4.3 NN

Exercise 1

Describe the topology for a multilayer network solving the problem of parity check with three coordinate vectors (3-3-1 Network).
Output z=1 (odd number of +1 in x)
Output z=-1 (odd number of +1)
Give the number of weights and propose some laws based on the hard limiting function: sign(x) to build the complete function.

Exercise 2

Show that the $\Delta\mathbf{w}[n]$ weight adaptation increment in the Levenberg-Marquadt algorithm is equivalent to $\Delta\mathbf{w}[n]$ that minimises the error function: $J[n+1] \simeq J[n] + \nabla J_{\mathbf{w}} \Delta\mathbf{w}[n] + \frac{1}{2}\Delta\mathbf{w}[n]^T \mathbf{H}\Delta\mathbf{w}[n] + \lambda\left(\left\|\Delta\mathbf{w}[n]\right\|^2 - K\right)$, where λ is a positive Lagrange Multiplier and K is a constant to control the trust-region where the approximation for $J[n+1]$ is valid.

Exercise 3 (Optativo)

En el diseño de redes neuronales basadas en funciones base radiales, demuestre que al minimizar la función de error $\frac{1}{2}Trace\left(\left(\Phi^{\mathbf{T}}(\mathbf{x})\mathbf{W}^{\mathbf{T}} - \mathbf{T}^{\mathbf{T}}\right)\left(\mathbf{W}\Phi(\mathbf{x}) - \mathbf{T}\right)\right)$ respecto a la matriz $\mathbf{W} \in \mathbb{R}^{Cxh}$, la solución obtenida es: $\mathbf{W}^T = \left(\Phi(\mathbf{x})\Phi^{\mathbf{T}}(\mathbf{x})\right)^{-1}\Phi(\mathbf{x})\mathbf{T}^{\mathbf{T}}$

Nota: Recuerde que $\nabla_{\mathbf{R}}\left(Trace\left(\mathbf{ARB}\right)\right) = \mathbf{A}^{\mathbf{T}}\mathbf{B}^{\mathbf{T}}$ with $\mathbf{A} \in \mathbb{R}^{axb}; \mathbf{R} \in \mathbb{R}^{bxc}; \mathbf{B} \in \mathbb{R}^{cxa}$

$$\nabla_{\mathbf{R}}\left(Trace\left(\mathbf{ARB}\right)\right) = \left(\nabla_{\mathbf{R}^{\mathbf{T}}}\left(Trace\left(\mathbf{ARB}\right)^{\mathbf{T}}\right)\right)^{\mathbf{T}}$$

$$\nabla_{\mathbf{R}^{\mathbf{T}}}\left(Trace\left(\mathbf{AR}^{\mathbf{T}}\mathbf{RB}\right)\right) = 2\mathbf{A}^{\mathbf{T}}\mathbf{B}^{\mathbf{T}}\mathbf{R}^{\mathbf{T}} \text{ with } \mathbf{A} \in \mathbb{R}^{axb}; \mathbf{R} \in \mathbb{R}^{cxb}; \mathbf{B} \in \mathbb{R}^{bxa}$$