Optimal manufacturing-remanufacturing policies in a lean production environment

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ABSTRACT

This study analyses a production-management model that considers the possibility of implementing a reverse-logistics system for remanufacturing end-of-life products in a lean production environment (as opposed to models that use EOQ approaches). Decision variables are identified (including manufacturing and remanufacturing capacities and return rates and use rates for end-of-life products) and optimal policies are determined. Moreover, the structure of these optimal policies is analysed. The conclusion drawn is that, in many realistic scenarios, mixed policies (that is, with return rates and use rates strictly between 0 and 1) can be optimal. This conclusion is contrary to results published in earlier studies, which are based on more restrictive assumptions.

Keywords: reverse logistics, lean production, remanufacturing, supply chain management.
1. INTRODUCTION

According to Dowlatshahi (2005), remanufacturing is a $53 billion industry in the US alone. Costs derived from reverse-logistics activities in the US exceed $35 billion per year. The rate at which consumers return used products will reach 15% of sales in the coming years and, in sectors such as catalogue sales and e-commerce, it could reach as much as 35%. Without a doubt, reverse logistics has become a matter of strategic importance—an element that companies must consider in decision-making processes concerning the design and development of their supply chains.

The following are the most often-cited reasons (Thierry et al., 1995; De Brito and Dekker, 2004; Ravi et al, 2005) as to why companies establish or participate in reverse-logistics systems:

1. Economic reasons, both direct (consumption of raw materials, reduction of disposal costs, recovery of the added value of used products, etc.) and indirect (an environmentally friendly image and compliance with current and future legislation).

2. Legal reasons, because current legislation in many countries (including, for example, members of the European Union) holds companies responsible for recovering or properly disposing of the products they put on the market.

3. Social reasons, because society is aware of environmental issues and demands that companies behave more respectfully towards the natural environment, especially with regard to issues like emissions and the generation of waste.

The legal reason has traditionally been viewed as having a negative effect on companies' ability to compete, due to the costs involved in adapting processes and industrial operations to comply with regulations. Nevertheless, according to Porter and van der Linde
(1995), properly designed environmental laws can spur innovations capable of compensating for the cost of compliance. These “innovation offsets” not only reduce the net cost of compliance but also generate sustainable competitive advantages by reducing overall manufacturing costs and time to market and increasing the value of the product for the consumer. Thus, in addition to companies’ legal responsibility, the potential for gaining competitive advantages by complying with environmental legislation is a further reason for adopting environmentally friendly policies such as reverse-logistics systems.

This interest in reverse logistics has attracted the attention of not only companies and professionals but also academia, which has been tackling this issue in recent years (Prahinski & Kocabasoglu, 2006). Many of the studies published on reverse logistics have focused on aspects of production planning and inventory management. Some of the most notable works have analysed the effects of the flow of returned products on traditional inventory-management models (see, for example, Fleischmann et al., 1997, De Brito and Dekker, 2003, Minner, 2003 and Fleischmann and Minner, 2004, for a review of the same).

However, these studies assume that such a return flow exists, without questioning whether its establishment is appropriate in economic terms and in light of the traditional producer-consumer logistical structure. It seems reasonable to assume that a company considering the development of a reverse-logistics system must first analyse how the introduction of such a recovery system would affect the capacity and resources of its current production system. Traditionally, the studies that have analysed these matters have predetermined the capacity of both the production systems (manufacturing and remanufacturing) and the systems that manage them (for example, EOQ models in constant-demand scenarios), thereby obtaining results that ultimately consist of optimal strategies that call for using all available capacity in either the manufacturing process or the remanufacturing process (Richter, 1996a, Dobos and Richter, 2003 and Dobos and Richter,
rather than adopting mixed strategies (manufacturing and remanufacturing) which are currently followed by many firms (see, for instance, Dowlatshahi, 2005).

Our study explores these matters through an analysis carried out in a deterministic environment in which we assume that market demand is constant. Under this assumption, we propose a model that could be described as lean or just-in-time (JIT) production, in contrast to other models covered in the literature that are based on economic order quantity (EOQ) or some variation of this model (Harris, 1913). The idea is for the manufacturing process to be able to adjust its capacity to the demand in order to avoid inventory generation and excess capacity. The main goal of this study is to analyse under what conditions (capacity, return rate, remanufacturing rate) a reverse-logistics system should be introduced at a company that uses a JIT production-management system. The next section briefly reviews some earlier studies that have analysed these matters. In subsequent sections we present our model, describe the assumptions and scenarios we use and illustrate them with a numerical exercise. The last section presents the main conclusions of this study.

2. LITERATURE REVIEW

As mentioned above, studies that analyse the effects of return flow on the traditional supply chain tend to look at its effects on inventory management, basing their analysis on the EOQ approach. In general, these studies can be classified according to whether the model used is deterministic or stochastic. Authors who use deterministic models, like ours, include Schrady (1967), who describes a traditional EOQ model with return flow that alternates between one production lot and several lots of remanufactured (repaired) used products ((1, R) policy) and obtains simple equations for the respective lots. Nahmias and Rivera (1979) generalise this model by considering a finite repair rate and Mabini et al. (1992) extend it to include multiple products, also with a finite remanufacturing rate. Koh et al. (2002) build
upon the model by Nahmias and Rivera (1979) by calculating the optimal lot sizes in (P, 1) and (1, R) policies and considering various scenarios for demand rates, return rates and economic-recovery rates. Teunter (2004) generalises the latter study by considering a finite manufacturing rate and compares the work of Koh et al. (2002) and Nahmias and Rivera (1979) for some of the proposed scenarios. Teunter (2001) includes a disposal option in the model by Schrady (1967) and generalises it using (P, 1) policies that alternate various production lots with one remanufacturing lot, thereby optimising the size of each lot. Dobos and Richter (2004) study a system that includes the option of disposing of returned products and they also consider finite manufacturing and remanufacturing rates, using a predetermined inventory-management policy that consists in preparing $m$ lots of remanufactured products and $n$ manufactured lots. Dobos and Richter (in press) extend this model for the case of quality consideration. As mentioned above, these authors draw conclusions similar to those of Richter (1996a) and Dobos and Richter (2003) with regard to the suitability of pure strategies (production or remanufacturing) as opposed to mixed strategies (production and remanufacturing).

Unlike these other studies, we propose a model that allows the manufacturing capacity and, if necessary, the remanufacturing capacity to be adjusted, under the assumption of a known and constant demand. Thus, we avoid inventory generation and contribute a new approach to the study of production-management systems that deals with the economic recovery (remanufacturing, recycling, reusing) of end-of-life products and encourages the sustainable management of the supply chain.

3. **EXPLANATION OF THE MODEL**

Let us assume that the market demand (D units/year) is currently satisfied by a manufacturing process (*Forward System*) whose capacity is perfectly suited to the demand
rate, which we assume to be deterministic. We wish to find out whether a reverse logistics system (Revlog System) can be implemented for the economic recovery of used products. The Revlog System (Fig. 1) consists of two different lines of production, which we refer to as the Manufacturing Line and the Remanufacturing Line. The Manufacturing Line manufactures “original” products that satisfy the demand that cannot be met by the Remanufacturing Line, which involves a process of economic recovery of end-of-life products. This process begins when products are returned at the return rate $r = \alpha D$. The returned products are inspected and their suitability for remanufacturing is determined. At a rate of $u = \delta r = \delta \alpha D$, the suitable items are sent to the remanufacturing facilities, where they are adapted to meet part of the market demand. Unsuitable items are discarded at the rate of $d = r - u$. The remanufactured products are identical to the original products in terms of quality, so there is no distinction between a manufactured and a remanufactured product.

**Parameters and variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$D$</td>
<td>Demand</td>
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<tr>
<td>$\alpha$</td>
<td>Rate at which end-of-life products are returned: $r = \alpha D$</td>
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<tr>
<td>$\delta$</td>
<td>Rate at which returned products are used: $u = \delta \alpha D$</td>
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<tr>
<td>$d$</td>
<td>Rate at which returned products are discarded: $d = r - u$</td>
</tr>
<tr>
<td>$c_m$</td>
<td>Variable unit cost of manufacturing original products</td>
</tr>
<tr>
<td>$c_w$</td>
<td>Variable unit cost of disposing of returned products</td>
</tr>
<tr>
<td>$I_R$</td>
<td>Annualised capacity cost of the end-of life product return facilities</td>
</tr>
<tr>
<td>$I_M$</td>
<td>Annualised capacity cost of the manufacturing facilities</td>
</tr>
<tr>
<td>$I_R$</td>
<td>Annualised capacity cost of the remanufacturing facilities</td>
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<tr>
<td>$I_M$</td>
<td>Annualised capacity cost of the manufacturing facilities</td>
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<tr>
<td>$TC_{FWD}$</td>
<td>Total cost of the Forward System</td>
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<tr>
<td>$TC_{RL}$</td>
<td>Total cost of the Revlog System</td>
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Initially, we assume that the capacity costs in the model are constant and that the capacity of each of the manufacturing and remanufacturing processes is sufficient and perfectly suited to the demand, regardless of the investments made in each of the facilities. We will later relax this assumption. We calculate the value of the total costs for each of the proposed systems as follows:

Forward System

\[ TC_{FWD} = I_M + c_m D \]

Revlog System

Let us assume that the process of returning end-of-life products is represented by the return-cost function \( \Phi_b(\alpha) = I_b + C_b D \phi_b(\alpha) \), where \( \phi_b(\alpha) \) is increasing and convex, in keeping with the assumption that increases in the return rate will go hand in hand with growing increases in the cost of returning used products (Fig. 2). It seems reasonable to assume that, because of location or transport, etc., it will be increasingly difficult for the
company to increase the number of returned products. Therefore, the cost of returning each additional used unit—the marginal return cost—continually increases. This assumption relative to the marginal return cost has been adopted in several recent studies (Savaskan et al., 2004; Savaskan and Van Wassenhove, 2006). Ferguson & Totkay (2005) provide cases where the collection cost is convex increasing in quantity.

We can represent this function as $\phi_B(\alpha) = \int_0^\alpha f_B(x)dx$, where $f_B(\alpha)$ (such that $\frac{df_B}{d\alpha} \geq 0$) is proportional to the marginal return cost for end-of-life products.

Similarly, the cost associated with the remanufacturing process will increase as the percentage of returned products considered suitable for remanufacturing ($\delta$) increases. We assume that, depending on the different levels of quality of the returned products, different activities may be required for their economic recovery (remanufacturing) and that therefore the costs will vary. Thus, as the percentage of used products that are suitable increases, the quality of the products will become less consistent and the remanufacturing process will
therefore become more complex and costly. In short, we are assuming that the marginal cost of remanufacturing increases monotonously. Ferguson & Totkay (2005) also point out that many firms experience a convex increasing processing cost and mention several studies that consider this assumption. Thus, we define the remanufacturing cost function as 

\[ \Phi_R(\alpha, \delta) = I_R + \alpha D \phi_R(\delta), \]

where \[ \phi_R(\delta) = \int_0^\delta f_R(x)dx, \]

with \[ \frac{df_R}{d\delta} \geq 0, \]

which means that \[ \phi_R(\delta) \]

is convex (but \[ \Phi_R(\alpha, \delta) \]
is not).

It must be pointed out that both \[ f_B \]

and \[ f_R \]

functions may be non-continuous functions (for instance, because \[ \phi_B \]

or \[ \phi_R \]

are step-wise linear functions).

Let us assume that the cost of disposal is directly proportional to the number of returned units that are ultimately not included in the remanufacturing process:

\[ \Phi_W(\alpha, \delta) = c_u(1 - \delta)\alpha D. \]

Finally, the original Manufacturing Line in the Revlog System is represented by the costs generated to satisfy the demand that cannot be met through the remanufacturing process:

\[ \Phi_M(\alpha, \delta) = I_M + c_m(1 - \delta\alpha)D. \]

Thus, we obtain a cost function for the Revlog System according to the percentage of end-of-life products that return to the system (\( \alpha \)) and the percentage of these products that are ultimately remanufactured (\( \delta \)):

\[
\begin{align*}
TC_{RL} &= \left[ I_M + c_m(1 - \alpha \delta)D \right] + \left[ (I_B + D \phi_B(\alpha)) + (I_R + \alpha D \phi_R(\delta)) + c_u(1 - \delta)\alpha D \right] \\
TC_{FWD} - TC_{RL} &= c_m \alpha \delta D - \left[ (I_B + D \phi_B(\alpha)) + (I_R + \alpha D \phi_R(\delta)) + c_u(1 - \delta)\alpha D \right]
\end{align*}
\]

Let \( Z \) be the difference between both costs:

\[
TC_{FWD} - TC_{RL} = c_m \alpha \delta D - \left[ (I_B + D \phi_B(\alpha)) + (I_R + \alpha D \phi_R(\delta)) + c_u(1 - \delta)\alpha D \right]
\]

The introduction of a reverse-logistics system is appropriate, from an economic point of view, when \( Z > 0 \) and inappropriate when \( Z < 0 \). To obtain the optimum value of \( Z \), \( Z^* \), one must solve the following optimisation problem:
where \( Z \) is not a concave function. However, for any value \( \delta \) (0 \( \leq \delta \leq 1 \)), \( Z(\delta) \) is a concave function of \( \delta \), so the optimisation problem can be solved as follows:

1. \( \frac{dZ(\delta)}{d\delta} = 0 \) (which, straightforwardly, implies \( f_R(\delta) = c_m + c_w \)) is a sufficient condition for optimum. Therefore:

   1.1. If \( c_m + c_w \leq f_R(0) \), then \( \delta^* = 0 \) (and \( \alpha^* = 0 \) and stop).

   1.2. If \( f_R(0) < c_m + c_w < f_R(1) \), then \( \delta^* = f_R^{-1}(c_m + c_w) \) if such \( \delta^* \) exists. Otherwise (which may happen only when \( f_R \) is not a continuous function) \( \delta^* \) must fulfill \( f_R(\delta) < c_m + c_w \) \( \forall \delta < \delta^* \) and \( f_R(\delta) > c_m + c_w \) \( \forall \delta > \delta^* \).

   1.3. If \( f_R(1) < c_m + c_w \), then \( \delta^* = 1 \).

2. Replacing \( \delta \) with \( \delta^* \) in \( Z(\delta) \) yields a concave function of \( \alpha, Z(\alpha, \delta^*) \) and \( \frac{dZ(\alpha, \delta^*)}{d\alpha} = 0 \) implies \( f_B(\alpha) = (c_m + c_w)\delta^* - (c_w + \phi_R(\delta^*)) \). Therefore:

   2.1. If \( (c_m + c_w)\delta^* - (c_w + \phi_R(\delta^*)) \leq f_B(0) \), then \( \alpha^* = 0 \).

   2.2. If \( f_B(0) < (c_m + c_w)\delta^* - (c_w + \phi_R(\delta^*)) < f_B(1) \) and \( \alpha^* \) exits, then \( \alpha^* = f_B^{-1}(c_m + c_w)\delta^* - (c_w + \phi_R(\delta^*)) \). Otherwise (\( f_B \) is not a continuous function) \( \alpha^* \) must fulfill \( f_B(\alpha) < (c_m + c_w)\delta^* - (c_w + \phi_R(\delta^*)) \) \( \forall \alpha < \alpha^* \) and \( f_B(\alpha) > (c_m + c_w)\delta^* - (c_w + \phi_R(\delta^*)) \) \( \forall \alpha > \alpha^* \).

   2.3. If \( f_B(1) < (c_m + c_w)\delta^* - (c_w + \phi_R(\delta^*)) \) then \( \alpha^* = 1 \)

Of course, \( \alpha^* = 0 \) means that the reverse logistics system have not be implemented, although when \( \alpha^* > 0 \) the Revlog System have to be implemented only if \( Z^* > 0 \).
We may conclude from our analysis that, in the assumed environment, the optimal approach may be a mixed strategy that combines manufacturing, partial recovery, disposal and remanufacturing. Therefore, the results of Richter (1996a), Dobos and Richter (2003) and Dobos and Richter (2004) cannot be extended beyond the EOQ model and the scenarios they consider.

Let us see an example in which we use quadratic functions for the costs of returning and remanufacturing. If we assume that \( \phi_B(\alpha) = 10\alpha^2 \) and \( \phi_R(\delta) = 50\delta^2 \) and that \( D = 1000, I_M = 11000, I_R = 5000, I_B = 2000, c_m = 65 \) and \( c_w = 25 \), we obtain the following optimal values:

\[
\alpha^* = 77.50\%, \quad \delta^* = 90.00\%, \quad TC_{RL}^{*} = 76,993.75, \quad TC_{FWD}^{*} = 76,000.00
\]

**Fig. 3. Model with constant capacity costs: Total Costs for \( \delta^* \)**

Therefore, the Revlog System performs worse than the Forward System in terms of cost and the introduction of the reverse logistics system is not a recommendable option under the cost criteria.
**Model with variable capacity costs**

In this section, we relax our prior assumption that the capacity costs—of both the manufacturing process and the remanufacturing process—are constant, and we assume that the company incurs certain costs and savings by adjusting its manufacturing and/or remanufacturing capacity in accordance with the lean production model in question. We could use the next expressions to represent our model:

**Forward System**

Let \( I_M(\alpha\delta) = \Gamma_M + \gamma_m(\alpha\delta) \) be the capacity cost of the manufacturing process. Then:

\[
TC_{\text{FWD}}^C = I_M(0) + c_m D
\]

where \( I_M(0) = \Gamma_M + \gamma_m(0) \) represents the capacity cost function of the manufacturing process in the Forward System.

**Revlog System**

We again consider a Manufacturing Line and a Remanufacturing Line for which we examine the cost functions generated in each process.

\[
TC_{\text{RL}}^C = \Phi_M^C(\alpha, \delta) + \left( \Phi_B^C(\alpha) + \Phi_R^C(\alpha, \delta) + \Phi_w(\alpha, \delta) \right)
\]

\[
\Phi_M^C(\alpha, \delta) = I_M(\alpha\delta) + c_m(1 - \alpha\delta)D,
\quad \text{where} \quad I_M(\alpha\delta) = \Gamma_M + \gamma_m(\alpha\delta)
\]

\[
\Phi_B^C(\alpha) = I_B(\alpha) + D\phi_B(\alpha),
\quad \text{where} \quad I_B(\alpha) = \Gamma_B + \gamma_B(\alpha)
\]

\[
\Phi_R^C(\alpha, \delta) = I_R(\alpha\delta) + \alpha D\phi_R(\alpha, \delta),
\quad \text{where} \quad I_R(\alpha\delta) = \Gamma_R + \gamma_R(\alpha\delta)
\]

In this scenario, the functions \( I_M, I_B, I_R \) represent the capacity costs function of the corresponding processes: manufacturing, collecting and remanufacturing.

Trying to choose specific functions for the variable capacity costs should certainly be a vain exercise because the nature of the capacity cost function depends on the type of adjustment that is considered (Van Mieghem, 2003) and, hence, there are not reasons to suppose any particular shape for the \( \gamma_i \ (i = M, B, R) \) functions. Therefore, only by way of
example, we will suppose that the capacity costs are linear functions of the corresponding variables.

Let $\gamma_m(\alpha \delta) = k_m (1 - \alpha \delta) D$; $\gamma_b(\alpha) = k_b \alpha D$; $\gamma_r(\alpha \delta) = k_r \alpha \delta D$, then we have:

$$TC_{FWD}^C = I_M + c_m D = (\Gamma_M + k_mD) + c_m D$$

$$\Phi^C_B(\alpha) = I_B(\alpha) + D\phi_B(\alpha) = (\Gamma_B + k_b \alpha D) + D\phi_B(\alpha)$$

$$\Phi^C_R(\alpha, \delta) = I_R(\alpha \delta) + \alpha D\phi_R(\delta) = (\Gamma_R + k_r \alpha \delta D) + \alpha D\phi_R(\delta)$$

$$\Phi^C_W(\alpha, \delta) = I_M(\alpha \delta) + c_m (1 - \alpha \delta) D = (\Gamma_M + k_m (1 - \alpha \delta) D) + c_m (1 - \alpha \delta) D$$

And taking into consideration the cost function for disposing of returned products $\Phi_W(\alpha, \delta)$, we determine the total-cost function of the Revlog System, thus:

$$TC_{RL}^C = \Phi^C_M(\alpha, \delta) + \left(\Phi^C_B(\alpha) + \Phi^C_R(\alpha, \delta) + \Phi_W(\alpha, \delta)\right)$$

Using the same functions $\phi_B(\alpha)$ and $\phi_R(\delta)$ that in the case of constant capacity costs and assuming that $\Gamma_M = 1000, \Gamma_B = 450, \Gamma_R = 815, k_m = 10, k_b = 2$ and $k_r = 6$ (increase one unit of capacity in manufacturing is more expensive than increase one unit of capacity in remanufacturing), we obtain:

$$f_R(\delta) = (c_m + c_w) + (k_m - k_r)$$

$$f_B(\alpha) = \left((c_m + c_w)\delta - (c_w + \phi_R(\delta))\right) + \left((k_m - k_r)\delta - k_b\right)$$

And, therefore:

$$\alpha^* = 85.90\%, \quad \delta^* = 94.00\%, \quad TC_{RL}^* = 69,886.19, \quad TC_{FWD}^* = 76,000.00$$

In this scenario, the incorporation of linear capacity costs in the proposed cost structure leads us to consider introducing the Revlog System, which would be rejected if the effect of the variations of capacity on the costs would have not taken into account. In our
example, this causes a “shift” towards the remanufacturing process. Of course, the opposite may occur in other scenarios.

4. SUMMARY AND CONCLUSIONS

This study proposes a model for analysing the decision to introduce a reverse-logistics system for remanufacturing used products. This model is considered in a lean production environment, unlike other models widely used in the literature that use approaches based on economic order quantity. We identified the decision variables (which include manufacturing and remanufacturing capacities, return rates, and use rates for end-of-life products) and determined the optimal policies. The analysis of these policies makes it clear that, in very general cases, they may be either pure or mixed. This conclusion is different from those of earlier studies, which start with assumptions that are more restrictive and probably unrealistic in many cases. In fact, many firms are currently following such mixed policies.

Our model allows us to examine the effects of modifying the capacity of the system by establishing a process of economic recovery of used products. By considering variable capacity costs, we can analyse the transfer of capacity between manufacturing and remanufacturing lines, which allows us to consider new production-management policy options.
References


