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MODEL ORDER REDUCTION OF SYSTEMS FOR ACTIVE VIBRATION AND NOISE CONTROL

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Abstract. This paper presents a framework for the design of active vibration control (AVC) and active noise control (ANC) systems. The framework is composed of a finite element (FE) model, model order reduction (MOR) methods and software for system-level simulations. The finite element method (FEM) is used to develop an experimentally verified model of the coupled structural, acoustic and piezoelectric problem. This model serves as an example for the study and discussion of modal and moment matching based MOR approaches. Finally, a reduced model is used to design an active control approach, which proves the feasibility of the framework.

1 INTRODUCTION

Many of today's structures, designed to reduce weight and material costs are more susceptible to vibrations than traditionally-designed structures. This lightweight design may cause increased noise, vibration and fatigue problems. The vibration and the sound radiation are often reduced by installation of active vibration and noise control. Piezoelectric sensors and actuators are linked with a controller that adapts itself to changing operating conditions. To be effective, there is a need to study different designs using numerical simulations in order to evaluate adaptive control strategies and feasible sensor or actuator concepts.
The FEM is a well established tool to set up customizable models of structures. It is then possible to create configurable dynamic models for the mechanical as well as the acoustic domains. Researchers proposed finite element formulations for piezoelectric [1]

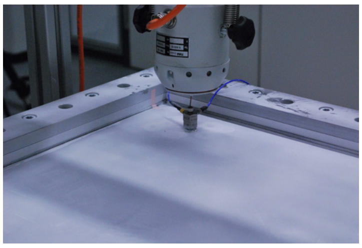
and acoustic problems which are implemented in commercial FEM codes by now.
To predict the real dynamic behaviour of the structure, it is effective to perform an experimental modal analysis (EMA). Based on these results, the FEM model can be tuned towards its real behavior. Unfortunately, the FEM models do not satisfy the requirements of the subsequent computer aided design of control systems which is carried out in the time domain. The dimension of these FEM models can be so large that time integration becomes inefficient or even prohibitive. An approach to solve this issue is by the application of model order reduction [2] methods. These techniques approximate the dynamical model by one of a smaller dimension while preserving its input-output behavior.
After a description of the demonstration object used for this paper, the set-up of its FE model is explained in Section 2. The model is verified using measured data. The test preparation and the results are presented in Section 3. Among the model order reduction approaches, a modal technique [3] for unsymmetric system matrices and a moment matching method via Krylov subspaces [4] by means of the Arnoldi process are introduced in Section 4. In the remainder of this paper, tests are executed in order to compare the performance of these methods with each other. Selected components of the acoustic box are reduced and the performances, as well as the results, are compared in Section 5. After this preprocessing, the reduced model is imported into the simulation software MATLAB/Simulink. The updated and reduced model is used to implement a control approach in order to show its capability. Within the MATLAB/Simulink environment, the interaction of structure, actuators, sensors and controller is optimized (Sec. 6) until the magnitude of vibration or sound radiation is minimal. Finally, the work is concluded and further research is outlined.

2 THE FINITE ELEMENT MODEL

In order to study and test the sound transmission, to develop reduction methods for sound radiation [5], an acoustic demonstrator was manufactured at the LOEWE-Zentrum AdRIA. The demonstrator consists of a cuboid box (Fig. 1a) with sound-reflecting walls. The top of the box is covered by a clamped elastic aluminium plate (Fig. 1b). The box is stiff, compared to its cover, and sound transmission through the box is negligible. Preliminary studies showed that this assumption holds true up to 500 Hz. It is possible to study and test acoustical behavior and smart structure systems for noise reduction in a frequency range from 0 to 500 Hz.

For this demonstration object FEM models were set up using the FE package ANSYS 12.1. Assuming that the acoustic fluid inside the box is incompressible, inviscid and that there is no mean flow of the fluid and density and pressure are constant throughout the fluid, the acoustic cavity was discretized using the 3-D acoustic element FLUID30. The element has eight corner nodes with four degrees of freedom (DOF) per node. These are the translations in the nodal x-, y- and z-directions and the pressure. The walls of the box are not modelled, because they are assumed to be stiff have no sound transmission. This effect is simulated when no absorption at the boundary is applied and the nodal

(a)



(b)

Figure 1: AdRIA acoustic box: a) general view, b) top view.

translation of the elements in the cavity is deactivated. The fluid-structure interaction (FSI) is induced by a layer of coupling elements between the cavity and the plate. Considering the partial differential equations (PDE) of this acoustics fluid-structure coupling, the discretization of this equation by means of the FEM yields a system of N ordinary differential equations:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{D}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{B^{in}}\mathbf{f}(t) \tag{1a}$$
$$\mathbf{y}(t) = \mathbf{B_1^{out}}\mathbf{q}(t) + \mathbf{B_2^{out}}\dot{\mathbf{q}}(t)\,. \tag{1b}$$

where $\mathbf{M}(t)$, $\mathbf{D}(t)$, $\mathbf{K}(t)$ are the system matrices, $\mathbf{B^{in}}\mathbf{f}(t)$ are the loads, and $\mathbf{q}(t) = (\mathbf{u}(t)\ \ \mathbf{p}(t))^T$ is a vector of unknown degree of freedom, where $\mathbf{u}(t)$ is the mechanical displacement and $\mathbf{p}(t)$ is the acoustic pressure. For the sake of simplicity, the time-dependence of the variables will be dropped from further calculation. The mass, damping and stiffness matrix $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}$ are assembled as follows:

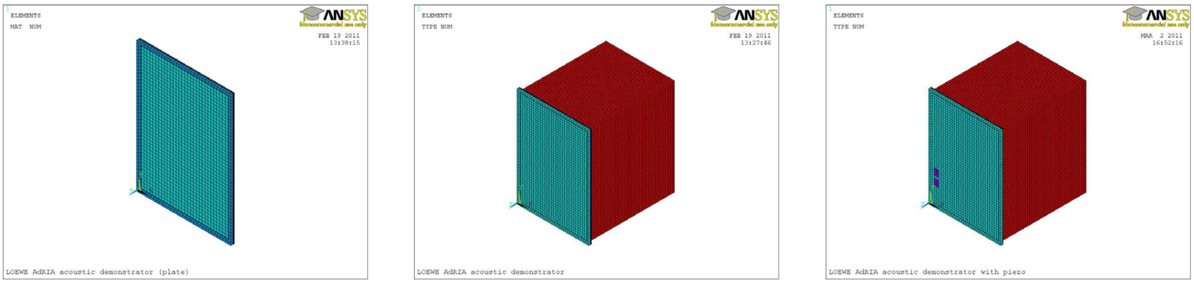
$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_{uu} & \mathbf{0} \\ \mathbf{M}_{pu}^{fs} & \mathbf{M}_{pp} \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \mathbf{D}_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{pp} \end{pmatrix}, \mathbf{K} = \begin{pmatrix} \mathbf{K}_{uu} & \mathbf{K}_{up}^{fs} \\ \mathbf{0} & \mathbf{K}_{pp} \end{pmatrix}. \tag{2}$$

The mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$ are both unsymmetric [6]. The loads are $\mathbf{B^{in}}\mathbf{f} = (\mathbf{F}\ \mathbf{0})^T$. When piezoelectric transducers are applied to the aluminum plate of the box, the FE formulation have to be extended. Adding the governing equations and the linear piezoelectric material law to the formulation yields [1]:

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_{uu} & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_{pu}^{fs} & \mathbf{M}_{pp} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \mathbf{D} = \begin{pmatrix} \mathbf{D}_{uu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{pp} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{\phi\phi} \end{pmatrix} \mathbf{K} = \begin{pmatrix} \mathbf{K}_{uu} & \mathbf{K}_{up}^{fs} & \mathbf{K}_{u\phi} \\ \mathbf{0} & \mathbf{K}_{pp} & \mathbf{0} \\ \mathbf{K}_{u\phi}^T & \mathbf{0} & -\mathbf{K}_{\phi\phi} \end{pmatrix}. \tag{3}$$

The mass, damping, and stiffness matrices from ANSYS 12.1 are non-symmetric and/or singular. The loads and the degree of freedom vector are assembled $\mathbf{B^{in}}\mathbf{f} = (\mathbf{F}\ \mathbf{0}\ \mathbf{Q})^T$ and $\mathbf{q} = (\mathbf{u}\ \mathbf{p}\ \phi)^T$ respectively. The matrix properties make demands on the MOR algorithms, which are discussed later in this paper (see Sec. 4).

To study the MOR techniques, three FEM models were set up. First, a pure mechanical



(a) (b) (c)

Figure 2: FEM models: a) plate, b) plate and cavity c) plate, cavity and piezo patches.

model of the aluminium plate was realized (Fig. 2a). The plate was discretized using eight nodes structural elements that exhibit linear displacement behavior. Certainly quadratic elements are more recommended due to their better strain approximation. However these set-up may require contact elements at the interface to the fluid in subsequent analyses, because no quadratic fluid elements are available. However such elements complicate the model but provide no additional intelligence. The linear elements were used accordingly. The support of the plate which consists of a frame and the rubber sealing (Fig. 1b) was modeled by means of linear elements as well.
For the analysis of the coupled structural acoustic behaviour the FE model was extended. This acousto-mechanical model is depicted in Fig. 2b. Assuming that the walls of the box are rigid (up to 500 Hz only), the volume of the cavity was modelled. The volume was discretized using FLUID30 elements which features fluid medium behaviour and the interface in fluid/structure interaction problems. The third model is identical to the latter, however piezoceramic patch actuators (Fig. 2c) were added at the left side of the plate in order to design active noise and vibration control.

3 EXPERIMENTAL VERIFICATION OF THE MODEL

In order to validate the coupled vibro-acoustical model, two EMA tests were carried out. In the first test the FE model without the acoustic cavity was validated using measured data which were available from previous studies [7]. The purpose of this was to find a feasible model for the plate clamping. Assuming that the clamping is roughly similar to a fixed support, the Young's modulus of the discretized sealing was adjusted [8] until the averaged relative error of the measured and calculated eigenfrequencies became 2.4 %. The second EMA was used to validate the coupled mechano-acoustical model. The quantities to be measured for the experimental vibro-acoustical modal analysis are the excitation of the structure and fluid as well as the displacement, the velocity or the acceleration responses. A force applied to the mechanical part of the structure or a defined volume displacement to the acoustical fluid are feasible excitations. In the present case, the excitation was realized by applying a point force with an electromechanical shaker (see Fig. 1b). The driving point spectra were captured with an impedance sensor, which allows the acquisition of force and acceleration simultaneously. The structure responses were measured with a laser Doppler vibrometer (LDV) at 1276 points of the plate, which are nearly coincident with the nodes of the FE mesh. Finally, the acoustic responses were recorded using a microphone inside the lower right corner of the cavity. A frequency domain multiple degree of freedom (MDOF) analysis using the PolyMAX algorithm, which leads to a modal model of the coupled system was conducted. There exist 15 structural and 13 acoustical modes in the frequency range up to 500 Hz. The 4,1 mode and the mode at 279.10 Hz were not detected using this set-up. The results of the measurement were used to tune the FEM model towards the real eigenfrequencies (Table 1). The relative

Table 1: Eigenfrequncies of the acousto-mechanical system.

No.	mode shape	f_{EMA} [Hz]	damp	f_{FEM} [Hz]	rel. error
1	1,1	61.63	1.71	60.51	1.824
2	1,2	98.839	1.2	98.09	0.757
3	2,1	144.888	0.89	140.61	2.953
4	1,3	165.332	0.97	163.23	1.271
5	2,2	182.849	0.81	182.15	0.382
6		197.449	0.14	198.76	-0.664
7		224.981	0.15	226.01	-0.457
8	2,3	246.434	0.8	247.16	-0.295
9	1,4	256.418	0.8	252.49	1.532
10	3,1	278.922	0.31	269.33	3.439
11				279.10	
12		297.964	0.21	299.97	-0.673
⋮	⋮	⋮	⋮	⋮	⋮
29	4,2	493.171	1.14	505.71	-2.543

error is almost less then 1%. For the comparison of experimental and numerical results, the modal assurance criterion (MAC) [9] of the plate displacement is depicted in Fig. 3. For the 1^{st} to the 5^{th}, the 8^{th}, the 9^{th} and the 17^{th} mode, the MAC value is 100% which implies very good correlation. The 6^{th}, the 7^{th} and the 16^{th} mode are acoustic resonances, therefore a MAC value of 100% is not mandatory. When the 12^{th} and 13^{th}, the 14^{th} and 15^{th}, or 18^{th} and 19^{th} mode is considered, one can see that their mode shapes are similar or even equal because of the structural-acoustic coupling. However, the correlation of the FEM model and the experiment are good and the model is valid for further studies.

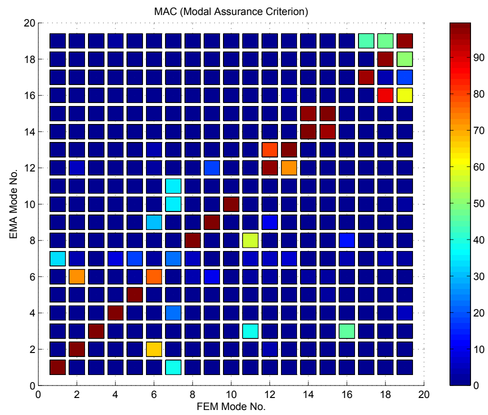


Figure 3: 2-D presentation of MAC Values.

4 MODEL ORDER REDUCTION

A common approach [2] to model reduction is to find a transformation $\mathbf{V}_n$ to a low-dimensional subspace. This transformation $\mathbf{q} = \mathbf{V}_n\mathbf{q}_n + \varepsilon$ should approximate the system behavior accurately within an error bound and project Eq. 1b onto that subspace. Therefore the reduced system becomes:

$$\mathbf{M}_n\ddot{\mathbf{q}}_n + \mathbf{D}_n\dot{\mathbf{q}}_n + \mathbf{K}_n\mathbf{q}_n = \mathbf{B}_n^{in}\mathbf{f} \tag{4a}$$

$$\mathbf{y} = \mathbf{B}_{1,n}^{out}\mathbf{q} + \mathbf{B}_{2,n}^{out}\dot{\mathbf{q}}, \tag{4b}$$

where $\mathbf{M}_n = \mathbf{V}_n^T\mathbf{M}\mathbf{V}_n$, $\mathbf{D}_n = \mathbf{V}_n^T\mathbf{D}\mathbf{V}_n$, $\mathbf{K}_n = \mathbf{V}_n^T\mathbf{K}\mathbf{V}_n$, and $\mathbf{B}_n^{in} = \mathbf{V}_n^T\mathbf{B}^{\mathbf{in}}$. The transformation matrix $\mathbf{V}_n$ can be determined by different methods. For this paper, the component mode synthesis (CMS) [10], a moment matching method via Krylov subspaces [11] and the modal reduction of non-symmetric systems [3] were studied.

4.1 Component Mode Systhesis

The component mode synthesis (CMS) was first proposed by Hurty [12] and further developed by Craig and Bampton [10]. The method has been developed with the purpose of analysing a complex structure as an assembly of less complex sub-structures. After reduction of the size of each sub-structure, all reduced models are then assembled into the global model, which has a much smaller size compared to the physical model. Considering a sub-structure A the physical DOFs are partitioned into boundary DOFs $\mathbf{q}_b^A$ and internal DOFs $\mathbf{q}_i^A$. The latter set is reduced by replacing it with the vector $\mathbf{p}_N$ of the generalised modal coordinates. The transformation matrix $\mathbf{V}_n$ for A is given by:

$$\mathbf{q}^A = \mathbf{V}_n\mathbf{p}^A = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{\Phi}_C & \mathbf{\Phi}_N \end{pmatrix} \begin{pmatrix} \mathbf{q}_b^A \\ \mathbf{p}_N^A \end{pmatrix} \tag{5}$$

In Eq. 5, $\mathbf{\Phi}_C$ is the matrix of the constraint modes of the sub-structure A. The matrix $\mathbf{\Phi}_N$ represents a truncated set of normal modes computed from the internal DOFs of A

when all boundary nodes kept fixed. This procedure is only applicable provided that the sub-structures system matrices are symmetric and positive semidefinite. Considering the discrete undamped equation of motion for the sub-structure A, after substitution of $\mathbf{q}_b^A$ the sub-system matrices $\mathbf{M}^A$, $\mathbf{K}^A$ and the load mapping $\mathbf{B^{A,in}}$ are transformed (Eq. 4b) into a low-dimensional subspace. The reduced model is generated, when this method is applied to different sub-structures of the demonstration object, like the plate, the cover frame or the cavity, and these are assembled.

4.2 Moment Matching

The concept of the projection-based moment matching approach is to find a projection matrix $\mathbf{V}_n$ so that the leading term of a Taylor series expansion of the transfer function matches for the reduced and the original system. An efficient method for engineering applications is moment matching via Krylov subspaces by means of either the Arnoldi or the Lanczos process. For this paper, a first order Krylov subspace was studied. Therefore The second order system can be converted into a descriptor first-order state space system of size $2N$:

$$\mathbf{C}\dot{\mathbf{x}} + \mathbf{G}\mathbf{x} = \mathbf{B}\mathbf{u} \tag{6a}$$

$$\mathbf{y} = \mathbf{L}\mathbf{x}\,. \tag{6b}$$

The state vector $\mathbf{x}$ is the concatenation of the first and second time derivative $\mathbf{x} = (\dot{\mathbf{q}}\,\mathbf{q})^T$ and the matrices $\mathbf{C}$, $\mathbf{G}$, $\mathbf{B}$, and $\mathbf{L}$ were assembled as follows:

$$\mathbf{C} := \begin{pmatrix} \mathbf{0} & \mathbf{X} \\ \mathbf{M} & \mathbf{D} \end{pmatrix}, \mathbf{G} := \begin{pmatrix} -\mathbf{X} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{pmatrix}, \mathbf{B} := \begin{pmatrix} \mathbf{0} \\ \mathbf{B^{in}} \end{pmatrix}, \mathbf{L} := \begin{pmatrix} \mathbf{B_2^{out}} & \mathbf{B_1^{out}} \end{pmatrix}, \tag{7}$$

where $\mathbf{X}$ is an arbitrary regular matrix. In order to obtain symmetric matrices $\mathbf{C}$ and $\mathbf{G}$ the matrix $\mathbf{X}$ is often set to $\mathbf{X} = -\mathbf{K}$ or $\mathbf{X} = \mathbf{M}$ in the case that $\mathbf{M}$ and $\mathbf{K}$ are symmetric. For this section the descriptor representation was chosen, because the matrices $\mathbf{C}$ and $\mathbf{G}$ are the input parameters of the utilized 1$^{\text{st}}$ order Krylov subspace method. The transfer function $\mathbf{H}(s)$ is developed by applying the Laplace transformation to (6) and eliminating the Laplace transform $\hat{\mathbf{x}}$ of $\mathbf{x}$ which results:

$$\mathbf{H}(s) = \frac{\hat{\mathbf{y}}(s)}{\hat{\mathbf{u}}(s)} = \mathbf{L}\left(\mathbf{G} + s\mathbf{C}\right)^{-1}\mathbf{B}\,. \tag{8}$$

The transfer function of the reduced-order model of size n that approximates the input-output behaviour of (8) is given by:

$$\mathbf{H}_n(s) = \mathbf{L}_n\left(\mathbf{G}_n + s\mathbf{C}_n\right)^{-1}\mathbf{B}_n\,. \tag{9}$$

The concept of the projection-based moment matching approach is to find a projection matrix $\mathbf{V}_n$ so that the leading term of a Taylor series expansion of $\mathbf{H}(s)$ and $\mathbf{H}_n(s)$ are matched. This can be done by means of a block Krylov-subspace method. Based on this, the transformation matrix $\mathbf{V}_n$ may be obtained by execution of the Arnoldi process listed in Algorithm 1 presented in [11]. Therefore, the matrices of the reduced-order model are defined as follows:

$$\mathbf{G}_n := \mathbf{V}_n^T \mathbf{G} \mathbf{V}_n,\ \mathbf{C}_n := \mathbf{V}_n^T \mathbf{C} \mathbf{V}_n,\ \mathbf{B}_n := \mathbf{V}_n^T \mathbf{B},\ \mathbf{L}_n := \mathbf{L} \mathbf{V}_n\,. \tag{10}$$

The aforementioned algorithm was implemented using the uBLAS C++ template class library and the parallel sparse direct solver MUMPS 4.8 in the MORAS software.

4.3 Modal reduction of non-symmetric systems

Classical modal reduction can be applied only to symmetric positive semidefinite systems with Rayleigh damping. Obviously, none of these properties is satisfied by the present system (3). Therefore, we have developed a generalized modal approach, which is based on the first order representation (6), projects the system on both, left and right eigenspaces, and accounts for higher order modes by static correction. More precisely, we construct reduced versions of (6):

$$\begin{aligned} \mathbf{C}_n\,\dot{\mathbf{x}} + \mathbf{G}_n\,\mathbf{x} &= \mathbf{B}_n\,\mathbf{u} \\ \mathbf{y} &= \mathbf{L}_{n,1}\,\mathbf{x} + \mathbf{L}_{n,2}\,\mathbf{u} \end{aligned} \tag{11}$$

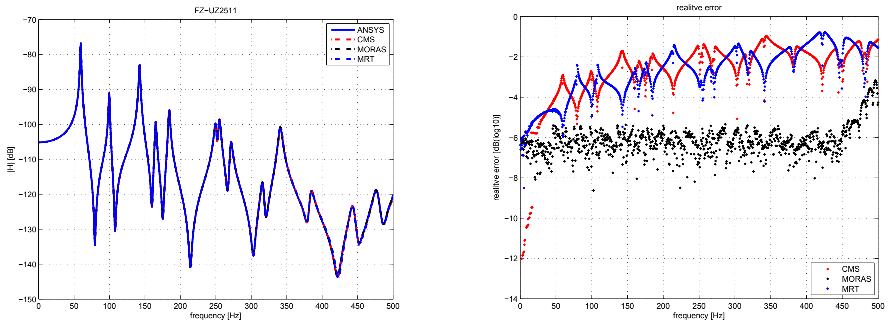
using the following algorithm:

$$\begin{aligned}
&\texttt{Choose shift } s_0 \texttt{ not being eigenvalue.}\\
&\texttt{Set } \mathbf{T} = -\left(\mathbf{G} + s_0\,\mathbf{C}\right)\ ,\ \mathbf{F} = \mathbf{T}^{-1}\,\mathbf{B}\ ,\ \mathbf{H} = \mathbf{T}^{-1}\,\mathbf{C}\ .\\
&\texttt{Compute incomplete Schur factorizations of desired size}\\
&\mathbf{H}\,\mathbf{V} = \mathbf{V}\,\mathbf{S}\ ,\ \mathbf{H}^*\mathbf{W} = \mathbf{W}\,\tilde{\mathbf{S}}\ .\quad \texttt{With } \mathbf{J} = \mathbf{W}^*\mathbf{V} \texttt{ set}\\
&\mathbf{C}_n = \mathbf{J}\,\mathbf{S}\ ,\ \mathbf{G}_n = -\left(\mathbf{J} + s_0\,\mathbf{C}_n\right)\ ,\ \mathbf{B}_n = \mathbf{W}^*\mathbf{F}\ ,\ \mathbf{L}_{n,1} = \mathbf{L}\,\mathbf{V}\\
&\mathbf{L}_{n,2} = \mathbf{L}_{n,1}\,\mathbf{J}^{-1}\,\mathbf{B}_n - \mathbf{L}\,\mathbf{F}\ .
\end{aligned} \tag{12}$$

In the Fraunhofer model reduction toolbox (MRT) [3], we use ARPACK [13] and the LU-decomposition of $s_0^2\,\mathbf{M} + s_0\,\mathbf{D} + \mathbf{K}$ in order to compute the Schur factorizations in an efficient way.

5 NUMERICAL RESULTS

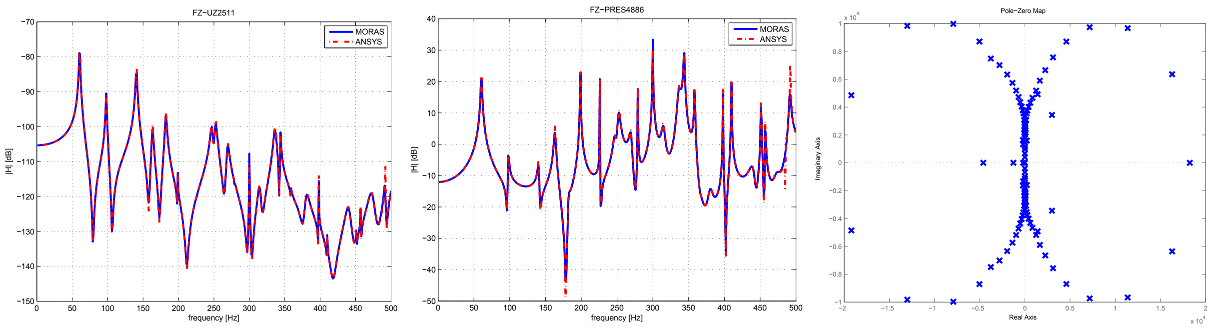
Generally, the CMS is applicable to acousto-mechnical problems [14] but not yet implemented in ANSYS. For this reason, the performance of the CMS, MORAS and MRT was compared when the FE model (11420 DOFs) of the pure mechanical plate is reduced to a 60 DOF first order system. The transfer functions (Fig. 4a) and the relative error computed against the full ANSYS model (Fig. 4b) are depicted in decibel scale. The best results for this model were achieved when using the Krylov subspace method (Fig. 4b).



(a) (b)

Figure 4: a) Transfer function of the mechnical model, b) error plot.

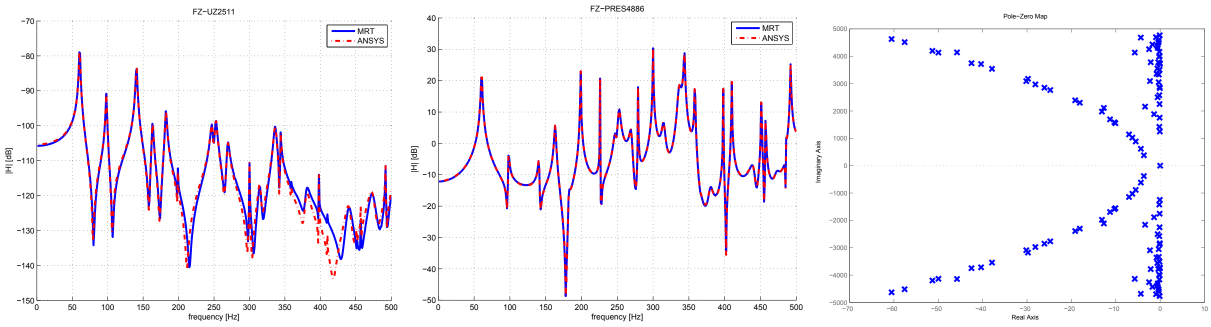
But considering the usual uncertainties of engineering applications, all MOR methods produce suitable results. Secondly, acousto-mechanical model (69020 DOFs) was reduced to 120 DOFs using moment matching via Krylov subspaces. In order to check the dynamic response the system was excited by a z-direction force in the lower right corner. The mechanical transfer function (Fig. 5a) and the acoustical transfer function (Fig. 5b) show a strong correlation up to 500 Hz, however when this method is applied the system becomes unstable, with poles appearing in the left plane of the pole plot (Fig. 5c). Proof



(a) (b) (c)

Figure 5: a), b) Transfer function of the acousto-mechnical model, c) pole map.

of this instability is presented by [11]. This paper proves that passivity and stability is guaranteed only if $\mathbf{G} + \mathbf{G}^T \geq 0$ and $\mathbf{C} = \mathbf{C}^T \geq 0$ are positive semidefinite and if matrix pencil $\mathbf{G} + s\mathbf{C}$ is regular. But the matrix $\mathbf{C}$ never becomes symmetric because of the unsymmetric mass matrix $\mathbf{M}$ [8] (Eq. 7 and Eq. 2). To overcome this issue, the modal reduction of non-symmetric systems was developed and this method was also applied to the model. The test used for the Krylov subspace method was executed using the Fraunhofer MRT code. The results are plotted in Fig. 6. The transfer functions are consistent up to 350 Hz and the systems remains stable (Fig. 6c). As a result, this model can be



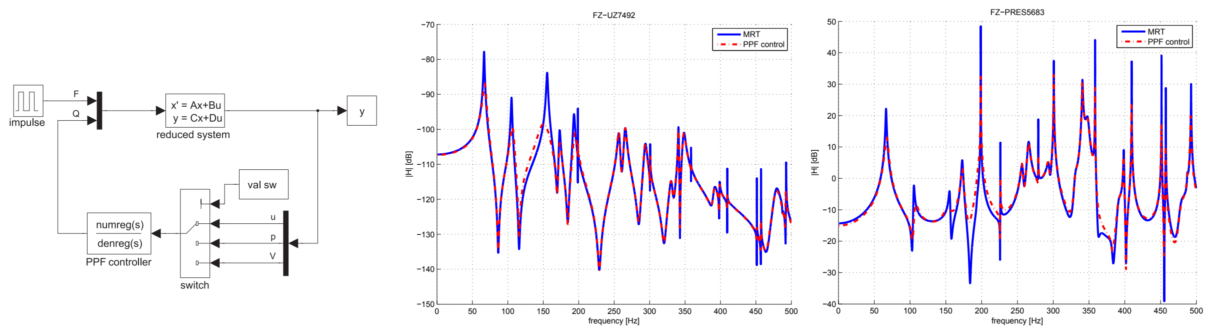
(a) (b) (c)

Figure 6: a), b) Transfer function of the acousto-mechnical model, c) pole map.

used for the controller implementation.

6 CONTROLLER IMPLEMENTATION

To show the capability of the reduced model for the development of active vibration control and active noise control, a system-level simulation was developed [8]. A FEM model with plate, cavity and piezo patches (98310 DOFs) was reduced to 60 DOFs and a six-mode positive position feedback (PPF) was implemented. The MATLAB/Simulink model, depicted in Fig. 7a, indicates the reduced system and the controller. The system was excited by a z-direction force impulse and the response recorded when the controller was switched on and off. With the active controller, the displacement response is decreased 7 dB, 8 dB, 14 dB, 3 dB and 4 dB for the 1st, the 2nd, the 3rd, the 4th and the 5th mode, respectively (Fig. 5a).



(a) (b) (c)

Figure 7: a) Simulink model, transfer function: b) force-displacement, c) force-pressure.

7 CONCLUSION

This paper has considered the MOR of the AdRIA acoustic demonstrator for design of active vibration control. Three different model order reduction techniques were evaluated, however only the modal reduction of non-symmetric systems was able to produce stable reduced models of the acoustic demonstration object. This model was used to set up a system-level simulation, which demonstrates the feasibility of this tool chain for the design of active vibration and active noise control.
This proposed framework enables researchers to efficiently model, simulate and study active structures, including acoustic cavities, with attached actuators and sensors. The MOR of the large FE model speeds up the simulations process, which helps to save significant time and costs.
The demonstration object used in this paper was covered by a plane plate. Currently, one-way and two-way curved shells covering acoustic cavities [5] are studied in order to minimize the of sound radiation of active structures. Therefore, parametric-reduced models of coupled mechanical, acoustic and electrical smart structure would be beneficial. Researcher [3, 15] have proposed promising approaches based on the interpolation of the system matrices. Research in the application of these methods to the demonstration object covered by curved shells will be the focus of future work.

8 ACKNOWLEDGMENT

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