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Improving the Robustness of Hamiltonian Passive Control *

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Abstract

Interconnection and damping assignment passivity based control (IDA-PBC) is a well known technique for port Hamiltonian dissipative systems (PHDS). In this paper we point out the kind of problems that can appear in the closed-loop structure obtained by IDA-PBC methods for relative degree one outputs, when nominal values are used in a system with uncertain parameters. In particular, we show that, in general, the positive semidefiniteness of the dissipation matrix breaks down, at least, in a neighborhood of the desired regulation point, preventing thus the use of LaSalle's theorem. Nevertheless, we present an example where the closed-loop system regulates to a fixed point, albeit different from the desired one. To correct this, we introduce an integral control, which can be cast into the Hamiltonian framework. Numerical simulations for our example show that the closed-loop system regulates to the desired point, although a rich dynamical behaviour is obtained when the feedback parameters are varied.

Keywords: *port Hamiltonian systems, passivity-based control, robust control.*

1   Introduction

The Interconnection and Damping Assignment-Passivity–based Control (IDA-PBC) is a technique based on the Port Hamiltonian Dissipative System (PHDS) formalism.

As discussed in [2][5][4] (and references therein) a large class of physical systems of interest in control applications can be modeled in PHDS framework. A general PHDS in explicit form is described by

$$\dot{x} = [J(x) - R(x)]\partial_x H + g(x)u \tag{1}$$

where x is the state in Hamiltonian variables, $H(x)$ is the total energy of the system[1], $J(x) = -J^T(x)$ is the interconnection matrix and $R(x) = R^T(x) \geq 0$ the dissipation matrix. It is easy to see that PHDS are passive with port variables $(u, g^T(x)\partial_x H)$, and storage function the total energy H.

The central idea of the IDA-PBC technique [9] is to, still preserving the PHDS structure, assign to the closed loop a desired energy function via the modification of the interconnection and dissipation matrices. That is, the desired target dynamics is a PHDS of the form

$$\dot{x} = [J_d(x) - R_d(x)]\partial_x H_d \tag{2}$$

where $H_d(x)$ is the closed-loop total energy and $J_d(x) = -J_d^T(x)$, $R_d(x) = R_d^T(x) \geq 0$, are the closed-loop interconnection and damping matrices, respectively. To achieve stabilization of the desired equilibrium point we impose

$$x^* = \arg\min H_d(x).$$

Equaling the open-loop and target closed-loop systems one gets

$$[J_d(x) - R_d(x)]\partial_x H_d = [J(x) - R(x)]\partial_x H + gu. \tag{3}$$

IDA-PBC techniques have been applied to a large class of physical systems, including electromechanical [10][1], mechanical underactuated [7], power electronics [3][11] and power system [6] models. The standard way to solve (3) is to fix the matrices $J_d(x)$ and $R_d(x)$—hence the name IDA—and then solve the PDE for $H_d(x)$. In general, solving the PDE is a very complicated task, which can be somehow eased by a judicious chose of J_d and R_d. Alternatively, one may try to fix H_d and solve the resulting algebraic equation for J_d and H_d. In any case, once the target system is obtained, asymptotic stability of x^* follows from $R_d \geq 0$ and the application of LaSalle's theorem to H_d, since the interconnection matrix does not contribute to the total energy variation and thus

$$\dot{H}_d = -(\partial_x H_d)^T R_d \partial_x H_d \leq 0, \tag{4}$$

and x^* is an invariant set of the closed loop dynamics.

Although the IDA-PBC method has some built-in robustness coming from its PHDS structure, the use of a nominal u for systems with uncertain parameters can give a closed-loop system which is not exactly PHDS. One may thing that for nominal parameters in a small neighborhood of the actual ones the "$J - R$" structure will not be destroyed; however, we will see that the resulting closed-loop system has interconnection and dissipation matrices depending on the state of the system, even if the closed-loop system for the actual parameter values does not; this has as a consequence that the effect of small parameter changes is not uniform in state space and, in particular, is unbounded in a neighborhood of the desired regulation point. In addition to this, the closed-loop system obtained with a nominal control does not have, in general, x^* as a fixed point. As is well known from elemental control theory, this last probelm can be corrected by adding control terms proportional to the integral of the error. Integral control has been considered in the

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[1]Gradients of functions are taken as column vectors to simplify the notation.

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literature in the PCHS setting in [8][12], where it is shown that adding as state variable the integral of the natural passive output of the closed-loop system yields a system which is again PCHS. However, to our knowledge, no detailed discussion of the robustness of the IDA-PBC method has been presented in the literature.

The paper is organized as follows. In Section 2 we present our main results for outputs of relative degree one, and introduce an integral control term to regulate to the desired point. In Section 3 we discuss in detail a simple 2-dimensional model, for which numerical simulations seem to point to better stability results that those which follow from the general discussion in Section 2. Finally, we state our conclusions and open problems in Section 4.

2 Relative degree one output regulation in the IDA-PBC framework

Consider a control system of the (non necessarily PHDS) form

$$
\begin{array}{rcl}
\dot{x}_i & = & f_i(x_i, x_u), \\
\dot{x}_u & = & f_u(x_i, x_u) + g(x_i, x_u)u,
\end{array}
\tag{5}
$$

where $x_i \in \mathbb{R}^i$, $x_u \in \mathbb{R}^u$, $u \in \mathbb{R}^u$ and $\det g \neq 0$, so that the x_u are relative degree one outputs which we want to regulate to desired values x_u^*. Given x_u^*, the fixed point values of x_i and u are obtained by equaling to zero the right-hand sides of (5).

Applying the IDA-PBC technique, we match the system to the desired PHDS

$$
\begin{pmatrix} \dot{x}_i \\ \dot{x}_u \end{pmatrix} = \begin{pmatrix} (J_d - R_d)_{ii} & (J_d - R_d)_{iu} \\ (J_d - R_d)_{ui} & (J_d - R_d)_{uu} \end{pmatrix} \begin{pmatrix} \partial_i H_d \\ \partial_u H_d \end{pmatrix}.
\tag{6}
$$

Equaling the first i rows of both systems yields the IDA-PBC matching equation

$$
f_i = (J_d - R_d)_{ii}\partial_i H_d + (J_d - R_d)_{iu}\partial_u H_d.
\tag{7}
$$

Assume that this equation can be solved, giving J_d, R_d and H_d satisfying the proper structural and control objective conditions. Substituting then into the equation coming from the last u rows, one gets the feedback control

$$
u = g^{-1}\left[(J_d - R_d)_{ui}\partial_i H_d + (J_d - R_d)_{uu}\partial_u H_d - f_u\right].
\tag{8}
$$

Assume now that the system (5) depends on some uncertain constant parameters ξ, for which we assume nominal values $\hat{\xi}$. The unknown parameters creep into the formalism through f_i (and f_u), making the solution to the matching equation (7) depend on them, and also through the desired values x_i^*, which appear in H_d and which may depend on ξ due to the fact that they must obey $f_i(x_i^*, x_u^*) = 0$. Hence, the nominal control is given by

$$
\hat{u} = \hat{g}^{-1}\left[(J_1 - R_1)\partial_i \hat{H}_d + (J_2 - R_2)\partial_u \hat{H}_d - \hat{f}_u\right],
\tag{9}
$$

where $J_1 - R_1$ and $J_2 - R_2$ are $(J_d - R_d)_{ui}$ and $(J_d - R_d)_{uu}$, respectively, computed with $\hat{\xi}$ instead of ξ.

The closed-loop system computed with the nominal control is

$$\dot{x}_i = (J_d - R_d)_{ii}\partial_i H_d + (J_d - R_d)_{iu}\partial_u H_d, \tag{10}$$

$$\dot{x}_u = f_u - g\hat{g}^{-1}\hat{f}_u + g\hat{g}^{-1}\left[(\hat{J}_d - \hat{R}_d)_{ui}\partial_i\hat{H}_d + \hat{J}_d - \hat{R}_d)_{uu}\partial_u\hat{H}_d\right]. \tag{11}$$

In the equation for x_i we can change H_d by $\hat{H}_d$ and put the balance terms into δ_i; denoting $\delta_u = f_u - g\hat{g}^{-1}\hat{f}_u$, we get a system of the form

$$\left(\begin{array}{c} \dot{x}_i \\ \dot{x}_u \end{array}\right) = \left(\begin{array}{cc} B_{ii} & B_{iu} \\ B_{ui} & B_{uu} \end{array}\right)\left(\begin{array}{c} \partial_i\hat{H}_d \\ \partial_u\hat{H}_d \end{array}\right) + \left(\begin{array}{c} \delta_i \\ \delta_u \end{array}\right). \tag{12}$$

The components of δ_i can be made proportional to components of $\partial_u\hat{H}_d$ by dividing by the corresponding factors; likewise, the components of δ_u can be made proportional to components of $\partial_i\hat{H}_d$ (one has a large amount of freedom in selecting the components of $\partial\hat{H}_d$ to which the extra terms are made proportional). After doing this, one gets

$$\left(\begin{array}{c} \dot{x}_i \\ \dot{x}_u \end{array}\right) = \left(\begin{array}{cc} B_{ii} & B_{iu} + \tilde{B}_{iu} \\ B_{ui} + \tilde{B}_{ui} & B_{uu} \end{array}\right)\left(\begin{array}{c} \partial_i\hat{H}_d \\ \partial_u\hat{H}_d \end{array}\right) \equiv \hat{A}_d\partial\hat{H}_d. \tag{13}$$

Notice that there are no singularities in the differential equations (13), since the singular terms in $\hat{A}_d$ are canceled by $\partial\hat{H}_d$.

Since any matrix can be decomposed into symmetric and skew-symmetric parts, we write

$$\hat{A}_d = \hat{J}_d - \hat{R}_d, \quad \hat{J}_d^T = -\hat{J}_d, \quad \hat{R}_d^T = \hat{R}_d. \tag{14}$$

Due to the $\tilde{B}_{ui}$ and $\tilde{B}_{iu}$ terms, the corresponding elements of $\hat{J}_d$ and $\hat{R}_d$ will contain terms which are singular at $x_i = \hat{x}_i^*$ or $x_u = x_u^*$. This is no formal problem for $\hat{J}_d$, but the presence of off-diagonal singular terms in $\hat{R}_d$ will destroy its positive semidefiniteness at least in a neighborhood of $(\hat{x}_i^*, x_u^*)$. Notice, however, that due to the presence of δ_i, δ_u the closed-loop system has fixed points which differ from $(\hat{x}_i^*, x_u^*)$; if $\hat{R}_d$ is positive semidefinite in a neighborhood of the closed loop fixed points, LaSalle's theorem can still be invoked to proof local asymptotic stability, albeit not for the desired regulation point.

In order to ensure the regularization objective in presence of the unknown parameter, an integral term is introduced in basic control theory. For relative degree one outputs, this can be given a Hamiltonian form as well [8].

First of all, we write $u = \hat{u} + v$ in the original system. This yields

$$\dot{x} = (\hat{J}_d - \hat{R}_d)\partial_x\hat{H}_d + gv. \tag{15}$$

Next, we assume that $\partial_u H_d = \gamma^{-1}(x_u - x_u^*)$, with γ^{-1} diagonal and positive definite, and enlarge the state space with $z \in \mathbb{R}^u$ so that

$$\dot{z} = -a\partial_u H_d = -a\partial_u\hat{H}_d, \tag{16}$$

with $a = a^T$ also diagonal and positive definite. This makes each component of z proportional to the integral of the error of the corresponding component of x_u. The closed-loop enlarged system can be written as

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The total system can be written as

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \hat{J}_d - \hat{R}_d & 0 \\ & a \\ 0 & -a & 0 \end{pmatrix} \begin{pmatrix} \partial_x \hat{H}_d \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ g \\ 0 \end{pmatrix} v + \begin{pmatrix} 0 \\ -az \\ 0 \end{pmatrix} \tag{17}$$

This is not in PCHS form due to the last term. However, choosing

$$v = g^{-1}az \tag{18}$$

yields a final closed-loop system of the form

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \hat{J}_d - \hat{R}_d & 0 \\ & a \\ 0 & -a^T & 0 \end{pmatrix} \partial \hat{H}_{dz}, \tag{19}$$

where

$$\hat{H}_{dz} = \hat{H}_d + \frac{1}{2} z^T z. \tag{20}$$

Due to the equation for $\dot{z}$, the only fixed points of the new closed-loop system are those with $x_u = x_u^*$. The equation for $\dot{x}_i$ determines then x_i^* in terms of x_u^* and the actual parameter values; finally, the equation for $\dot{x}_u$ sets the equilibrium value of z, z^*, in terms of the nominal parameter values.

We have now obtained a closed-loop system which has the desired regulation point as a fixed point for any nominal value of the parameters; however,

$$\hat{R}_{dz} = \begin{pmatrix} \hat{R}_d & 0 \\ & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{21}$$

has the same singularity problems that $\hat{R}_d$ in a neighborhood of x_u^*, and a proof of stability based on LaSalle's theorem cannot be given. Nevertheless, we will present an example in the next Section where the desired regulation point seems to be asymptotically stable.

3 A toy model example

To illustrate the quite general remarks of the previous Section, consider the following 2-dimensional nonlinear control system

$$\begin{aligned} \dot{x}_1 &= -x_1 + \xi x_2^2, \\ \dot{x}_2 &= -x_1 x_2 + u, \end{aligned} \tag{22}$$

where $\xi > 0$ is an uncertain parameter. This can be cast into PCHS form

$$\dot{x} = (J - R)\partial_x H + gu \tag{23}$$

with

$$J = \begin{pmatrix} 0 & x_2 \\ -x_2 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$H(x) = \frac{1}{2}x_1^2 + \frac{1}{2}\xi x_2^2, \quad g = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The control objective is to regulate x_2 to a desired value v_d. The equilibrium of (22) corresponding to this is given by

$$x_1^* = \xi v_d^2, \quad u^* = \xi v_d^3.$$

Using the IDA-PBC technique, we match (23) to

$$\dot{x} = (J_d - R_d)\partial_x H_d \tag{24}$$

with

$$J_d = \begin{pmatrix} 0 & \alpha(x) \\ -\alpha(x) & 0 \end{pmatrix}, \quad R_d = \begin{pmatrix} -1 & 0 \\ 0 & r \end{pmatrix}, \tag{25}$$

and

$$H_d(x) = \frac{1}{2}(x_1 - x_1^*)^2 + \frac{1}{2\gamma}(x_2 - v_d)^2, \tag{26}$$

where $\alpha(x_1, x_2)$ is a function to be determined by the matching procedure and $\gamma > 0$, $r > 0$ are adjustable parameters.

From the first row of the matching equation $(J - R)\partial_x H + gu = (J_d - R_d)\partial_x H_d$ one gets

$$-x_1 + \xi x_2^2 = -(x_1 - x_1^*) + \frac{\alpha}{\gamma}(x_2 - v_d),$$

from which

$$\alpha(x_1, x_2) = \frac{\gamma}{x_2 - v_d}(\xi x_2^2 - x_1^*) = \gamma\xi(x_2 + v_d). \tag{27}$$

Substituting this into the second row of the matching equation

$$-x_1 x_2 + u = -\alpha(x_1 - x_1^*) - \frac{r}{\gamma}(x_2 - v_d),$$

yields the feedback control law

$$u = x_1 x_2 - \gamma\xi(x_1 - x_1^*)(x_2 + v_d) - \frac{r}{\gamma}(x_2 - v_d). \tag{28}$$

This control law yields a closed-loop system which is Hamiltonian with (J_d, R_d, H_d), and which has (x_1^*, v_d) as a globally asymptotically stable equilibrium point. However, if we use an estimated value $\hat{\xi}$ of the uncertain parameter ξ, the feedback control is

$$\hat{u} = x_1 x_2 - \gamma\hat{\xi}(x_1 - \hat{x}_1^*)(x_2 + v_d) - \frac{r}{\gamma}(x_2 - v_d), \tag{29}$$

where

$$\hat{x}_1^* = \hat{\xi} v_d^2 = \frac{\hat{\xi}}{\xi}x_1^*.$$

For later convenience, we also define

$$\hat{\alpha} = \gamma\hat{\xi}(x_2 + v_d). \tag{30}$$

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Using this $\hat{u}$, the closed-loop system equation for $\dot{x}_2$ is

$$
\begin{aligned}
\dot{x}_2 &= -\gamma\hat{\xi}(x_1 - \hat{x}_1^*)(x_2 + v_d) - \frac{r}{\gamma}(x_2 - v_d) \\
&= -\hat{\alpha}(x_1 - \hat{x}_1^*) - r\frac{1}{\gamma}(x_2 - v_d) \\
&= -\hat{\alpha}\partial_1\hat{H}_d - r\partial_2\hat{H}_d,
\end{aligned}
\tag{31}
$$

where

$$
\hat{H}_d = \frac{1}{2}(x_1 - \hat{x}_1^*)^2 + \frac{1}{2\gamma}(x_2 - v_d)^2.
\tag{32}
$$

The equation for $\dot{x}_1$ is not changed by the feedback, but can be rewritten as

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + \xi x_2^2 \\
&= -(x_1 - \hat{x}_1^*) - \hat{x}_1^* + \hat{\xi}x_2^2 + (\xi - \hat{\xi})x_2^2 \\
&= -\partial_1\hat{H}_d + \hat{\xi}(x_2 + v_d)(x_2 - v_d) + (\xi - \hat{\xi})x_2^2 \\
&= -\partial_1\hat{H}_d + \hat{\alpha}\frac{1}{\gamma}(x_2 - v_d) + (\xi - \hat{\xi})x_2^2 \\
&= -\partial_1\hat{H}_d + \hat{\alpha}\partial_2\hat{H}_d + (\xi - \hat{\xi})x_2^2.
\end{aligned}
\tag{33}
$$

These two equations can be cast into Hamiltonian form as

$$
\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -1 & \hat{\alpha} + \gamma(\xi - \hat{\xi})\frac{x_2^2}{x_2 - v_d} \\ -\hat{\alpha} & -r \end{pmatrix} \begin{pmatrix} \partial_1\hat{H}_d \\ \partial_2\hat{H}_d \end{pmatrix} = \hat{A}_d\partial\hat{H}_d = (\hat{J}_d - \hat{R}_d)\partial\hat{H}_d,
\tag{34}
$$

where $\hat{J}_d$ is the skew-symmetric part, giving the closed-loop interconnection matrix, and

$$
\hat{R}_d = -\frac{1}{2}(\hat{A}_d + \hat{A}_d^T) = \begin{pmatrix} 1 & -\frac{\gamma}{2}\frac{(\xi - \hat{\xi})x_2^2}{x_2 - v_d} \\ -\frac{\gamma}{2}\frac{(\xi - \hat{\xi})x_2^2}{x_2 - v_d} & r \end{pmatrix}.
\tag{35}
$$

One has

$$
\begin{aligned}
\operatorname{tr}\hat{R}_d &= 1 + r > 0, \\
\det\hat{R}_d &= r - \frac{\gamma^2}{4}\frac{(\xi - \hat{\xi})^2}{(x_2 - v_d)^2}x_2^4.
\end{aligned}
$$

Hence, in order to ensure that $\hat{R}_d \geq 0$, it is necessary that

$$
\frac{(x_2 - v_d)^2}{x_2^4} \geq \frac{\gamma^2(\xi - \hat{\xi})^2}{4r},
\tag{36}
$$

which is globally true if $\hat{\xi} = \xi$ but fails in a neighborhood of $x_2 = v_d$, as well as for $|x_2|$ large enough, if $\xi \neq \hat{\xi}$.

Notice that, for $\hat{\xi} \neq \xi$, the closed-loop system does not have $x_2 = v_d$, $x_1 = \hat{x}_1^*$ as a fixed point, even though these are critical points of $\hat{H}_d$, due to the $1/(x_2 - v_d)$ term in $\hat{A}_d$.
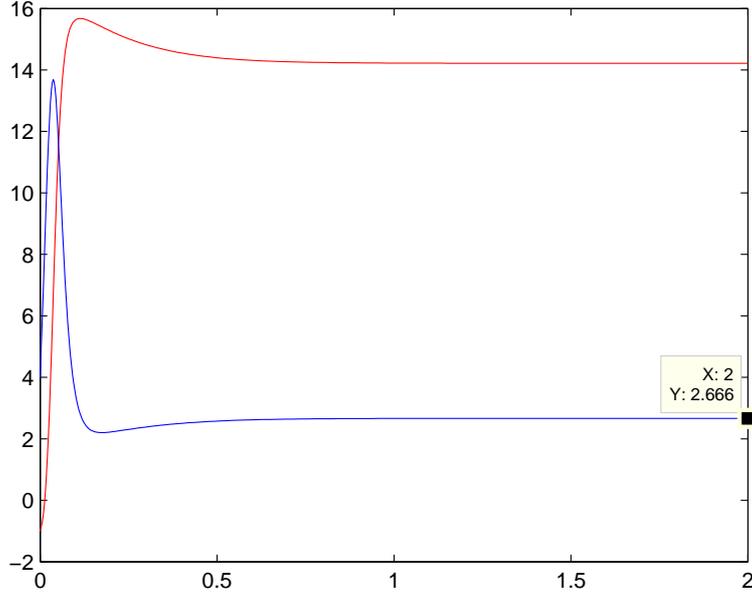
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Figure 1: Simulation for $v_d = 2$, $\xi = 2$, $\hat{\xi} = 4$, $\gamma = 1$, $r = 50$ and initial condition $(-1, 4)$.

In general, due to the state dependence of $\hat{A}_d$, other solutions may appear anyway. In fact, computing the fixed points yields the relation (depending only on the actual value of ξ)

$$x_1 = \xi x_2^2 \tag{37}$$

while the value of x_2 comes from the solutions to

$$0 = \gamma^2 \hat{\xi}(\xi x_2^2 - \hat{x}_1^*)(x_2 + v_d) + r(x_2 - v_d). \tag{38}$$

If $\hat{\xi} = \xi$, one gets

$$\gamma^2 \xi^2 (x_2^2 - v_d^2)(x_2 + v_d) + r(x_2 - v_d) = 0$$

which only has a real solution, namely $x_2 = v_d$. For $\xi \neq \hat{\xi}$ one has, in general, three solutions, at least one of them real, all different from v_d.

Figure 1 shows a simulation of the controller. The asymptotic value of x_2 is ~ 2.666 instead of $v_d = 2$, while x_1 goes to $\xi \times (2.666)^2$, as expected. As discussed in the previous Section, local asymptotic stability can be proved using LaSalle's theorem, but extensive simulations with very broad initial conditions seem to indicate that the stability is in fact global.

Following the general theory, an integral term is introduced next, so that the equation for x_2 gets modified by an az term while the dynamics of z is

$$\dot{z} = -a\frac{1}{\gamma}(x_2 - v_d). \tag{39}$$

All the fixed points of the closed-loop system have $x_2 = v_d$; from the equation for $\dot{x}_1$, one gets again $x_1 = x_1^* = \xi v_d^2$. Finally, the equation for $\dot{x}_2$ determines now the fixed point

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value of z, z^*, which depends on the nominal value $\hat{\xi}$, instead of determining the fixed point for x_2.

Figure 2 shows a simulation of the new controller, for the same parameter values than the simulation for the old controller and $a = 50$. The variable z, the integral of the error in x_2, starts from zero an goes asymptotically to z^*. A longer transitory appears, as is characteristic of integral controllers. Simulations with initial values in a wide range of points, seem to point to the global stability of the closed loop system.

However, if r is decreased oscillations do appear. For instance, for $r = 20$ and the same values of all the other parameters, one gets the response displayed in Figure 3. The disappearance of the oscillations when r is increased corresponds to a (reversed) Hopf bifurcation. In fact, linearizing the closed loop system around $(\xi v_d^2, v_d, z^*)$ yields a system which is asymptotically stable as long as

$$\frac{r}{\gamma} + \gamma \hat{\xi} v_d^2 (\xi - \hat{\xi}) - (\xi - \gamma \hat{\xi})^2 v_d^2 > 0, \tag{40}$$

which is true for r sufficiently large. Numerical simulations seem to imply that the fixed point of the nonlinear system is globally asymptotically stable. Computing the time derivative of $\hat{H}_{dz}$,

$$\frac{\mathrm{d}}{\mathrm{d}t} \hat{H}_{dz} = -(x_1 - \hat{x}_1^*)^2 + (\xi - \hat{\xi}) x_2^2 (x_1 - \hat{x}_1^*) - \frac{r}{\gamma^2} (x_2 - v_d)^2, \tag{41}$$

it can be seen that the region where (41) is nonpositive is much larger than what is implied by (36), due to the state-space dependence of the closed-loop dissipation matrix; in fact, for r large enough, the nonpositive region is pushed away from the desired regulation point, except for a bounded shrinking region whose boundary contains the later and which contains most of the periodic orbit. Although the details are quite particular to this example, we hope to obtain some insight into any existing mechanism which could be generalized.

4   Conclusions

We have studied the effect of uncertain parameters in the IDA-PBC method. For regulation of relative degree one outputs, it has been shown that, generally, the resulting closed-loop system contains a dissipation matrix which losses its nonnegativity in a neighborhood of the regulation point. Adding an integral control proportional to the error of the output corrects the fixed point for any nominal value of the uncertain parameters and can be cast into the PHDS framework, but the problems associated to the loss of nonnegativity remain. Nevertheless, simulations performed with a simple model seem to point out that (global) asymptotic stability is not lost in the process, although LaSalle's theorem cannot be invoked. Work is in progress to obtain more general results for integral controllers in the IDA-PBC setting as well as to address the additional problems related to higher relative degree outputs. We are also considering ways to reduce the dependence of the controller on the uncertain parameters, without integral terms, by choosing a simultaneous $J_d - R_d$ structure in the matching equation and then decomposing it into skew-symmetric and (minus) symmetric nonnegative parts.
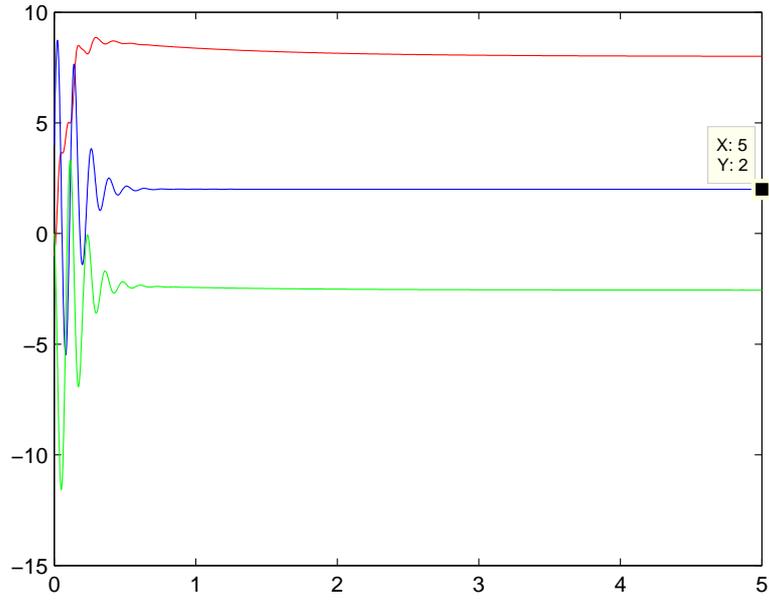
Figure 2: Simulation of the IDA-PBC+integral controller for $a = 20$, $v_d = 2$, $\xi = 2$, $\hat{\xi} = 4$, $\gamma = 1$, $r = 50$ and initial condition $(-1, 4, 0)$.
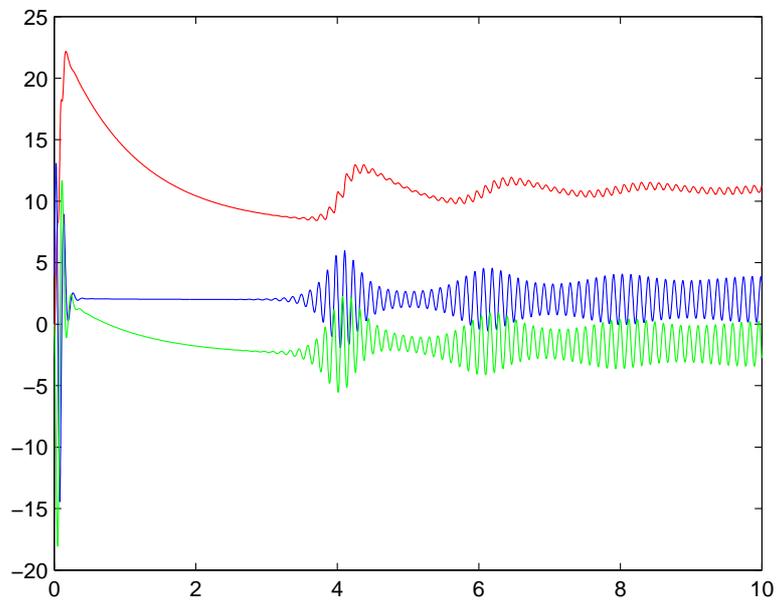

Figure 3: The same simulation as the one in Figure 2, but now with $r = 20$.

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