ADAPTIVE CALIBRATION OF A NONLOCAL COUPLED DAMAGE PLASTICITY MODEL FOR ALUMINIUM ALLOY AA6082 T0

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Abstract. Continuum Damage Mechanics (CDM) accounts for material degradation (softening and ultimately failure) by modifying the load-bearing properties of the material (stiffness and strength) through a special state variable referred to as damage. Damage is typically represented by a scalar or a higher dimension object (such as vector or tensor) with values between zero for virgin material and unity for the material that lost all its bearing capacity. Considered in this way, damage becomes an additional field quantity that needs to be considered along with strain and stress, and can be computed either incrementally, or as a certain function of a suitable physical parameter such as inelastic strain. The advantage of enriching the formulation of a continuum deformation problem with a damage parameter is that it allows considering the material post-critical behaviour, i.e. its response under deformations exceeding those when the maximum load-bearing capacity is reached. Typically, this post-critical behaviour is associated with strain localisation, initiation, growth and interaction of discontinuities, and final fracture. Within the CDM framework, cracks are represented by diffuse regions of material damaged so that it lost all its strength in at least one direction. Computationally, modelling the post-critical (softening) behaviour of material represents a challenge in terms of the numerical stability of algorithms. Nonlocal description of damage appears to offer a rational route towards stable modelling. Nonlocal averaging of the plastic strain for the evaluation of damage also renders CDM models independent of the mesh size and orientation, and helps overcome numerical instabilities. The formulation that emerges can be referred to as coupled nonlocal damage-plasticity modelling [1, 2].

An important challenge remains, however, in developing this general approach into a flexible and material-specific modelling tool. This concerns the need to calibrate a large number of material parameters that emerge in this formulation. In order to address this challenge, recently we developed an approach for the calibration of CDM models of ductile materials that we propose to refer to as adaptive calibration. The calibration of the damage function is accomplished by matching the model prediction to the experimental data obtained from a single tensile test with multiple gauge length extensometry [3] used to capture strain localisation and size effects. We describe the application and validation of this approach to the damage function parameter calibration for the aluminium alloy AA6082 T0. Excellent agreement with experimental measurements is obtained.
1 INTRODUCTION

The approaches to the issue of structural analysis in the aeronautical industry have evolved over the decades. Original methodologies relied on the principle of avoiding crack initiation in order to ensure fatigue resistance and durability. Later, damage tolerant design principles were introduced, in which the presence of small defects (cracks) was accepted as unavoidable, and emphasis was placed on ensuring that they do not grow to critical lengths within the exploitation intervals between inspections. In the current view, not only crack initiation, but also crack propagation and trajectory analysis are very important for ensuring safe design and operation. Aeroengine components are subjected to complex loading induced by the combination of mechanical loading, changing temperatures and thermal gradients, inducing plastic deformation and creep that ultimately may lead to crack initiation and propagation. Along with an increasing proportion of composite materials, many aerospace components continue to be made from ductile metallic materials, such as nickel-base superalloys, titanium and aluminium alloys. In these materials rupture (material separation) is preceded by damage that finds its physical representation in void nucleation, growth and coalescence. The development of populations of defects in ductile materials is responsible for material softening that is played out in competition with the usual strain hardening behaviour often observed when ductile metal alloys are subjected to (tensile) loading. Modelling the behaviour of these materials in the post-peak softening regime and the correct and reliable prediction of crack propagation even today represent a serious challenge.

Predicting efficiently crack propagation rates and directions has been the aim of many researches since the emergence and the development, in the 1970’s, of numerical methods such as Finite Elements (FE). The traditional approach consists in representing the crack as a discontinuity of the modelled structure, for example, introducing cohesive zone elements [4] on the assumed crack path or using the Extended Finite Element Method (XFEM) [5]. This so-called discontinuous approach to crack problems is well-suited to describing the failure process of brittle materials in which the crack path is known in advance (e.g. laminate composites). However, if one wants to model a failure process (e.g. ductile failure) happening in a less sudden manner, in which the deformation process is more homogeneously distributed and for which the crack path is not known \textit{a-priori}, then discontinuous crack approach becomes problematic. Therefore, simultaneously with the development of discontinuous crack modelling methods, a second approach, called Continuous Damage Mechanics (CDM) [6, 7, 8] has been proposed and explored. Material degradation is taken into account through the use of a state variable whose value varies between zero (for virgin material) and unity (for fully damaged material) and that modifies the material stiffness and strength. Coupling between damage and plasticity is introduced by making the damage variable dependent on the equivalent plastic strain. The method is particularly attractive in the sense that it allows to take into account some of the most fundamental aspects of the failure process that are particularly important in the context of ductile rupture (e.g. as material softening, strain localization and size effects) and also to grow a crack without knowing the path in advance. However, in the course of advancing this approach to modelling both crack initiation and propagation in an internally consistent fashion, two major shortcomings of the CDM method have been identified:
1. Material softening and strain localization that occur at the late stage of the deformation process (i.e. when the damage becomes high enough to overcome the strain hardening) are responsible for numerical instabilities and loss of ellipticity of the governing equation [9].

2. In terms of the effect of the finite element mesh size, it is observed that the energy dissipated during the failure process is reduced with mesh refinement. In the extreme case of an infinitely fine mesh, the classical CDM models predict no energy dissipation during the failure process; a conclusion that is clearly physically unrealistic. This is a manifestation of the phenomenon known as mesh-dependency.

A further characteristic feature of CDM-type models is that they do not capture the discontinuous nature of cracks precisely in the form of interfaces that open up between adjacent volumes of material. Instead, cracks are represented as diffuse regions (bands) of damaged material. The damage variable reaches the ultimate value of unity at the mid-line of these bands, so that the mid-line can be associated with the crack line.

Over the years, various modifications have been elaborated aiming at making CDM models more reliable. A classical solution consists in replacing the “conventional” equivalent plastic strain by its weighted average over a sphere of radius $R$ (the nonlocal equivalent plastic strain) in the damage function formulation [10]. Nonlocal models have been first applied to describe the failure behaviour of brittle materials [10, 11]. Recently, the authors of the present paper proposed a model aiming at describing the failure behaviour of ductile materials [1, 2, 12]. Until now, most efforts have been directed to resolving the numerical issues in the 1D [1] and 2D (plane stress) [2, 12] formulations. In the present contribution, a closer look is taken at the identification of the material parameters used by the model.

In the approach described here, the model parameter calibration is realized using experimental data obtained from a single tensile test in which multiple gauge length extensometry is used to capture the strain localisation and size effects. The material considered in the present study is a ductile aluminium alloy AA6082 T0 [3]. The material plastic hardening behaviour is described by a combination of linear and Chaboche isotropic hardening law. The damage behaviour is captured using a novel approach called adaptive calibration that allows to extract the material behaviour in the post-peak softening regime without recourse to complex or iterative parameter identification procedures (e.g. Markovian optimisation [13], least squares fitting [14], Levenberg-Marquardt algorithm [15]).

Firstly, the idea of adaptive damage calibration method [16] is briefly introduced and reviewed. Secondly, the damage law (the damage evolution as a function of the nonlocal equivalent plastic strain) that is obtained by the adaptive calibration procedure is approximated as a simple piece-wise function in order to ease its use within the finite element implementation. Finally, advantages and drawbacks of the method are discussed.
2 EXPERIMENTAL PROCEDURE

Flat dogbone specimens of the shape shown in Figure 1 were prepared from an aluminium alloy AA 6082 T0 by mechanical milling of 3 mm-thick sheets. The T0 heat treatment was chosen so as to bestow low strength and high ductility on the material. The specimens were solution treated at 570°C for various amounts of time depending on their thickness, and air cooled afterwards.

![Figure 1: Dog-bone specimen design (from [3]).](image)

Tensile testing until complete failure by rupture was performed using a screw-driven Hounsfield universal testing machine with Servocon control and acquisition system (see Figure 2). The crosshead speed was set to 2 mm/min. As illustrated in Figure 2(b), white bands of flexible paint were printed on the specimen surface so as to obtain a screen of contrasting markers. The positions of band boundaries were continuously measured with a Fiedler Optoelektronik laser scanning extensometer (with a spatial resolution of about 0.5 µm). Continuous recording of band boundaries positions allows the determination of extension (and average strain) between any two band boundaries at any time during the deformation process. Laser extensometry thus provides the records of load-extension curves for multiple gauge length specimens within one experiment.

![Figure 2: Laser extensometry](image)

Figure 2: Laser extensometry a/ Experimental setup - b/ Dog-bone specimen: markers and the multiple gauge length definition (from [3]).
The experimental technique chosen for this study thus allows plotting multiple Load-Extension curves based on the data collected from a single dog-bone specimen experiment. As illustrated in Figure 2(b), a marker located at some small stand-off distance above the fracture line can be chosen as reference. All other markers located below the fracture line are used to define “sub-specimens” of gauge lengths $h_i$. Knowing the time evolution of marker positions allows to calculate the extension of every "sub-specimen" (of initial length $h$) at any time during the deformation process. The interpretation of laser extensometry results gives rise to experimental curves such as those presented in Figure 3. It also makes it possible to calculate the strain anywhere in the bar and at any time during the deformation process. This particular capability is of crucial importance for the determination of the characteristic material length (the nonlocal radius $R$).

![Figure 3](image.png)

**Figure 3:** Load-extension curves of a 3 mm-thick dog-bone specimen obtained for sub-specimen of initial lengths: $h_1$=23 mm, $h_2$=17 mm, $h_3$=11 mm, $h_4$=7 mm, and $h_5$=3 mm.

As stated in previous publications on the subject [17, 18], the information needed to calibrate CDM models (i.e. the characteristic material length, the damage function shape and the associated parameters values) must be contained in the Load-Extension curves presented in Figure 3. Providing that a suitable calibration procedure for the model parameters is used, a successful finite element simulation of the tensile test on a dog-bone specimen must provide simultaneous satisfactory agreement with all of the individual stress-strain curves presented in Figure 3.
3 DUCTILE FAILURE MODELLING

3.1 The model

The multiple gauge length tensile test described in Section 2 above was first represented in the form of an 1D FE model. The model consisted of a bar with one extremity kept fixed whilst the other one was subjected to incremental outward displacement (see [1]). The deformation of the material elements along the bar that ensued is described using the 1D version of the coupled nonlocal damage-plasticity model. The simulation involves sequential calculation of increments of stress, strains, damage within each element, and the tangential stiffness that must show agreement with the experimental results. This provides the basis for determining the damage function evolution with deformation by the means of the adaptive calibration procedure described below.

3.2 The adaptive calibration method

For the purpose of parameter identification, the assumption is made that at early stage of the deformation process, the damage has very little effect on the material stress-strain response. The Young's modulus $E$ of 73 GPa was found by fitting the initial linear behaviour by a straight line, the material yield stress at 0.2% of plastic strain, $\sigma_Y$, was fixed at 50 MPa. An Excel spreadsheet was setup in order to vary the description of the plastic behaviour by a combined linear and Chaboche type isotropic hardening function [19, 16] (see (1) below). The best fit material parameters of the model were found to be $h=50$ (to account for the linear contribution to strain hardening), and $b=37.5$ and $Q=67.5$ (Chaboche function parameters), leading to the overall expression:

$$\sigma_f = \sigma_y + h\varepsilon_p + Q(1-e^{-b\varepsilon_p})$$  \hspace{1cm} (1)

In equation (1), $\sigma_f$ and $\varepsilon_p$ stand respectively for the current yield stress (i.e. modified by the deformation history) and the plastic strain at instant $t$. An optimal fit of the experimental curve can be achieved using automatic methods (e.g. least-squares) for the determination of the parameters $h$, $Q$ and $b$.

During testing, the continuous recording of paint band boundary positions allowed following the evolution of the strain distribution along the specimen. At the early stages of the deformation process, the strain in the bar remained low and homogeneously distributed. During the elastic stage of the deformation process, and all the way to the critical point of maximum overall load borne by the specimen, the strain distribution remained uniform. This part of the process corresponds to the conditions when strain “levelling” effects due to hardening dominate over localization (promoted by damage). The localization of plastic strain, into an area of approximately 4 mm, happened at a relatively late stage of the deformation process, once the critical elongation was exceeded, and necking (damage and strain localization) began. Previous analysis [16] of the effect of the nonlocal radius on the strain distribution just before failure suggest that $R$ should therefore be set to 4 mm.
The concept of adaptive damage calibration was described extensively in [16]. The idea of the adaptive damage function simulation procedure is as follows. Firstly, let us note the similarity between the incremental description of the processes of damage and plastic deformation. For a given material volume at any stage in its deformation history, the prior behaviour is already known and stored in the computer memory. The subsequent behaviour is found in a stepwise fashion, at each stage only for a short time period corresponding to a small increment of deformation. Wishing to apply this principle to the description of the damage process, we note that at any particular stage in the damage process it is not necessary to know the entire damage function. Rather, it suffices to know the instantaneous slope of the damage function. This allows the calculation of the effective tangent stiffness, and hence the determination of the subsequent deformation increment.

Therefore, the following procedure for adaptive damage function calibration was put forward. In the first instance, the threshold value of plastic strain was chosen beyond which the damage process was likely to begin. As will be seen further, guessing a value for this parameter that is too low is not detrimental to the accuracy, and may only result in some slowing down of the algorithm. Other aspects of the calculation also need to be set at this stage, namely, the strain increment to be used, and the initial guesses for the slope of the damage function (these can be conveniently chosen to be zero and a non-negative number, e.g. 0.1). The calculation of the next segment of the stress-strain curve is then carried out twice: without damage (or, more precisely, with the damage function slope of zero), and with the damage switched on. This results in two predictions in the form of linear segments describing the stress-strain curve over the next increment. The data available from the experimental stress-strain curve is then used to guide the correct selection of the damage function to deliver the best match. This process can be accelerated by using linear interpolation, and repeated incrementally, until certain tolerance is attained. The procedure is then repeated for the next and subsequent increments. In this way, a piecewise linear description of the damage function dependence on the plastic strain is built up. One remarkable aspect of this approach is that this piecewise linear description readily affords itself to be used as a lookup table for further calculation. Therefore only the damage in the most deformed element need to be adjusted. Damage in the other elements just follows the evolution that has already happened and been recorded in the most deformed element.

The procedure described here is adaptive in the sense that it determines the damage function profile in a way that ensures incremental adjustment so as to preserve the agreement with the experimental data throughout the deformation history. The results of the application of this procedure are illustrated in Figure 4 for the rising (box 1) and falling (box 2) parts of the stress-strain curve. It is apparent that the step size and initial guesses for the damage function slope could be chosen to deliver satisfactory agreement for both parts of the experimental curve.
The adaptive damage calibration method makes the assumption that the damage function is piecewise linear. Two initial guesses are made for the initial slope of the damage function, and computation is carried out based on both guessed values. The intermediate slope of the damage function that allows good match to the experimental data is then determined by linear interpolation.

Figure 5(a) illustrates the damage function curve that was obtained after the adaptive simulation was carried out until complete sample failure. It is worth noting here that an artificial modification was applied to the damage curve for the range of damage parameter values $0.4 < \alpha_d < 1$. When the damage approaches 0.4, unstable and rapid failure takes place. This process is too fast for the quasistatic loading machine to capture correctly, so that the experimental points obtained beyond this stage are not fully meaningful any more. Subsequent damage evolution to the value of unity is modelled by prescribing a very high slope to the damage function. This smooth approach to unity is dictated by numerical stability reasons.

Figure 5(b) illustrates the quality of the resulting simulation. The stress-strain curves for all the sub-specimens considered are captured equally well, indicating that the size effect and the energetic characteristics of the plastic rupture process are adequately represented by the model. It is worth noting here that the adaptive damage process has been applied only to the analysis of the load-displacement response of the longest sub-specimen. The observed high quality agreement between the model and the experimental data for all other sub-specimens thus furnishes a good evidence of the accuracy of the nonlocal characteristic length determination, and of the overall coupled nonlocal damage model.
Figure 5: a) Damage function obtained from the adaptive damage calibration procedure – b) Load-extension curves for a 3 mm-thick dog-bone specimen obtained for sub-specimens of initial lengths: $h_1=23$ mm, $h_2=17$ mm, $h_3=11$ mm, $h_4=7$ mm, and $h_5=3$ mm. Model predictions are continuous curves superimposed on the experimental data (markers).

3.3 Damage function: a simple analytical formulation

Although the match between the model and the experimental data looks satisfactory, the adaptive damage calibration method suffers from one major drawback: the resulting damage function is stored in the form of a table corresponding to a certain number of data points. This makes the use of the damage function in an FE code (e.g. in 2D plane stress) somewhat tedious. We therefore propose to fit the obtained damage function by a piece-wise combination of two functions, one for the range $0<\alpha_d<0.4$, and the other for the range $0.4<\alpha_d<1$. Figure 6 illustrates that satisfactory description is obtained by using the following equation:

$$f = \begin{cases} 
0.5785 \times (\bar{\varepsilon}_p - 0.1293) & \text{if } 0 < \alpha_d < 0.4 \\
1 - 8 \times (\bar{\varepsilon}_p - 1.3)^4 & \text{if } 0.4 \leq \alpha_d < 1
\end{cases}$$

(2)

Figure 7 illustrates the load-extension curves obtained from the simulation based on the approximation of the damage function described above. The load-extension curves for all sub-specimens considered appear to have been captured relatively well. It is clear that equation (2) provides only an approximation of the correct description of the damage function evolution. However, the accuracy obtained appears to be good enough to justify using this formulation to streamline and speed up FE simulations of the damage process. It can be concluded, furthermore, that abrupt change in the damage function slope introduces a similar discontinuous slope feature in the simulation. Consequently, it appears logical to adopt a rule that relative changes in the damage function slope must be restricted to ensure smoothness of the simulated load-displacement curves.
Figure 6: The damage function obtained using the adaptive calibration method (in blue) is fitted by a linear function (in pink). Instable crack propagation is modelled by a quadratic function (in red).

Figure 7: Load-extension curves of a 3 mm-thick dog-bone specimen obtained for sub-specimen of initial lengths: \( h_1 = 23 \text{ mm} \), \( h_2 = 17 \text{ mm} \), \( h_3 = 11 \text{ mm} \), \( h_4 = 7 \text{ mm} \), and \( h_5 = 3 \text{ mm} \). Model predictions are superimposed on experimental data. The damage function proposed in Figure 7 (green line) is used.
4 DISCUSSION AND CONCLUSION

The adaptive damage calibration procedure combined with the approximate analytical description of the damage function provides a convenient framework for simulating post-critical softening. Once the calibration is obtained, the damageable material model can be run efficiently within the finite element framework.

It is apparent from Figure 3 and Figure 7 that the post-critical part of the load-displacement curve is only captured well until the damage parameter value reaches the value of 0.4 or so. In order to capture the subsequent evolution of the load-displacement curves beyond this stage, high rate acquisition of displacement and load is required.

Accomplishing both of these tasks may be possible by employing a suitably fast imaging camera, such as the Vision Research Phantom v7.3 high speed camera capable of over 6000 frames per second at full resolution of 800×600 pixels (and up to 0.5M frames per second at reduced resolution). However, it is important to note that dynamic deformation effects would need to be taken into account, so that an explicit Finite Element formulation must be used.

REFERENCES


