varphi_n \to 0 \text{ in } W^1, \tag{9}$$

$$i\rho J\lambda_n \varphi_n - (a_0 \phi_{n,xx} - \mu_0 u_{n,x} - \mu_2 T_{n,x} + \beta_1 \theta_n - \xi\phi_n) \to 0 \text{ in } L^2, \tag{10}$$

$$ia\lambda_n \theta_n + \beta_0 v_{n,x} + \beta_1 \varphi_n - k\Upsilon(\theta_n)_{xx} - k_1 \Upsilon(T_n)_x \to 0 \text{ in } L^2, \tag{11}$$

$$ib\lambda_n T_n + \mu_2 \varphi_{n,x} + k_3 \Upsilon(\theta_n)_x - (k_4 \Upsilon(T_n)_{xx} - k_2 \Upsilon(T_n)) \to 0 \text{ in } L^2, \tag{12}$$

In view of the dissipation and the assumptions on the coefficients we see that $\vartheta_x, \vartheta_{xx}, S, S_x$ and S_{xx} tend to zero in L^2 and, therefore, θ and T also tend to zero in L^2.

If we divide convergence (12) by λ_n and we multiply it by $\phi_{n,x}$, we find

$$ib\langle T_n, \phi_{n,x}\rangle + i\mu_2 ||\phi_{n,x}|| - k_4 \langle \lambda_n^{-1} \Upsilon(T_n)_{xx}, \phi_{n,x}\rangle - k_2 \langle \lambda_n^{-1} \Upsilon(T_n), \phi_{n,x}\rangle \to 0.$$

As $T_n, \Upsilon(T_n)_{xx}$ and $\Upsilon(T_n)$ tend to zero, $\phi_{n,x}$ is bounded and μ_2 is assumed different from zero we obtain that $\phi_{n,x} \to 0$ in L^2. If we multiply convergence (10) by ϕ_n we see

$$-J||\varphi_n||^2 + a_0||\phi_n||^2 - \mu_2\langle T_n, \phi_{n,x}\rangle - \beta_1\langle\theta_n, \phi_n\rangle + \xi||\phi_n||^2 \to 0.$$

As $\phi_{n,x} \to 0$ in L^2 it follows that φ_n also tends to zero in L^2. In a similar way, if we divide convergence (11) by λ_n and we multiply it by $u_{n,x}$ we obtain that

$$ia\langle\theta_n, u_{n,x}\rangle + i\beta_0||u_{n,x}|vert^2 - k\langle\lambda_n^{-1}\Upsilon(\theta_n)_{xx}, u_{n,x}\rangle - k_1\langle\lambda_n^{-1}\Upsilon(T_n)_{xx}, u_{n,x}\rangle \to 0.$$

As $\theta_n, \Upsilon(\theta_n)_{xx}$ and $\Upsilon(T_n)$ tend to zero, $u_{n,x}$ is bounded and β_0 is different from zero we also see that $u_{n,x} \to 0$ in L^2. After the multiplication of convergence (8) by u_n and following a similar way to the one used to prove the convergence of φ_n we get that $v_n \to 0$ in L^2. It contradicts the assumption that the elements of the sequence have unit norm, so we conclude that $i\mathbb{R} \subset \rho(\mathcal{A})$.

Now, we want to prove that condition (6) also holds. To this end we can assume that this condition does not hold. Therefore, there exist a sequence of real numbers λ_n such that $|\lambda_n| \to \infty$ and a sequence of unit norm vectors $U_n = (u_n, v_n, \phi_n, \varphi_n, \theta_n, T_n)$ in the domain of the operator satisfying (7)- (12). In this situation we can repeat the analysis proposed to show that the imaginaty axis is contained at the resolvent because the key point is to note that the sequence λ_n does not tend to zero. Thus, we arrive to a contradiction and condition (6) also holds.

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