

SOME CONSEQUENCES OF THE QUALITATIVE ANALYSIS OF THE POINT-SYMMETRIC COUPLED CONSOLIDATION MODELS

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Summary. *The point-symmetric linear coupled consolidation models, known from the theory of the oedometric testing and from dissipation testing, can be summarized into a single mathematical model in the function of the embedding space dimension m ([1]). When a set of boundary conditions is specified equally for the 1, 2 and 3 dimensional models (i.e. oedometric, cylindrical and spherical models) then a family of related model: a “model-family” is obtained.*

Some inferences of the results of the qualitative analysis of two model-families are presented and discussed in this paper. These are (i) the similarity of the solution within a model-family, (ii) a direct proof that the uncoupled consolidation theories cannot be considered as a special case of the coupled consolidation theories and (iii) the interesting fact that an instantaneous dissipation may be predicted for the uniform initial pore water pressure distribution if the displacement is specified at both boundaries.

1 INTRODUCTION

Consolidation models are commonly employed in the theory of the oedometric testing and dissipation testing. These may differ in terms of the formulation of the constitutive equations and the assumed boundary conditions ([1]). In particular, models often differ in the nature of the condition specified at the outer boundary of the consolidating zone, which can be in terms of either a prescribed radial displacement (v) or a prescribed volumetric strain (ϵ).

In this paper, a single point-symmetric, coupled, linear consolidation constitutive model is considered in a generalised form that can be expressed as a function of the embedding space dimension, m . When the same set of boundary conditions is adopted for the 1, 2 and 3 dimensional models (i.e. oedometric, cylindrical and spherical models), then a family of related models, a “model-family”, is obtained. Two model families (referred to as coupled 1 and coupled 2) are considered here, based on different boundary condition assumptions. A summary of existing consolidation models and their classification within this framework is given in Table 1.

Table 1 Summary and classification of existing consolidation models.

V or ε boundary condition	1D point-symmetric (Oedometric models)	2D point-symmetric (Cylindrical pile models)	3D point-symmetric (Spherical pile models)
no (uncoupled)	Terzaghi [2]	Soderberg [5]	Torstensson [8]
v - v (coupled 1)	Imre [3]	Imre & Rózsa [6]	Imre & Rózsa [9]
v - ε (coupled 2)	Biot [4]	Randolph et al [7]	Imre & Rózsa [10]

2 MODEL

The basic units of the model-families (coupled 1 and 2), differing in one boundary condition, are presented in this section. The models are one dimensional with embedding space of various dimensions, m , as shown in Figure 1.

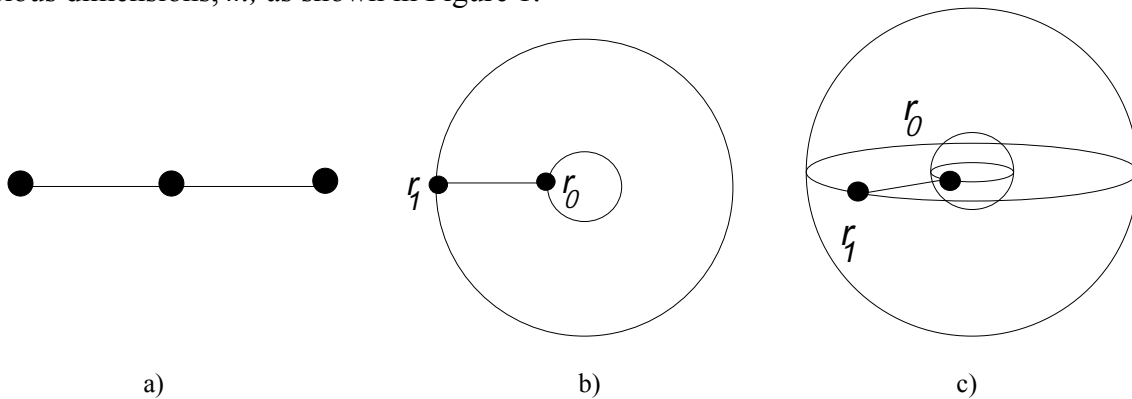


Figure 1. The displacement domain for the point-symmetric models bounded by a (a) 0 dimensional sphere, (b) 1 dimensional sphere, (c) 2 dimensional sphere.

2.1 System of differential equations

Two equations are derived ([1]): one from the equilibrium condition and one from the continuity condition. Equation (1) combines the equilibrium condition, the effective stress equality, the geometrical conditions and the constitutive equations:

$$E_{oed} \frac{\partial \varepsilon}{\partial r} - \frac{\partial u}{\partial r} = 0 \quad (1)$$

Equation (2) compiles the continuity equation, Darcy's law and the geometrical arrangement:

$$-\frac{k}{\gamma_w} \Delta u + \frac{\partial \varepsilon}{\partial t} = 0 \quad (2)$$

In equations (1) and (2), u is the pore water pressure (neglecting the gravitational component of the hydraulic head), ε is the volumetric strain, r and t are the space and the time co-ordinates respectively, k is the coefficient of permeability, γ_w is the unit weight of water, Δ is the Laplacian operator and E_{oed} is the oedometric modulus, which is expressed as:

$$E_{oed} = \frac{2G(1 - \mu)}{1 - 2\mu} = \frac{E(1 - \mu)}{(1 + \mu)(1 - 2\mu)}, \mu < 0.5 \quad (3)$$

In equation (3), G is the shear modulus, E is the Young's modulus, μ is the Poisson's ratio in terms of the effective stress σ' ($\sigma' = \sigma - u$ where σ is the total normal stress). The volumetric strain and the Laplacian operator, containing the dimension m of the embedding space, and expressed in terms of the radial displacement v , are as follows:

$$\varepsilon = \frac{1}{r^{m-1}} \frac{\partial}{\partial r} \left(r^{m-1} v \right) \quad (4)$$

$$\Delta = \frac{1}{r^{m-1}} \frac{\partial}{\partial r} \left(r^{m-1} \frac{\partial}{\partial r} \right) \quad (5)$$

2.2 Boundary conditions

The two model families considered here have three of their boundary conditions in common. These are (for $m=2$)

(1) The (common) *boundary condition #1*: zero pore pressure at distance r_1 .

$$u(t, r) |_{r=r_1} = 0 \quad (6)$$

(2) The (common) *boundary condition #2*: zero gradient in the pore pressure at distance r_0 .

$$\frac{\partial u(t, r)}{\partial r} |_{r=r_0} \equiv 0 \quad (7)$$

(3) The (common) *boundary condition #3*: constant, non-zero radial displacement at distance r_0 .

$$v(t, r) |_{r=r_0} \equiv v_0 > 0 \quad (8)$$

The coupled 1 and coupled 2 model families differ, however, in the nature of their fourth boundary condition. For the coupled 1 models, the condition is

(4a) *Boundary condition #4a* for the coupled 1 models: zero radial displacement at distance r_1

$$v(t, r) |_{r=r_1} \equiv 0 \quad (9)$$

(4b) *Boundary condition #4b* for the coupled 2 models: constant, non-zero volumetric strain at distance r_1

$$\varepsilon(t, r) |_{r=r_1} \equiv \varepsilon_1 > 0. \quad (10)$$

3 QUALITATIVE ANALYSIS

The transient part of the solutions, that is, the solution of the system of differential equations (1) and (2) with the homogeneous form of the boundary conditions, is qualitatively analysed in this section, without actually determining it.

3.1 Analysis of Equation (1)

Explicit expressions are derived for v and u by integrating the equilibrium equation (1) with respect to r subject to boundary condition #1:

$$u(t, y) = E_{oed} \varepsilon(t, r) - E_{oed} \varepsilon(t, r) \Big|_{r=r_1} \quad (11)$$

The boundary condition function can be determined by further integrating the Equation (11) with respect to r using boundary condition #3 and boundary condition #4a for the coupled 1 models:

$$\varepsilon(t, r) \Big|_{r=r_1} = - \left(\frac{\mathbf{u}_{mean}(t)}{E_{oed}} \right) \quad (12)$$

$$\mathbf{u}_{mean}(t) = \frac{r_0 \int_{r_1}^{r_1} r^{n-1} u(t, r) dr}{\int_{r_0}^{r_1} r^{n-1} dr} \quad (13)$$

Equation (11) can also be integrated with respect to r using boundary condition #3 and the homogeneous form of boundary condition #4b for the coupled 2 models, which results in the zero function.

Coupled 1 models

Using Equation (11) and the boundary condition function:

$$\varepsilon(t, r) = \frac{1}{E_{oed}} \left[u(t, r) - \mathbf{u}_{mean}(t) \right] \quad (14)$$

$$v(t, r) = \frac{I}{r^{n-1} E_{oed}} \left(\int_{r_1}^r r^{n-1} u(t, r) dr - \mathbf{u}_{mean}(t) \int_{r_1}^r r^{n-1} dr \right) \quad (15)$$

The initial condition functions for u and v^t have the following relationships:

$$u_0(r) = E_{oed} \mathcal{E}_0^t(r) - E_{oed} \mathcal{E}_0^t(r) \Big|_{r=r_1} \quad (16)$$

$$v_0^t(r) = \frac{I}{r^{n-1} E_{oed}} \left(\int_{r_1}^r r^{n-1} u_0(r) dr - u_{0,mean} \int_{r_1}^r r^{n-1} dr \right) \quad (17)$$

It is apparent that the $v_0^t(r)$ is the zero function when the initial pore water pressure function $u_0(r)$ is uniform.

Coupled 2 models

Using Equation (11) and the boundary condition function:

$$\varepsilon(t, r) = \frac{u(t, r)}{E_{oed}} \quad (18)$$

$$v(t, r) = \frac{1}{E_{oed} r^{n-1}} \int_{r_0}^r r^{n-1} u(t, r) dr \quad (19)$$

The initial condition functions for u and v^t have the following relationships:

$$u_0(r) = E_{oed} \mathcal{E}_0^t(r) \quad (20)$$

$$v_0^t(r) = -\frac{1}{r^{n-1} E_{oed}} \int_{r_0}^r r^{n-1} u_0(r) dr \quad (21)$$

3.2 Analysis of Equation (2)

By integrating Equation (2) twice with respect to r using the homogenous form of the boundary conditions #2 and #1, the following explicit expression is derived for the pore water pressure:

$$u(t, r) = \frac{k}{\gamma_v} \int_{r_1}^r \frac{1}{r^{n-1}} \int_{r_0}^x r^{n-1} \frac{\partial \varepsilon^t}{\partial t} dr dx \quad (22)$$

By further integration with respect to t between 0 and ∞ to give $A(r)$, the area of the subgraph of the dissipation curve $u(t, r)$ can be expressed for any fixed r as follows, for the coupled 1 and 2 models, respectively:

$$A(r) = \int_0^\infty u(t, r) dt = \frac{1}{c} \int_{r_1}^r \frac{1}{t^{n-1}} \int_{r_0}^x x^{n-1} [u_0(x) - u_{0,mean}] dx dt \quad (23)$$

$$A(r) = \int_0^\infty u(t, r) dt = \frac{1}{c} \int_{r_1}^r \frac{1}{x^{n-1}} \int_{r_0}^x x^{n-1} [u_0(x)] dx dt \quad (24)$$

It can be observed that the $A(r)$ is equal to 0 for the coupled 1 models if the initial pore water pressure function $u_0(r)$ is uniform. It follows that dissipation must be instantaneous in this case.

4 DISCUSSION

4.1 Constructing the analytical solution

The analytical solution for the total and effective stress components can be constructed using the analytical solution of the pore water pressure and the displacement or volumetric strain, on the basis of the constitutive equations and the explicit expressions derived from Equation (1). As an example, we consider the cylindrical case here. The constitutive equations valid for embedding space dimension $m=2$ ([7]) are

$$\sigma'_r = -\frac{2G}{1-2\mu} \left[(1-\mu)\varepsilon - (1-2\mu)\frac{v}{r} \right] \quad (25)$$

$$\sigma'_\phi = -\frac{2G}{1-2\mu} \left[\mu\varepsilon + (1-2\mu)\frac{v}{r} \right] \quad (26)$$

$$\sigma'_z = -\frac{2G\mu}{1-2\mu} \varepsilon \quad (27)$$

For the coupled 1 model, the transient components of the effective stresses are

$$\sigma'_r(t, r) = -\frac{1}{1-\mu} \left[(1-\mu)[u(t, r) - \mathbf{u}_{mean}(t)] - \frac{1-2\mu}{r^2} \left(\int_{r_1}^r ru(t, r) dr - \mathbf{u}_{mean}(t) \int_{r_1}^r r dr \right) \right] \quad (28)$$

$$\sigma'_\phi(t, r) = -\frac{1}{1-\mu} \left[\mu[u(t, r) - \mathbf{u}_{mean}(t)] + \frac{1-2\mu}{r^2} \left(\int_{r_1}^r ru(t, r) dr - \mathbf{u}_{mean}(t) \int_{r_1}^r r dr \right) \right] \quad (29)$$

$$\sigma'_z(t, r) = -\frac{\mu}{1-\mu} [u(t, r) - \mathbf{u}_{mean}(t)] \quad (30)$$

For the coupled 2 model, the transient components of the effective stresses are

$$\sigma'_r(t, r) = -\frac{1}{1-\mu} \left[(1-\mu)u(t, r) - \frac{1-2\mu}{r^2} \int_{r_0}^r ru(t, r) dr \right] \quad (31)$$

$$\sigma'_\phi(t, r) = -\frac{1}{1-\mu} \left[\mu u(t, r) + \frac{1-2\mu}{r^2} \int_{r_0}^r ru(t, r) dr \right] \quad (32)$$

$$\sigma'_z(t, r) = -\frac{\mu}{1-\mu} u(t, r) \quad (33)$$

4.2 Radial stress behavior

For the coupled 1 models, the transient component of the radial effective stress is

$$\sigma'_r(t, r) = -\frac{2G}{1-2\mu} \left[(1-\mu)\varepsilon - (n-1)(1-2\mu)\frac{v}{r} \right]$$

$$= -[u(t, r) - \mathbf{u}_{mean}(t)] + \frac{(n-1)}{(1-\mu)} \frac{1}{r^n} \left[\left(\int_{r_1}^r r^{n-1} u(t, r) dr - \mathbf{u}_{mean}(t) \int_{r_1}^r r^{n-1} dr \right) \right] \quad (34)$$

$$\sigma_r^t(t, r_0) = \mathbf{u}_{mean}(t) - u(t, r_0) \quad (35)$$

For the coupled 2 models, the transient component of the radial effective stress is

$$\sigma_r^t(t, r) = -\frac{2G}{1-2\mu} \left[(1-\mu)\varepsilon - (n-1)(1-2\mu)\frac{v}{r} \right] = -u(t, r) + \frac{(n-1)}{(1-\mu)} \frac{1}{r^n} \int_{r_0}^r r^{n-1} u(t, r) dr \quad (36)$$

$$\sigma_r^t(t, r_0) = -u(t, r_0) \quad (37)$$

During dissipation, the radial total stress at r_0 decreases with time by the initial value of the mean pore water pressure for the coupled 1 models, but it is constant for the coupled 2 models. The radial effective normal stress at r_0 increases during dissipation, by the difference between the initial pore water pressure and the initial mean pore water pressure for the coupled 1 models, and by the initial pore water pressure for the coupled 2 models.

4.3 A theoretical consequence of the analysis of Equation (1)

For certain boundary conditions, a coupled model with irrotational displacement may have the same pore water pressure solutions as an uncoupled model (which can be derived under the assumption that the total stress state is constant: see e.g. [11]). The additional boundary condition constraint adopted by [3] can be summarized as follows. The equilibrium equation (1) is integrated to give

$$E_{oed}\varepsilon - u = K(t) \quad (38)$$

Using this, equation (2) can be expressed in terms of u .

$$-\frac{k}{\gamma_v} \Delta u = \frac{\partial u}{E_{oed} \partial t} + \frac{\partial K(t)}{E_{oed} \partial t} \quad (39)$$

Equation (2) reduces to Terzaghi's uncoupled consolidation equation (except that instead of the bulk modulus the oedometric modulus occurs) if the term $K(t)$ is constant for one value of the space coordinate: a condition that is met for the coupled 2 model-family.. However, despite this, the uncoupled model cannot be considered as a special case of the coupled 2 model.

We can derive an explicit expression for the first invariant of the transient part of the total stress tensor for the coupled 2 models using Hooke's law, the effective stress equality and the explicit expression derived for the volumetric strain on the basis of Equation (1) in terms of the pore water pressure. It is

$$\sigma_1^t(t, r) = \sigma_1^d(t, r) + 3u(t, r) = E_{oed} \frac{1+\mu}{1-\mu} \varepsilon(t, r) + 3u(t, r)$$

$$= \left[-\frac{1+\mu}{1-\mu} + 3 \right] u(t,r) = 2 \frac{1-2\mu}{1-\mu} u(t,r) \quad (40)$$

This term is constant if μ (the Poisson's ratio in terms of the effective stress) is equal to 0.5 which is physically impossible in case of consolidation (it implies the soil is incompressible).

4.4 A theoretical consequence of the analysis of Equation (2)

Although the time for consolidation is infinite, using Equation (2), a finite-valued function can be derived in terms of the initial condition for each model-family to characterize the rate of the dissipation at any point r (i.e. as the area of the subgraph of the dissipation curve $u(t,r)$ for fixed r).

According to these expressions, for the coupled 1 models the dissipation time depends on the difference between the initial pore water pressure and the initial mean pore water pressure and it has two zeros. For the coupled 2 models, the dissipation time depends on the initial mean pore water pressure and it has one zero. It follows from the derived expression for the coupled 1 models that the consolidation from a uniform initial pore water pressure distribution is instantaneous since the initial transient volumetric strain is identically equal to zero in this case. By contrast, for the coupled 2 models, the greatest possible dissipation times are predicted for consolidation from a uniform initial pore water pressure distribution.

According to the Zeroth law of Thermodynamics ([12]) The transport of any extensive quantity implies the existence of an intensive quantity, the homogeneous distribution of which is a precondition for equilibrium, and according to the Second law, the movement of an extensive quantity is caused by the inhomogeneous distribution of the intensive quantity, which tends to be eliminated (Theorem 2 of Thermodynamics). In consolidation phenomena, the extensive variable for seepage is the water mass or volume. The intensive variable for seepage is the total hydraulic head of the water phase. These are related through

$$h = z + \frac{u}{\gamma_w} \quad (41)$$

where z is the vertical distance from an arbitrary datum. In the models presented here, the effect of z was neglected assuming that $h = u/\gamma_w$.

The instantaneous dissipation phenomenon indicates that the intensive variable of seepage cannot be the hydraulic head alone. The initial transient effective stress determined from the equilibrium equation gives a precise answer whether a seepage occurs or whether dissipation is instantaneous.

4.5 The stress drop phenomenon in some experimental tests

These results can be considered in the contexts of two types of experimental tests with (basically) constant displacement boundary conditions. One is the oedometric relaxation test where the total stress and the pore water pressure are measured after a fast load imposition. The second is the simple rheological-type cone penetrometer test where the local side friction and cone resistance are measured after the steady penetration ceases. In the latter case the effect of

a stress release, which occurs as the elastic deformation of the rod is recovered when steady penetration is stopped, makes the boundary condition approximate, as the rod diameter slightly decreases and the rod length slightly increases.

Some typical oedometric relaxation test results showing ‘instantaneous stress drop’ can be seen in Figure 2, where t' is a modified time variable indicating the time elapsed since loading ceased ([13]). The rate of the stress decrease is constant, and about equal to the rate of the load imposition, with both being limited by the mechanical characteristics of the constant power servo-system used in the test (which may allow some partial unloading, also).

Figure 3 shows some rheological-type cone penetrometer test results, sampled over a large area and averaged for the various soil types. These also display some instantaneous drop in stress ([14]).

According to the theoretical results, instantaneous dissipation may occur in the tests with constant displacement boundary conditions, (irrespective of the space dimension m) if the initial pore water pressure distribution is constant. The initial pore water pressure distribution may be considered to contain a constant component on the condition that the load imposition is very fast. However, very fast loading rates may invoke a slightly different, more pronounced time dependency in the rheological response that is not described by the constitutive laws derived for well-known models ([15], [16]). Rapid loading may induce a discontinuity when load imposition ceases (i.e. stress drop phenomenon), also. For example, fast initial stress drop during soil relaxation tests with fast, partly-drained load imposition was reported by Whitman [15].

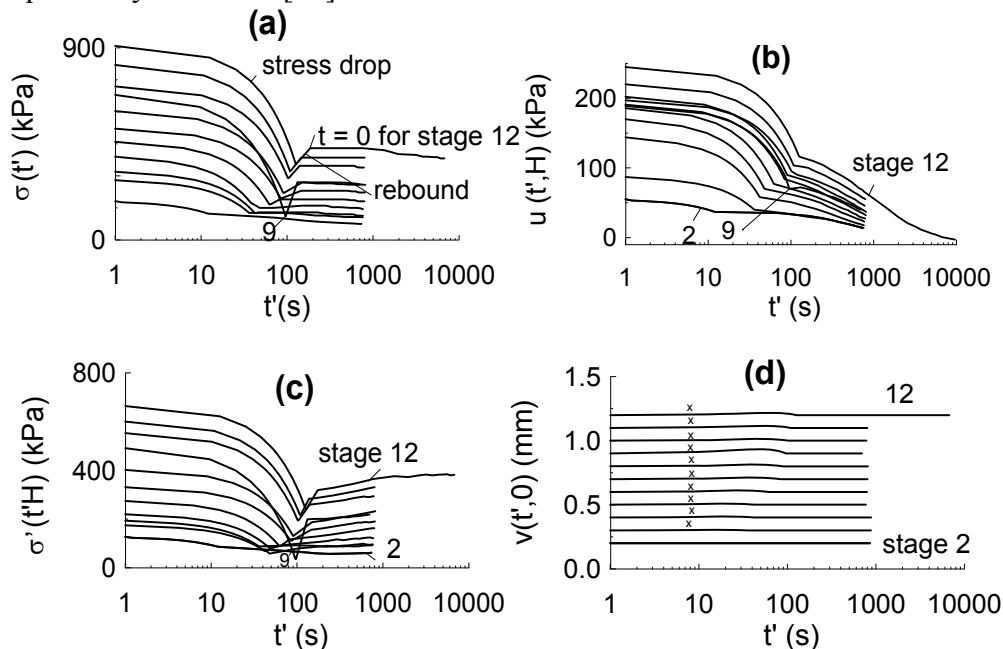


Figure 2. Oedometric relaxation data for a soft clay soil indicating the initial stress drop. (a) Measured total stress, (b) measured pore water pressure, (c) computed effective stress, (d) measured displacement [3]

The instantaneous dissipation response is not easily measured, since pore water pressure measurement is prone to error and the data measured during the stress drop cannot be considered as reliable for the oedometric relaxation test. The measured values are underestimated and there is delay in the measuring system ([17], [18]).

Two time dependent constitutive models that do accommodate some initial stress drop upon relaxation condition are Kelvin's model [20] and Leroueil et al's experimental model [16]. The reason for the sudden stress drop in the case of Kelvin's model is that the dashpot does not store energy, whereas in the case of Leroueil et al's model, it is because the strain rate of the sample changes discontinuously at the end of the loading period from a finite value to zero, while the mean strain is constant. These models fail to simulate the actual stress relaxation, since the stress is constant after the initial stress drop.

The compression curve constructed on the basis of multistage oedometric relaxation test is not the virgin compression curve unless the load imposition is fully drained. If the load imposition consists of a drained and undrained component, it can be assumed that the stress response relates to the drained part. According to some experimental results, the deviation is generally small for clays and may be considerable for freshly deposited loose silts. Larger deviation of the compression curve for loose soils can be experienced probably due to the fact that the effective stress may be negative within the sample (and local hydraulic fracturing may take place) if the preconsolidation pressure of the soil is small [19].

In the case of the rheological-type cone penetrometer test, it is very probable that an amount of stress drop also occurs due to the effect of stress release, as the elastic deformation of the rod is recovered and the rod diameter decreases when steady penetration ceases. However, this is not the case for the measured cone resistance.

In the light of the foregoing comments, the stress drop phenomenon can be attributed to

- the instantaneous dissipation of a constant component of the initial pore water pressure distribution if such component does exist,
- the change in the time dependency of the constitutive law,
- the fact that the soil may store a definite amount of energy,
- the stress release of the rod which modifies the boundary condition (in the case of the rheological type cone penetrometer tests).

5 CONCLUSIONS

The solution of the linear, point-symmetric, coupled consolidation models for two different sets of boundary conditions was qualitatively characterized. The same explicit expression could have been derived for the pore water pressure, strains or the displacements from Equation (1) for a given model-family.

From these, some explicit expressions can be derived for the total and effective stress components which are generally the same within a model family. It follows from this similarity that a point-symmetric problem with a specified embedding space dimension can be studied on the basis of a test related to different embedding space dimension.

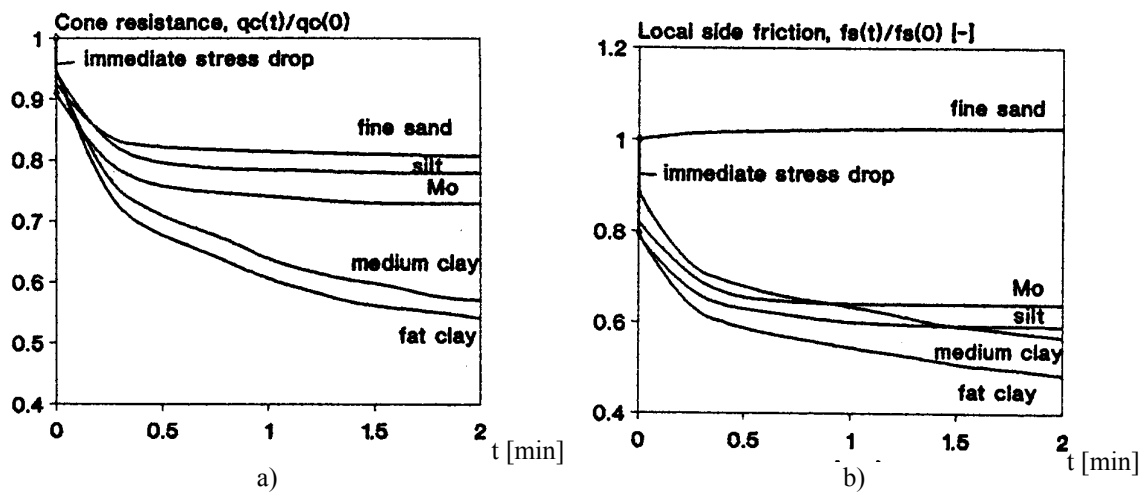


Figure 3. Results of simple rheological type CPT test indicating the immediate stress drop. (a) Local side friction vs. time data. (b) Cone resistance vs time data.

There are two special theoretical inferences coming from the qualitative analysis which can be concluded as follows. The pore water pressure solutions of uncoupled models and, the coupled 2 models are basically the same. However, the former cannot be derived from the latter, in the sense that the assumption of a constant total stress state of the uncoupled models cannot be verified.

In the case of the coupled 1 model-family, if the initial pore water pressure distribution is constant then the transient component of the initial volumetric strain is identically equal to zero, resulting in an instantaneous dissipation. This phenomenon indicates that the intensive variable of seepage, – which is a thermodynamic process – cannot be the total hydraulic head alone. If the distribution of the total hydraulic head is not uniform then the initial transient effective stress function, which can be determined from the equilibrium differential equation, gives a precise answer to whether there is a seepage or whether an instantaneous dissipation takes place.

It follows that some stress drop may theoretically occur for the tests with a constant displacement boundary condition if the initial pore water pressure distribution has a constant component. In a large number of tests with constant displacement boundary conditions, some kind of stress drop has been detected at the beginning of the test, which probably cannot be attributed to measurement errors, and its rate of decrease seems to be limited by the control system. However, further research is needed on the components and the cause of these stress drops since the measured records may entail some measuring errors.

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