

Stopover and hub-and-spoke shipment strategies in less-than-truckload carriers

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Abstract

This paper presents a methodology to identify when freight consolidation strategies are cost-efficient in the less-than-truckload carriers operations. Shipments are assigned based on proximity and cost criteria to build an initial long-haul shipment solution. This initial solution is later improved by the implementation of Tabu Search algorithm. The proximity criterion takes into account the spatial distribution of shipments loads among centers. The results show that the proposed methodology may reduce the transportation cost by 20% compared to the solution of those heuristics only considering cost criterion.

Keywords: Stopover; Long-haul shipments; less-than-truckload carriers; hub-and-spoke systems.

1. Introduction

In recent years, the evolution of logistics has promoted the outsourcing of transportation operations to the “for hire” carriers. These carriers provide transportation services for several suppliers so that they have to adjust their regular schedules to satisfy the maximum number of customers. When the size of the shipments is small, these companies must consolidate multi-shipment freight in vehicles to increase their load factor. These kinds of companies are known as less-than-truckload (*LTL*) carriers. Their shipments usually involve different origins and destinations constituting a many-to-many network.

The network of *LTL* carriers is divided into two hierarchical levels. Firstly, the transportation of small shipments from suppliers’ origins to carrier’s distribution centers constitutes the local network level. In its general definition, the vehicles depart from the distribution center and visit supplier points picking up the goods to be shipped. When a vehicle has visited its associated region of service it returns to the distribution center at full capacity. Basically, the order to visit the customer points, the vehicle capacity, technology and time restrictions determine the distribution cost (see Robusté *et al.* 1990).

Subsequently, the freight associated to the inbound routes of a local network is consolidated in a distribution center. Then, goods are loaded in vehicles of higher capacity for being transported to other distribution centers. The routes among distribution centers constitute the long-haul network level. This network is composed by many origins and destinations (O-D) scattered in the region of service. Finally, the delivery process where goods are transported from outbound distribution centers to the final customers is also made in the local network level. In that case, the routes start from the distribution center at full capacity to deliver goods sequentially to the customers of the parcel.

Although the management of a long-haul network would be basically carried out using direct shipments between O-D distribution centers, their loads difficultly fit the potential capacity of the vehicles. This fact is a key inefficiency for the carrier that needs to be addressed in order to be competitive. Hence, carriers usually use several strategic nodes that group and consolidate the load associated to different origins and destinations acting as distribution centers. The operation in these nodes known as *hubs* causes new handling costs and increases the distance traveled between O-D distribution centers. However, it can reduce the number of shipments (vehicles) in the whole region of service as well as the time of the total transportation chain for O-D pairs due to terminal consolidation. In fact, the advantages of *hub and spoke* networks have transformed the strategic and operational planning of transportation companies from the early 80s, causing major profits as it is reported in Hall (1987a) and Chestler (1985). A lot of research has been done about the optimal location of hubs and long-haul network design. A wide approach of strategic, tactical and operational planning of *LTL* carriers is addressed in Daganzo (2005), whereas in Crainic (2003) a compilation of available methodologies about the long-haul network design problem is provided. More specifically, Hall (1987b) and Blumenfeld *et al.* (1985) compare analytically the relative efficiency between *direct* and *hub and spoke* shipments. The basic formulation for the uncapacitated multiple allocation hub location problem is shown in Gelareh and Nickel

(2011) suitable for a wide range of transportation modes. Moreover, in Alumur et al. (2012), the hierarchical multimodal hub location problem is presented where two types of hubs and hub links for ground and air transportation are considered.

However, there is an additional way to increase vehicle load factor based on the introduction of intermediate stops over distribution centers in an existing O-D route. The load contribution of those intermediate distribution centers may keep the vehicle load factor above a minimal profitable threshold in a relevant part of the shipment length.

This strategy called *stopover* is the basis to fill the whole capacity of the vehicles in one- to-many networks. Burns *et al.* (1985) proposed the *Economic Order Quantity (EOQ)* analytic model to evaluate the optimal shipment size and the number of stops considering both transportation and inventory costs for general carriers. This model is focused on the service of one-to-many networks in which deliveries have to be made in parcels far from the centers (long-haul). This strategy is also analyzed in Daganzo (1988) for one terminal to other centers of less hierarchy level in order to reduce the freight inventory cost. Nevertheless, the development of a specific methodology focused on *LTL* express carrier network design problems has been only proposed in Lin (2001), Lin and Chen (2004) and Lin and Chen (2008) where both *hub and spoke* and *stopover* strategies are considered. In these contributions, the space–time network configuration is formulated as an integer and time constrained multicommodity min-cost flow problem. Apart from that, Mesa-Arango and Ukkusuri (2013) analyze the benefits of consolidation strategies when a shipper invites a set of carriers to submit bids for freight lane. These bids allow carriers increasing truck payload utilization in intermediate links of their routes, fostering the in-vehicle consolidation.

The little number of contributions about a *stopover* strategy in the long-haul network level (many-to-many networks) is due to two major reasons. Firstly, there is a potential inability to constitute one route linking many intermediate distribution centers without violating the time constraints of the long-haul network. Secondly, the strategic planning of the long-haul network has received much attention with regard to the optimal location of hubs and other facilities. These contributions consider the goods assignment as a multi-commodity flow problem. The nature of these analyses does not allow the trip-based approach needed to identify the sequence of stops visited in a particular vehicle route. Nevertheless, the study of *stopover* strategy in ground-exclusive long-haul networks serving small regions can produce relevant cost savings to *LTL* carriers complementary to the *hub and spoke* configurations. In fact, one of the major Spanish *LTL* carriers shows that 66% of its daily long-haul routes visit more than two distribution centers (origin and destination), i.e. they are operated with a *stopover* strategy.

In this research, the effectiveness of *stopover* and *hub-and-spoke* strategies in the long-haul routing design problem is studied for *LTL* carriers. The aim of this paper is to propose a methodology that evaluates under which conditions the implementation of consolidation strategies may reduce the total transportation cost.

In Section 2, the long-haul route design problem is defined as well as the available strategies of shipments. A cost analysis of freight consolidation implementations are carried out in Section 3. In Section 4, we explain how the *stopover* and *hub-and-spoke*

strategies have been incorporated into a heuristic algorithm to solve the long-haul network design problem. In addition to that, the efficiency of this algorithm has been assessed in a set of test instances in Section 5. Finally, Section 6 provides conclusions and future research directions.

2. Long-haul routing design problem

The long-haul routing design problem (*LRDP*) consists of finding the least cost vehicle routes that serve shipments among distribution centers within an established amount of time in a region R_s . The transportation network is represented as a complete graph $G = (N, A)$, where $N = \{1 \dots n\}$ is the set of distribution centers and $A = N \times N$ is the set of arcs linking distribution centers. For each node pair (i, j) , a demand flow w_{ij} is given which represents the daily freight volume or weight between these distribution centers. For the sake of simplicity, we assume that freight demands between distribution centers are expressed in terms of volume units. We define the set S composed by all shipment volumes w_{ij} (m^3) > 0 , $\forall i, j \in N$ and the integer functions $or(q)$ and $des(q)$. These functions return respectively the origin $i \in N$ and destination node $j \in N$ of the q -th shipment element $s_q = w_{ij}$, $s_q \in S$.

Additionally, the d_{ij} matrix is provided determining the distance between center pairs (i, j) . It is supposed that the fleet operating the long-haul network is homogeneous so that each vehicle presents a fixed capacity C (m^3). Moreover, the subset $H \subset N$ is formed by all p distribution centers considered as hubs in the long-haul network ($p < n$). The location of these hubs is supposed to be an input of the problem. Hence, the route design process will determine a set of routes R where each route $r \in R$ is defined by a sequence of b_r nodes, $r = \{n_r(1), \dots, n_r(b_r)\}$.

The problem considered in this paper aims at minimizing the total transportation cost of the system given a specific delivery time for each (i, j) distribution center pair. There are three sorts of shipment strategies to serve each (i, j) flow: *direct*, *hub and spoke*, and *stopover*. The components of the total cost (Z) incurred by the *LTL* carriers are evaluated in Equation (1) based on the formulations proposed in Daganzo (2005). In the case of this study, the inventory and other temporal costs are not taken into account. It is supposed that this inventory cost is incurred by the supplier when it selects the delivery time of service. The first term of Equation (1) captures the distance cost, where the parameter c_d (Euros/km) is the unit distance cost and d_r (km) is the total distance traveled in route $r \in R$. The second term reproduces the cost of operations to be made at each stop in the route $r \in R$, where the parameter c_s (Euros/stop) is the unit stopping cost and n_r the total number of stops in $r \in R$. Moreover, the third term captures the fixed cost of each route $r \in R$ where F (Euros/veh) means the dispatching cost of a vehicle in a day of service. Furthermore, the fourth term is only incurred by those shipments assigned with a *hub and spoke* strategy due to the extra handling operations at *hubs*. In fact, c_t (Euros/ m^3) is the volume unit transfer cost and W_r (m^3) the total amount of inbound loads transshipped at the *hubs* of route $r \in R$. Finally, Equation (2) states that the freight volume on the i -th arc of route r (q_r^i) does not exceed the vehicle capacity. In addition, Equation (3) obliges that the vehicle arrival time to node j of route r ($t_r^{n_r(j)}$) must happen before a predefined time $T_{n_r(j)}$.

$$\min Z = \sum_{r \in R} (c_d d_r + c_s n_r + F + c_t W_r) \quad (1)$$

$$q_r^i \leq C \quad \forall r \in R, i = 1, \dots, b_r - 1 \quad (2)$$

$$t_r^{n_r(j)} \leq T_{n_r(j)} \quad \forall r \in R, \forall j = 2, \dots, b_r \quad (3)$$

The long-haul route and network design is a *Np-Hard* problem and existing resolution methods are extremely time-consuming. In Crainic (2003) there is a wide analysis of the available solution techniques for this problem. The initial exact approaches have been focused in *EOQ* models and mixed-integer formulations. The latter are completed by constraint relaxation techniques or branch and bound algorithms. However, the inclusion of a large set of variables and constraints for adapting the formulations to real-size problems has justified the use of heuristics. In this way, heuristic algorithms have become approximate resolution techniques that provide good quality solutions in a reasonable amount of computation time. These techniques have evolved from greedy iterative algorithms that constructed routes by local optimum approach to computational procedures in which the solution search is led by probabilistic criteria. An implementation of metaheuristics (Tabu Search) on the network design of *LTL* carriers is provided in Estrada and Robusté (2009) and Estrada (2007).

3. Cost analysis of consolidation strategies

In this section, an analytical procedure is developed to identify the recommended typology of shipment strategy for a flow between two pair centers (i, j) among *direct*, *hub-and-spoke* and *stopover strategy*. This analysis is based on the average cost per unit of volume of each consolidation strategy with regard to the *direct* shipment. It may complement the comparative breakdown between direct and terminal (*hub*) routing criterion established in Hall (1987b).

Consider a set of R routes and a pending shipment between distribution centers i and j with volume $w_{ij} = \alpha C$ ($0 < \alpha < 1$) to be assigned to any route $r \in R$.

Figure 1a depicts the route scheme of *direct* shipment strategy for load w_{ij} . The unit cost of direct shipment strategy between (i, j) distribution centers ($c_{T,d}(i, j)$) is presented in Equation (4). This strategy minimizes the travelled distance between centers due to the triangular inequality. However, the provision of an exclusive freight vehicle (F) for a low shipment size (αC) may result in a high dispatching cost. In this case, alternative consolidation strategies would become a potential way to increment the efficiency of the resources deployed to carry the shipment w_{ij} .

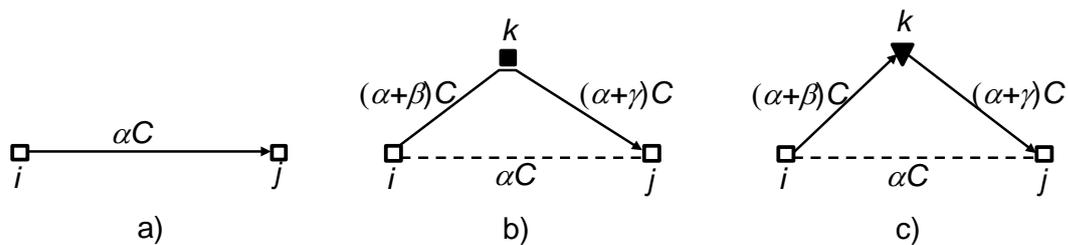


Figure 1. Shipment strategies between distribution centers (i, j)

$$c_{T,d}(i, j) = [c_d D_{ij} + F + c_s] / \alpha C \quad (4)$$

$$c_{T,s}(i, j, k) = [c_d (D_{ik} + D_{kj}) + F + 2c_s] / [(\alpha + \varepsilon)C] \quad (5)$$

$$c_{T,h}(i, j, k) = [c_d (D_{ik} + D_{kj}) + 2F + 2c_s + c_r \alpha C] / [(\alpha + \varepsilon)C] \quad (6)$$

Hence, the *stopover strategy* of shipment w_{ij} can be performed when there is a distribution center k close enough to nodes i and j (Figure 1b). This center k should present a direct shipment between (i,k) or (k,j) pairs. We assume that the shipment size in links (i,k) and (k,j) are respectively $w_{ik} = \beta C$ and $w_{kj} = \gamma C$; $\beta, \gamma \in [0,1)$. If $w_{ik} \neq 0$ and $w_{kj} = 0$, the vehicle operating the link (i,k) may be loaded with the shipment w_{ij} at node i and may continue its route from node k to node j (Figure 1b). On the other hand, if $w_{ik} = 0$ and $w_{kj} \neq 0$ the vehicle operating the existing route between (k,j) nodes will depart from node i to carry the shipment w_{ij} . For the sake of simplicity, the existence of shipments w_{ik} and w_{kj} should be disjunctive. The case when $w_{ik} = 0$ and $w_{kj} = 0$ implies the creation of a new route visiting the chain $i-k-j$ with a load factor α . Due to the triangular inequality, the cost of *stopover strategy* in that situation will always be higher than the corresponding cost of *direct strategy* so that, it will never be chosen. On the other hand, if $w_{ik} \neq 0$ and $w_{kj} \neq 0$, it means that there are two existing vehicles operating the links (i,k) and (k,j) . Although in this situation *stopover strategy* may increase the vehicle load factor in both routes, it would imply the transshipment of the load between vehicles at node k . This operation is only considered to be performed in those distribution centers acting as hubs so that it will be analyzed later in the *hub-and-spoke shipment strategy*.

Therefore, the average cost of *stopover strategy* ($c_{T,s}(i, j, k)$) for shipment w_{ij} on distribution center k is evaluated by Equation (5). It includes the distance cost, the dispatching cost of a new vehicle and the stopping cost at node k or j . In this equation, the variable ε accounts for the existing relative load carried in link (i,k) or link (k,j) ; i.e. $\varepsilon = \beta + \gamma$. The implementation of this strategy must satisfy the capacity constraint of vehicles, so that $\varepsilon + \alpha \leq 1$.

Finally, the *hub-and-spoke strategy* consists of routing the pending shipment w_{ij} through a hub k located in the service area. We assume that there are two independent routes, one with a *direct* shipment between (i,k) nodes and a complementary route with a shipment in the (k,j) link. Therefore, the load w_{ij} will need a transfer operation at hub k from the vehicle operating the link (i,k) to the route (k,j) as it is depicted in Figure 1c. The existing shipment loads in the links of study are respectively $w_{ik} = \beta C$ and $w_{kj} = \gamma C$, $\beta, \gamma \in [0,1)$. In that case, Equation (6) estimates the unit cost of *hub-and-spoke strategy* ($c_{T,h}(i, j, k)$). As in the *stopover strategy*, the variable $\varepsilon = \beta + \gamma$ represents the sum of the existing shipment loads in the affected routes by *hub-and-spoke strategy*. We also allow the implementation of this strategy even if $w_{ik} = 0$ or $w_{kj} = 0$, although link (i,k) or (k,j) needs the deployment of a new vehicle. The situation when $w_{ik} = 0$ and $w_{kj} = 0$ is not considered since its corresponding average cost will always be more expensive than the direct shipment strategy. This strategy also has to verify the capacity constraint in both routes (i.e. $\alpha + \beta \leq 1$ and $\alpha + \gamma \leq 1$).

The selection of the most cost-efficient strategy for shipment w_{ij} should be made in terms of the unit cost presented in Equations (4)-(6). These analytic expressions are similar to those proposed in Burns *et al.* (1985) but they do not take into account the inventory costs. Generally, the *stopover* and *hub-and-spoke* strategies for shipment w_{ij} present a trade-off between the incurred extra distance traveled for visiting distribution center (or hub) k and the increment of the vehicle load factor. Therefore, given the pending shipment w_{ij} , it is necessary to analyze under which spatial and load conditions these consolidation strategies are more efficient than the basic *direct* shipment strategy.

3.1. Identification of the center location domain for performing consolidation

The objective of this section is the selection of those distribution centers k (Figure 1b and 1c) that allow the performance of *stopover* or *hub-and-spoke* strategies in order to minimize unit distribution cost of a pending shipment. Let $w_{ij} = \alpha C$ be a pending shipment between the center pair (i, j) and k ($k \in N$) a distribution center with a given shipment load ε ($0 < \varepsilon < 1$) to perform a consolidation through it. Two key conditions have to be accomplished for guaranteeing the cost-efficiency of these shipment strategies. Firstly, the complementary flows w_{ik} or w_{kj} should be as much equal as $(1-\alpha)C$ in order to maximize vehicle occupancy. Secondly, the distribution center k must be close enough to distribution centers i and j to avoid covering an excessive extra distance. Therefore, distribution centers k should be chosen with regard to the load variable ε and the distance $(D_{ik} + D_{kj})$.

We can define the distance $d' = (D_{ik} + D_{kj})'$ that equals the unit cost of *any consolidation strategy* (Equation 5 or 6) through a distribution center $k \in N$ with the corresponding value of *direct* shipment strategy between centers (i, j) (Equation 4). In any ellipse, the sum of the distances from any point k located at the ellipse contour (l) to the two foci is constant and it equals two times the major semi-axis a . Therefore, it may be stated that the locus of distribution centers k located at the boundary of the cost-efficient area for performing consolidation strategies is defined by an ellipse whose foci are exactly points i and j and the major semi-axis is $a = (D_{ik} + D_{kj})/2$, $\forall k \in l$ (see Figure 2). It allows the definition of Equations (7) and (8) that determine the major semi-axis of the ellipse region where the *stopover* and *hub-and-spoke* shipment show equal average cost than *direct shipment* respectively (Figure 2).

$$d' = 2a = (1 + \varepsilon / \alpha)D_{ij} + (\varepsilon / \alpha)F / c_d + (\varepsilon / \alpha - 1)c_s / c_d \quad (7)$$

$$d' = 2a = (1 + \varepsilon / \alpha)D_{ij} + (\varepsilon / \alpha - 1)(F + c_s) / c_d - \alpha C(c_t / c_d) \quad (8)$$

Moreover, the ellipse eccentricity (e) is equal to the ratio between the length of the segment defined between the two foci and two times the length of the major semi-axis a (Equation 9). This property leads to the determination of the minor semi-axis (b) for a given distance between distribution centers i and j (Equation (10)).

$$e = \frac{D_{ij}}{2a} = \sqrt{1 - (b/a)^2} \quad (9)$$

$$b = +\left(a^2 - D_{ij}^2 / 4\right)^{1/2} \quad (10)$$

The major achievement of these formulas is that those distribution centers or hubs k located out of the corresponding ellipse should not be considered as an efficient candidates for its insertion as an intermediate stop in the link (i,j) with a consolidation strategy. In these situations, direct shipment strategy between (i,j) would imply lower average cost. As it is depicted in Figure 2, the shipment between (i,j) pair stopping over a distribution center P (or a hub P) may even justify the development of consolidation strategies (one-dimensional problem) in spite of the backtracking movement in the segment (i,P) .

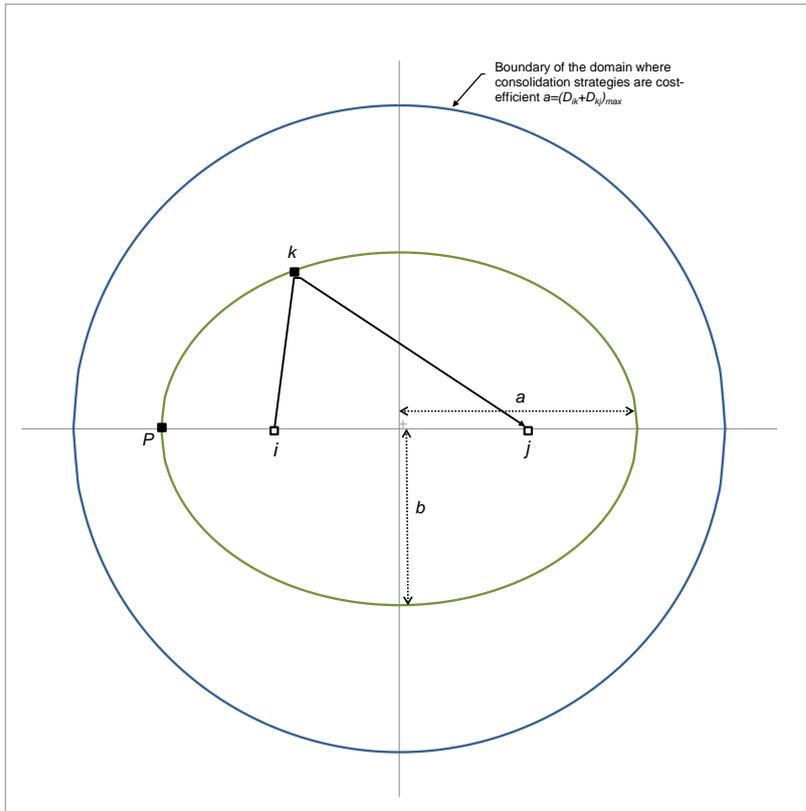


Figure 2. Domain where distribution centers k can be included in the existing route (i,j) with a consolidation strategy

The major and minor semi-axes of this ellipse (variables a and b respectively) are specific for each distribution center k to be included in the route depending on the corresponding load variable ε and the input cost parameters. This relation is defined in Figure 3, where the ellipse semi-axis a grows linearly as a function of the shipment size ε corresponding to distribution center k . As the vehicle load factor of a route is increased by the freight ratio ε of distribution center k , this center may be located further of points (i,j) and the corresponding ellipse size increases. We may note that the ellipse size of the *stopover* strategy is higher than the corresponding size of *hub-and-spoke* strategy for the same input parameters due to the extra handling cost incurred in hub terminals.

We may even define the furthest point k from nodes i and j to be included with *stopover* strategy using $\varepsilon_{max}=(1-\alpha)C$ in Equations (7) and (10) (or Equations (8) and (10) in the case of *hub-and-spoke* strategy). This calculation determines the maximal spatial domain defined by an ellipse whose major semi-axis is $a_{max}=(D_{ik}+D_{kj})_{max}$ (see Figure 2).

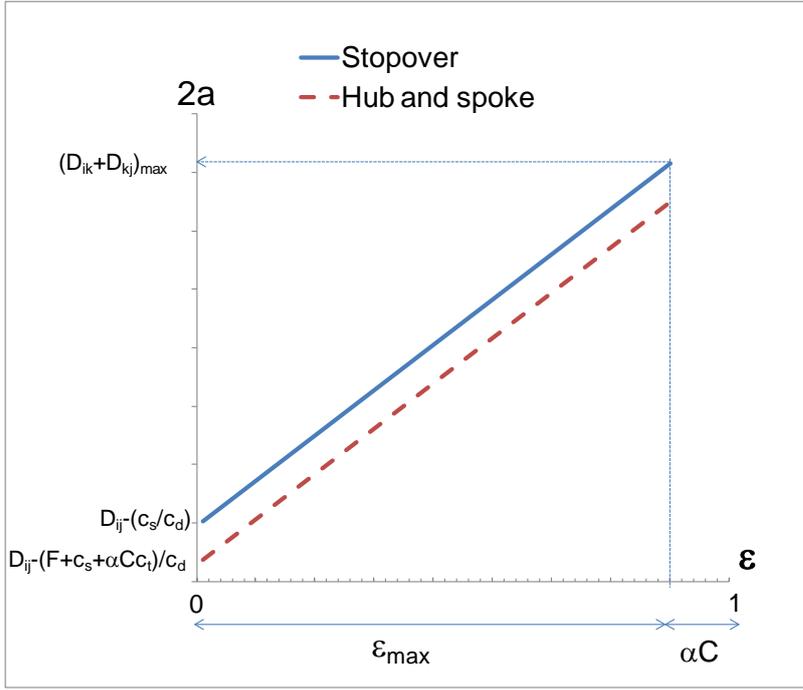


Figure 3. Relation between existing shipment load in the route (ε) and the semiaxis of the ellipse (a) guarantying the efficiency of consolidation strategy with regard to direct shipment.

4. Selecting the best shipment strategy in the route construction process

In the last section, an analysis of the shipment strategy of lowest cost for a single load between (i, j) distribution centers was presented. We considered a complementary distribution center or hub k in an existing route $r \in R$ ($s \neq r$) as a candidate for implementing consolidation strategies.

However, in any stage of the route construction process for serving all transportation customer demands, there would be multiple combinations of pending shipments (i, j) with other distribution centers $q \in N - \{k\}$ in alternative routes $s \in R$ ($s \neq r$). This allocation may fill better the vehicle capacity and may result in a lower system cost. Therefore, it is necessary to provide an efficient methodological approach to determine under which situations the implementation of consolidation strategies for a specific load at a defined route is the best among other potential candidates. To do so, a sequential procedure is developed to perform an intermediate stop at distribution center or hub k , $k \in N$, between a direct (i, j) shipment. It is based on two main criteria: i) the definition of the cost-efficient region where distribution centers or hubs k should be located (proximity criterion) and ii) selecting the distribution center or hub k producing the lowest operational cost in the previous region (cost saving criterion).

The criterion i) should be based on the definition of the corresponding ellipse previously defined. For a single pending shipment w_{ij} , all points k located inside the ellipse defined by $\varepsilon = \varepsilon_{max}$ whose foci are distribution centers i and j can be considered as a potential candidates to perform *stopover strategy*. Therefore, this region can be determined evaluating the semiaxis (a_{max}, b_{max}) defined by Equation (7) and (10) when $\varepsilon = \varepsilon_{max}$.

Similar procedure can be defined for the performance of *hub-and-spoke shipment strategy* on hub k for a shipment between (i,j) pairs. In that case, the domain region is defined by an ellipse with the same properties except for the semiaxe a that is defined using Equation (8).

The criterion ii) concerning the selection of the most cost-efficient distribution center k with a load ε in the available region must be made depending on the proper balance between the consolidation rate and the relative proximity of this point k to point i and j . Depending on the unit cost parameters and the value of ε_{max} , the area of the maximal ellipse may be critically huge and it may contain several node candidates where a stop over this point will be effective. In the following subsections, the criterion ii) is formulated in two scenarios assuming different hypotheses with regard to the location of distribution centers and the freight flows among them.

4.1. Random location of centers and uniform distribution of flows

In this section, we assume that distribution centers are uniformly scattered in the region of service and the shipment size w_{ij} between distribution centers i,j ($\forall i,j \in N$) follows a uniform distribution in $(0,C)$. Under these circumstances, we define a probabilistic criterion to select a complementary center k that shows a shipment size $\varepsilon \leq (1-\alpha)C$. These complementary centers k for implementing *stopover strategy* have a spatial density $\delta_{1-\alpha} = N(1-\alpha)/|R_s|$ and their distribution over a region of area A may be approximated by a spatial Poisson distribution of parameter $\delta_{1-\alpha}A$. When the *hub-and-spoke strategy* is considered, the spatial density of hub distribution should be $\delta_{1-\alpha} = p(1-\alpha)/|R_s|$.

In this way, the probability of encountering n^* potential distribution centers with a shipment size equal or less than $(1-\alpha)C$ in an area A is defined by Equation (11).

$$P(n = n^*) = (\delta_{1-\alpha}A)^{n^*} \exp(-\delta_{1-\alpha}A) / n^*! \quad (11)$$

$$A^* = \pi a^* b^* = \frac{n^*}{\delta_{1-\alpha}} \quad (12)$$

$$a^{*4} - a^{*2} D_{ij} / 4 - (\delta_{1-\alpha} \pi)^{-2} = 0 \quad (13)$$

$$(D_{ik} + D_{kj}) \leq 2a^* \quad (14)$$

Hence, the probabilistic criterion aims at identifying the critical ellipse whose area $A^* = \pi a^* b^*$ maximizes the probability of encountering $n^*=1$ centers k with load $\beta \leq (1-\alpha)$. The maximization of $P(n=1)$ establishes an analytical relation of its semiaxis a^* and b^* through Equation (12). Finally, the critical ellipse semiaxe a^* can be univocally determined in Equation (13) using some algebra in the formulas presented in Equation (10) and (12). Therefore, if a distribution center k is located inside the critical ellipse whose foci are nodes i and j , this center k should be added to the route with a consolidation strategy. This statement is accomplished when the total distance of this center k to the distributions centers (i,j) (the ellipse's foci) verifies Equation (14). On the contrary, if distribution center k is out of this ellipse, we may expect that other distribution centers k' will be closer to the i and j centers providing less operational cost.

Eventually, the probabilistic methodology accepts the *stopover* or the *hub-and-spoke* strategy visiting the distribution center k or hub k between distribution centers i,j if the following criteria are guaranteed:

- i. the distribution center k is contained in the ellipse of semi-axis a and b (Equations (7) and (10) for *stopover strategy* and Equations (8) and (10) for *hub- and- spoke*) whose foci are the i,j distribution centers
- ii. the distance between distribution center k and the (i,j) foci of the critical ellipse verifies Equation (14).

This procedure is not time-consuming since the validation of shipment w_{ij} with consolidation strategies for its inclusion in the existing routes is made without calculating other potential candidates.

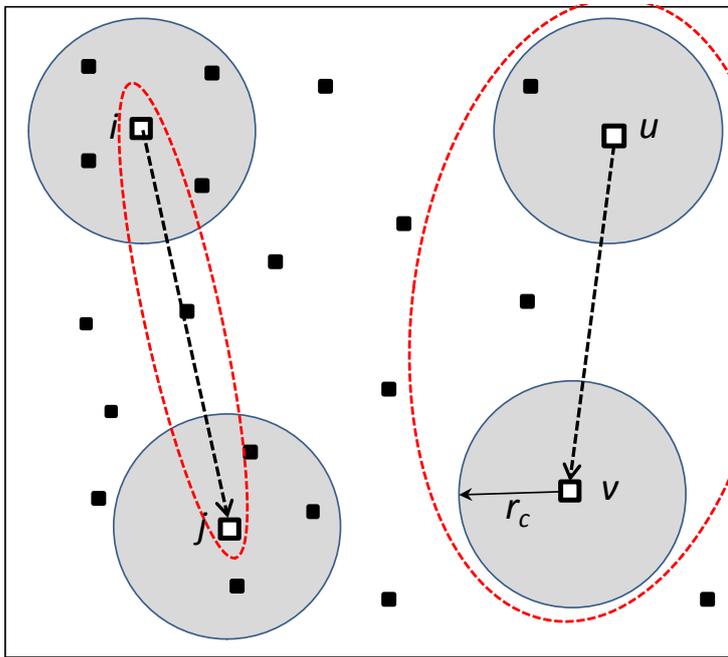
4.2. Unknown spatial distribution of centers over the region of service

When the assumption regarding the random location of centers and the uniformity of flows among centers may not be accepted, a complementary criterion to perform consolidation strategies should be addressed. Given a pending shipment $w_{ij} = \alpha C$, we propose a discretization of the spatial density of those distribution centers presenting complementary shipments of size $(1-\alpha)C$ in different zones of the service region to maximize vehicle load factor. Hence, for each distribution center $i \in N$, we identify the number of other distribution centers (m) located in a circle of radius r_c concentric to node i .

Therefore, the estimation of the spatial density of distribution centers in the neighborhood of node i is made by Equation (15). The numerator is the sum of the total k centers located inside the circle of radius r_c . However, each center k in this circle is weighted by a factor ϕ_k since the distribution of flows among centers is not uniform. Let $F(\xi)$ be the accumulative distribution function of the load factor portion ξ , that determines the probability of selecting a shipment with a load α^*C ($\alpha^* \leq \xi$) among all distribution center pairs in the area of service. The function $F(\xi)$ should also include those distribution centers pairs that do not present a daily shipment (i.e. $w_{ij}=0$). Moreover, for each distribution center $i \in N$, we can estimate the corresponding accumulative distribution function $F_1(\xi)$ of the load factor portion ξ of those shipments whose origin or destination point is center $i \in N$. The factor ϕ_k of point k is defined as the ratio between the probability that center $i \in N$ presents a shipment with $\varepsilon \leq (1-\alpha)$ and its average value for all N distribution centers. Thus, given the pending shipment of load $w_{ij} = \alpha C$, the factor ϕ_k weights the consideration of center k in terms of the amount of inbound or outbound existing shipments that will fit the vehicle capacity. The denominator of Equation (15) represents the searching area in the neighborhood of points $i \in N$. The determination of the r_c value should be equal to $k_r |R_S|^{1/2}$, where k_r is a constant to be defined.

$$\frac{\delta_{i,1-\alpha}}{\pi r_c^2} = \frac{\sum_{k=1}^m \phi_k}{\pi r_c^2} \quad \text{where } \phi_k = \frac{F_k(1-\alpha)}{F(1-\alpha)} \quad (15)$$

Therefore, under these assumptions, the selection of the candidate distribution center k with relative load $\varepsilon \leq (1-\alpha)$ to implement a *stopover* or *hub-and-spoke* strategy between centers (i,j) can be also made using the previous criteria i) and ii). However, in that case, the variable $\delta_{1-\alpha}$ of Equation (13) should be replaced by $(\overline{\delta_{i,1-\alpha}} + \overline{\delta_{j,1-\alpha}})/2$. The estimation of $F_k(1-\alpha)$ and $F(1-\alpha)$ can be made easily before the route construction process. Hence, the area of the critical ellipse containing the candidates for inclusion with consolidation strategies in a route is adaptive to the different spatial concentration of centers and the unbalanced freight shipment loads among them. Figure 4 shows the analysis of the *stopover* shipment strategy implementation of pending shipments $s_m=w_{ij}$ and $s_n=w_{uv}$ ($s_m, s_n \in S$, $i,j,u,v \in N$) with an intermediate distribution center k . The neighborhoods of nodes i and j defined by a circle of radius r_c contain more distribution centers in comparison to nodes u and v .



- Origin-destination of shipment to be analyzed
- Candidate DC with complementary shipment loads to perform consolidation strategies

Figure 4. Identification of the corresponding ellipses for two shipments between (i,j) and (u,v) pairs considering the dispersion of distribution centers in the region of service

If we assume that $\phi_k = 1 (\forall k \in N)$, the shipment w_{ij} in Figure 4 would present a higher spatial center density in its surroundings $[(\overline{\delta_{i,1-\alpha}} + \overline{\delta_{j,1-\alpha}}) \gg (\overline{\delta_{u,1-\alpha}} + \overline{\delta_{v,1-\alpha}})]$. Therefore, the estimation of the corresponding ellipse size for shipment w_{ij} through Equation (14) would result in small ellipse's semiaxe a . In the case of shipment w_{uv} , the ellipse size would be higher due to the low concentration of complementary centers k . Therefore, the spatial domain of distribution center candidates k is amplified at the expenses of incurring in a major travelled distance. However, the size of these ellipses would be relevantly different when the assumption $\phi_k = 1 (\forall k \in N)$ is not considered, i.e. there is an unbalance distribution of flows among distribution centers. In this situation, the

ellipse semiaxis a for shipment w_{ij} would expand if distribution centers located in the surroundings of points i and j do not present complementary shipments of load $\beta \leq (1-\alpha)$. The maximal value of this semiaxe a is constrained to guaranty that consolidation strategy produces lower operational costs than direct shipment, as it was depicted in Figure 3.

5. Effectiveness assessment of *consolidation* strategies for solving LRDP

The procedure defined in Section 4 is aimed at developing a methodology to decide optimally the load assignment of a single O-D pair to one of the existing routes in the long-haul network. In this section, this strategy has been complementarily incorporated in sequential heuristic algorithms, where the configuration of the routes is iteratively constructed. Routes are created or modified as new shipments are being considered. The basic problem of this approach is to prevent the high-cost assignment of any shipment at current iteration when it may fit better the vehicle capacity in other routes in the next iterations. In this way, the methodology defined in Section 4 of this paper may overcome this difficulty. The method assesses the optimality of a *stopover* and *hub and spoke* strategy of a single shipment looking ahead. It analyzes the likelihood of other future shipments pending to be assigned that would fill better the vehicle capacity and minimize the operational cost.

In Estrada and Robusté (2009), a sequential heuristic has been used to design the long-haul routes for *LTL* carriers allowing *direct*, *stopover* and *hub and spoke* shipment strategies. Here, this heuristic has been modified showing two main variants. On the one hand, we created the *P-LTL* algorithm that incorporates the *consolidation* strategies defined in the previous Section. It only accepts these strategies if and only if conditions i) and ii) are satisfied so that direct shipment is implemented in other circumstances. On the other hand, the second variant known as *L-LTL* algorithm always implements the *stopover* or *hub and spoke* strategy if it provides the minimum average cost of shipment insertion at current iteration. Therefore, the *L-LTL* algorithm provides local optimal solutions since it does not consider the total amount shipments to be assigned. This procedure is equivalent to identify the insertion of node k^* with load ε that produces the maximal distance savings with regard to the insertion of one point located in the contour of the ellipse defined by the semiaxis a (Equation 7 or 8) and b (Equation 10). This ellipse represents the boundary of the region of points k where *stopover* or *hub-and-spoke* strategy is efficient. This criterion for the selection of k center is defined by Equation (16).

$$\max_k (2a - D_{ik} - D_{kj}; 0) \quad (16)$$

Both heuristics have been complemented with a fine-tuning optimization technique based on the Tabu Search algorithm as it is defined in Estrada and Robusté (2009). The efficiencies of both *P-LTL* and *L-LTL* are assessed in a set of problems covering a wide range of scenarios.

5.1. Heuristic procedures to solve LRDP

The basic heuristic is articulated in two key steps: a) the determination of an initial suboptimal solution and b) the fine-tuning procedure based on Tabu Search heuristic.

The basic procedure shared by *P-LTL* and *L-LTL* algorithms builds iteratively the long-haul routes by a cost reduction criterion as it is depicted in Figure 5. In Step 1, the basic procedure creates a second set of shipments $S'=S=\{w_{ij}\}, (i,j \in N)$ that will be modified during the iterative process. We rank the elements of the shipment set S' in a decreasing order of shipment load w_{ij} . In Step 2, the iterative procedure to define the route structure is addressed. At each iteration m ($m=1, \dots, |S'|$), the current route configuration is modified to allocate the m -th element of S' ($s'_m \in S'$) with one of the following shipment strategies: *direct*, *hub and spoke* and *stopover* strategy.

In the *L-LTL* algorithm, the procedure evaluates the whole feasible assignment of the shipment s'_m to each route in the current solution using all shipment strategies. Note that steps 2.2 and 2.3. of *L-LTL* algorithm imply the solution enumeration of the implementation of a consolidation strategy between nodes $or(m)$ and $des(m)$ with the rest of distribution centers ($N-\{or(m), des(m)\}$). The selection of the shipment strategy and the complementary route is made with regard to the minimum average cost.

On the other hand, the *P-LTL* evaluates the assignment of shipment s'_m to one route r using the criteria i) and ii) associated to the *consolidation strategies* defined in Section 4. For each consolidation strategy, the algorithm firstly identifies the maximal area where complementary distribution centers would be located in order to provide cost savings in comparison to *direct* shipment strategy. Given the shipment load $s'_m = \alpha C$ between $or(m)$ and $des(m)$ nodes, this area corresponds to the ellipse of maximal size whose semiaxes are defined by Equations (7)-(10) (*stopover strategy*) or Equations (8)-(10) (*hub and spoke strategy*) when $\varepsilon=(1-\alpha)$. Then, we search the whole distribution centers k located in the maximal ellipse that constitute the subset K . The subset H formed by hubs located inside the corresponding ellipse is also created. Finally, the first distribution center $k \in K^*$ or hub $h \in H^*$ that verifies criteria i) and criteria ii) is chosen to perform *stopover* or *hub-and-spoke* strategy and the parameter f is updated ($f=1$). If these strategies do not succeed ($f=0$), the algorithm assigns the shipment load s'_m with a direct strategy between distribution centers $or(m)$ and $des(m)$.

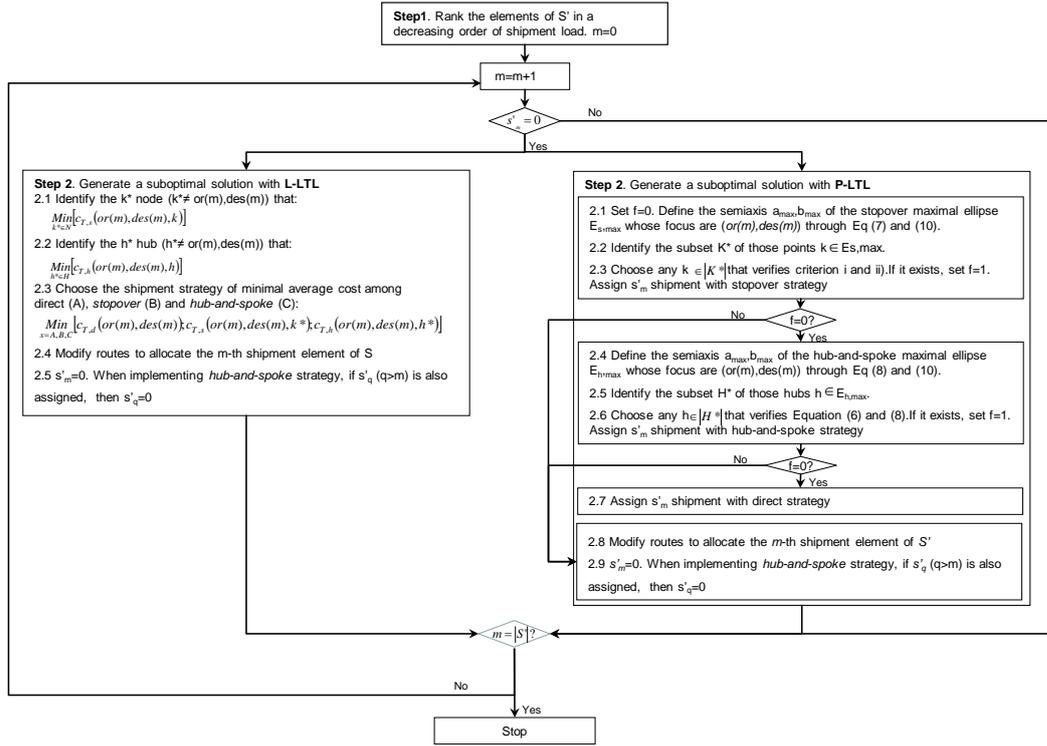


Figure 5. Scheme of the route creation process in P-LTL and L-LTL heuristic algorithms.

Once all shipments s'_m ($m=1, \dots, |S'|$) have been assigned to routes and the total long-haul network defined, the heuristic procedure stops and a fine-tuning technique based on the Tabu Search algorithm is called (Figure 6).

The Tabu Search algorithm (TS) may improve the solution of both *L-LTL* and *P-LTL* adapting the contents developed in Glover and Laguna (2002) and Golden *et al.* (2002). TS is an iterative procedure that defines a set $M(x)$ of potential movements to modify the attributes of the current solution x at each iteration k . The implementation of $M(x)$ to the solution x generates the neighborhood $N^*(x)$, where the solution of minimum cost $x' \in N^*(x)$ will be selected for the next iteration $k+1$. The available movements considered in this research encompass the *insertion movement* of the links of route r at the end of another route s , the *swapping movement* of link chains between routes, and two movements for strengthening the *hub and spoke* and *stopover* strategy as well (see Estrada and Robusté, 2009).

The goodness of the new solution x' is evaluated using the function $f(x')$ of Equation (17) proposed in Cordeau and Laporte (2003). It considers the transportation cost $c(r)$ defined in Equation (1). As the procedure may also incorporate cost-efficient route configurations that violates temporal constrains, a penalty factor $p_l(r)$ is also considered in Equation (17) for those infeasible solutions. The term $p_l(r)$ is evaluated in Equation (18) as a function of the excess arrival or departure time $t_{e,j}(r)$ of route $r \in R$ in each node j . Parameter ε_l will be equal to 1 at first iteration whereas at iteration k^* it will be evaluated using Equation (19). Parameters $k1$ and $k2$ are respectively the number of feasible and unfeasible solutions enumerated up to the current iteration ($k^*=k1+k2$).

$$f(x') = \sum_{r=1}^R [c(r) + p_l(r)] + g_f(x') \quad r \in R \quad \forall x' \in X \quad (17)$$

$$p_l(r) = \varepsilon_1 \sum_{\forall j \in N} t_{e,j}(r) \quad (18)$$

$$\varepsilon_1 = \max \left(\min \left(\frac{(1+\delta)^{k_2}}{(1+\delta)^{k_1}}; 100 \right); 0,01 \right) \quad (19)$$

At each iteration k , the *TS* algorithm chooses only one typology of available movements $M(x)$ as it is depicted in Figure 6. This movement is implemented into a subset V of different node pairs belonging to the current solution. In order to avoid the creation of a huge size of solutions $N(x)$ (that would result in an excessive computational time) we just consider 10 candidate node pairs to be modified ($|V|=10$). For each candidate pair and the selected movement typology, the cost of the potential reassignment movement of this node pair with the rest of available routes is enumerated. Finally, the solution x' that presents the minimal value of the function $f(x')$ detailed in Equation (17) is chosen in the search domain $N^*(x)$. The algorithm allows the acceptance of feasible solutions even if they would worsen the cost of the network in the previous iteration (i.e. $f(x') > f(x)$).

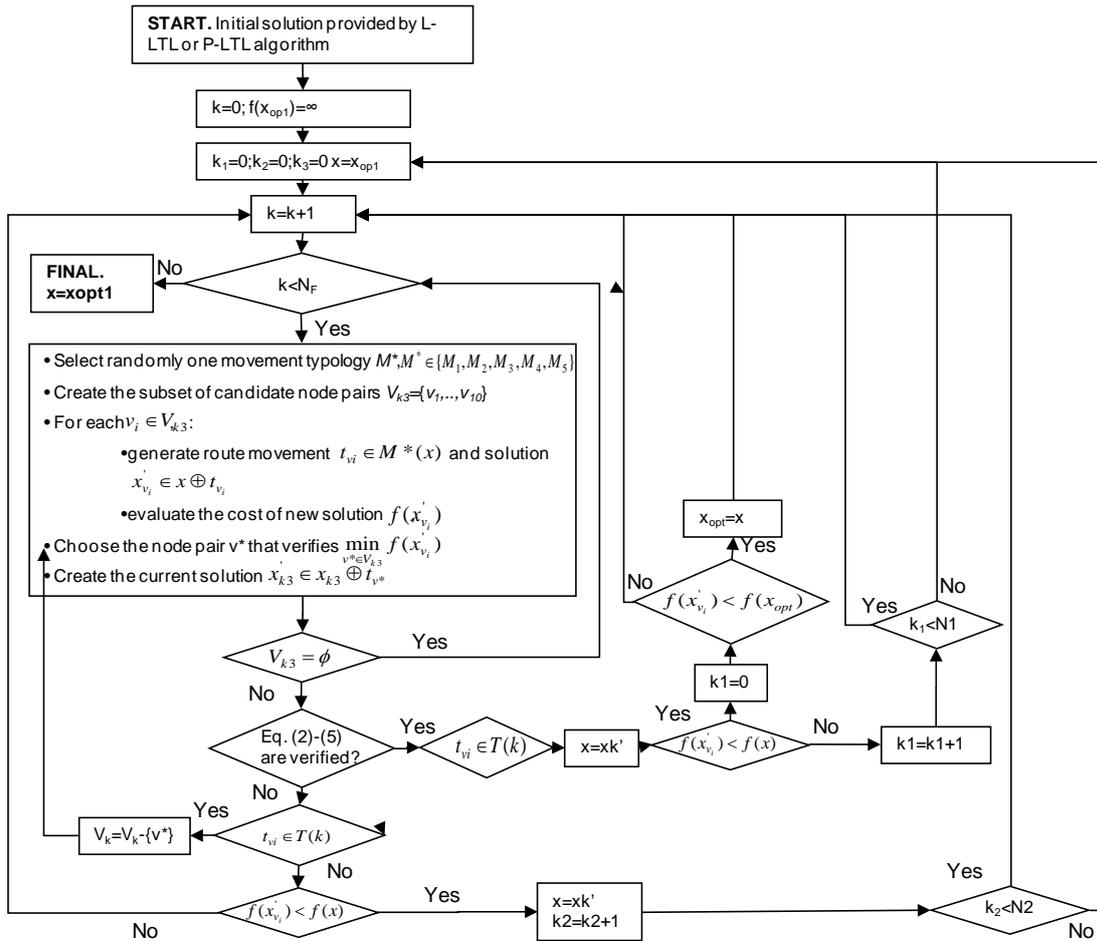


Figure 6. Scheme of the fine-tuning process.

In order to prevent a cyclic performance, the TS algorithm encompasses two strategies for diversifying the search process. This methodology has been partially adopted from Cordeau *et al.* (2003). On one hand, the short term memory criterion defines a “Tabu” list of node pairs that can not be modified in the iteration. Those elements that have been modified in the solution at iteration k are incorporated in the Tabu list and they will maintain the Tabu Status during the next θ iterations. Although we may define a different tabu tenure for elements removed (θ_m , mortality) or added (θ_l life) in the solution, we established $\theta = \theta_l = \theta_m = 100$ iterations. On the other hand, the long term memory aims at diversifying the search in those elements that presented a lowest frequency of insertion in the current solution. In fact, the last term $g_f(x')$ of Equation (17) is responsible for diversifying the solution domain in the long-term search (Equation 20). The term ρ_{mr} in Equation (20) is the number of iterations that shipment s_m is added or removed from route r , λ the current number of iterations and $c(x')$ the actual cost of the problem evaluated by Equation (1). In addition, $|R|$ is the total number of routes, l the number of candidate shipments to be moved, and parameter γ is a factor to be calibrated.

$$g_f(x') = \begin{cases} \gamma \sqrt{N_T |R| l} \cdot c(x') \frac{\rho_{mr}}{\lambda} & \text{if } f(x') > f(x) \\ 0 & \text{if } f(x') \leq f(x) \end{cases} \quad (20)$$

The searching process is finally stopped when N_F iterations have been evaluated. However, the TS may restart the iterative process from a pre-saved low-cost configuration when the current solution has not been improved in N_R iterations ($N_R \ll N_F$). In the calibration process conducted in Estrada and Robusté (2009), the highest savings in terms of transportation cost are obtained when $N_R = 0.8 N$. It has been pointed out that $N_F = 4000 N$ determines a good balance between the optimization results and computational time.

5.2. Test instances and real carrier network problem

Two differential sets of random problems have been created representing the domestic ground-exclusive long-haul routes of an *LTL* carrier. The first set contains scenarios with different numbers of distribution centers, different numbers of hubs and three possible domains of shipment size. The region of service is square-shaped; with an edge length of 500 km. Basically, the aim of this set is to assess the efficiency of the *consolidation* strategies in a variable shipment volume keeping constant the operational service attributes. Hence, this sensitivity analysis will be made assuming the daily shipment volume between all (i,j) distribution center pairs may range in the domains: $s_m \in [0;80] \text{ m}^3$, $s_m \in [0;20] \text{ m}^3$ or $s_m \in [20;40] \text{ m}^3$. The vehicle fleet is assumed to be homogeneous with a volume capacity of $C=80 \text{ m}^3$, a unit distance cost of $c_d=0.4 \text{ €/km}$ and a dispatching vehicle cost of $F=260 \text{ €/day}$. These attributes correspond basically to the major available truck in Spain. In addition to that, these problems are generated with a unit stopping cost of $c_s=15 \text{ €/stop}$ and a transfer cost $c_t=2 \text{ €/m}^3$ in hubs.

On the other hand, the second set of problems has been generated to analyze the *consolidation* strategy performance in the same problem when both fleet and cost

parameters are changed. The physical system is defined by 25 distribution centers scattered in a square region whose side is 500 km long. In this region, there are five distribution centers that act as hubs whereas the flow between each distribution center pair (i,j) varies in $w_{ij} \in [0;80] \text{ m}^3$. In this way, three categories of vehicle capacity are selected corresponding to the available commercial fleet for intercity shipments: vehicles with a capacity $C=80 \text{ m}^3$, $C=52.5 \text{ m}^3$ and finally a vehicle capacity of $C=33.5 \text{ m}^3$ (each of them with an associated unit distance cost and daily fixed cost). The cost parameters have been consulted in a permanent freight transportation cost observatory in Spain (Generalitat de Catalunya, 2006). Furthermore, the stop cost coefficient has been selected to be $c_s=\{5,50\}(\text{€stop})$ whereas the transfer cost has been $c_t=\{0.1,1,5,10\}(\text{€m}^3)$.

Finally, the line-haul network design of the largest Spanish freight carrier has been analyzed for ground-exclusive domestic one-day services. This carrier owns 45 inland distribution centers and hubs scattered in an area of approximately 600,000 km² (Figure 7a). The service of a representative day of year 2006 involves 717 shipments among distribution centers and it is carried out with 256 vehicles. The fleet characteristics and the cost parameters are assumed to be the same of those shown in the first set of test instances. This problem was selected to analyze the performance of the algorithms when the distribution of centers are not located randomly and the shipments loads does not follow a uniform distribution in $[0,C]$. Here, distribution centers are located according to the economic activity of each region. In Figure (7b), the horizontal axis represents the accumulative distribution function of the 45 centers whereas the vertical axis corresponds to the accumulative weight of the daily shipments managed by each center. The 20% of the major centers (hubs) are responsible for roughly the 70% of the total shipments.

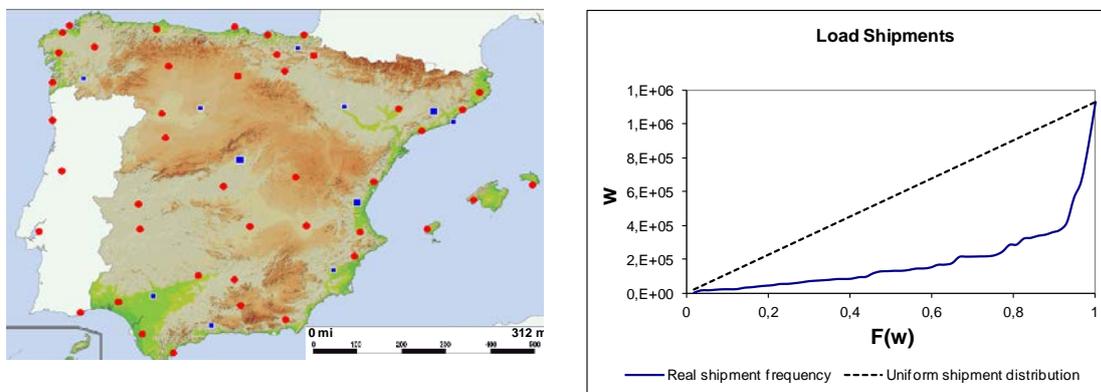


Figure 7. a) Spatial location of distribution centers over Iberian peninsula b) Annual accumulative distribution of shipment volumes (m^3) among distribution centers.

5.3. Results

Both *L-LTL* and *P-LTL* algorithms and the fine-tuning subroutine are programmed in VisualBasic 6.0 and run in a PC equipped with an Intel Core2 Duo 530MHz chip 1024Mb (RAM). The results obtained in the first set of problems are summarized in Table 1. The values find out that the final cost obtained with the *probabilistic*

consolidation strategy (*P-LTL*) is always less than the value associated to the basic *L-LTL* algorithm after the implementation of the Tabu Search algorithm.

Table 1. Comparative result analysis of L-LTL and P-LTL algorithm varying the network configuration and the volume of shipments among distribution centers (first set of instances).

N	hubs	Shipment Volume limits (m ³)			L-LTL algorithm			P-LTL algorithm		
		Lower	Upper	Shipments	Initial	Final cost	# Routes	Initial	Final cost	# Routes
					Solution Cost (€)	with T.S. (€)		Solution Cost (€)	with T.S. (€)	
17	4	0	20	250	42,515.1	31,042.0	50	32,522.0	28,440.6	46
		20	40	270	52,028.1	47,653.7	83	42,428.4	39,176.9	66
		0	80	271	50,646.6	44,373.4	77	42,563.4	40,845.6	71
38	6	0	20	722	128,128.2	95,393.8	163	97,082.7	83,167.8	132
		20	40	737	143,589.0	136,770.6	237	142,608.6	130,390.7	225
		0	80	751	115,061.0	108,100.9	195	120,015.0	112,536.4	203
66	8	0	20	641	115,397.1	100,550.2	170	120,162.2	95,563.2	162
		20	40	656	116,656.2	111,813.0	197	121,096.6	109,673.0	192
		0	80	654	142,126.6	133,361.3	241	142,132.9	132,394.8	239
25	5	0	20	388	62,341.7	49,821.9	85	49,459.0	41,162.8	67
		20	40	384	70,912.8	66,657.7	115	63,063.3	56,553.0	95
		0	80	393	74,307.6	67,752.6	121	62,450.4	59,817.2	106
41	6	0	20	555	77,451.7	58,427.9	98	51,265.7	46,951.9	77
		20	40	554	82,543.1	80,370.3	140	60,234.6	56,900.2	95
		0	80	566	88,229.2	80,352.0	146	73,795.7	70,159.9	125

It has to be highlighted that the most efficient implementation of *P-LTL* compared to *L-LTL* algorithm has been identified in problems where the average shipment volume is significantly lower than the vehicle capacity, i.e. in typical less-than-truckload problems. In the first test instance, the average cost reduction and the ratio of saved routes are 10% after the implementation of Tabu Search algorithm. Additionally, the improvements in the initial solution derived from the application of *P-LTL* (before fine-tuning process) are significant, obtaining an average cost reduction of 13% regarding the *L-LTL* algorithm. It means that the proximity criteria provides better solutions than the algorithm that only considers the cost savings to allocate pending shipments to existing routes.

Especially, when $w_{ij} \in [0;20]m^3$ (extreme case generated of less-than-truckload shipments) the average solution of *P-LTL* is 20% less than *L-LTL*. The high number of consolidation chances in *L-LTL* problems (with *hub and spoke* and *stopover* strategies) limits the efficiency of the Tabu Search algorithm with regard to *P-LTL* problems. Although the Tabu Search algorithm can iteratively reduce the total transportation cost, the inclusion of *probabilistic stopover strategy in origin or destination* is essential to guarantee a good solution and a reasonable computational time.

Apart from that, the results of the second set of problems (summarized in Table 2) show that the efficiency of the *stopover* and *hub-and-spoke* strategies depends slightly on the stopping cost. In those problems with low stopping cost ($c_s=5\text{€/stop}$), the relative efficiency of *P-LTL* is significant. In this situation, the average reduction percentage in final cost is 14.9% while the corresponding value when $c_s=50\text{€/stop}$ is 13.4%.

Table 2. Comparative result analysis of L-LTL and P-LTL algorithm varying the vehicle capacity and unit transportation cost (second set of instances).

Type of fleet	Unit transfer cost	Unit stop cost cs(€)	L-LTL algorithm			P-LTL algorithm			
			Cost Initial solution (€)	Final cost-T.S. (€)	# Routes	Cost Initial solution (€)	Final cost-T.S. (€)	# Routes	
V1 (C=80m ³)	0.1	5	68,781.7	66,919.6	137	57,259.1	56,137.3	110	
		50	90,733.9	86,446.4	131	75,668.7	73,733.3	108	
	1	5	69,906.5	64,992.3	127	57,733.4	56,611.6	110	
		50	91,201.9	87,611.7	132	76,159.6	74,224.3	108	
	5	5	71,403.2	69,452.0	137	58,081.1	58,081.1	110	
		50	93,281.9	89,691.7	132	78,341.6	76,666.6	109	
	10	5	74,078.2	72,127.0	137	60,491.1	59,290.4	106	
		50	95,881.9	92,291.7	132	81,069.1	79,394.1	109	
	V2 (C=52.5m ³)	0.1	5	90,088.7	85,623.2	167	73,244.8	71,495.6	132
			50	119,262.7	114,911.7	170	99,439.4	97,854.1	134
1		5	90,657.6	86,192.0	167	73,578.5	71,829.3	132	
		50	119,821.6	115,470.6	170	100,266.8	99,956.8	139	
5		5	92,977.4	90,227.9	172	75,061.5	73,312.3	132	
		50	122,305.6	117,954.6	170	99,631.5	98,081.5	133	
10		5	95,989.9	93,240.4	172	76,915.3	76,120.3	135	
		50	125,410.6	121,059.6	170	107,514.9	105,964.9	144	
V3 (C=33.5m ³)		0.1	5	115,322.8	113,364.9	215	102,594.6	100,613.9	186
			50	159,712.0	155,569.6	226	138,620.2	137,380.2	189
	1	5	115,578.1	113,620.5	215	102,778.4	100,797.7	186	
		50	160,506.0	153,932.4	220	138,816.2	137,576.2	189	
	5	5	116,714.4	114,765.5	215	103,595.4	101,614.7	186	
		50	161,630.4	157,488.0	226	139,687.2	138,447.2	189	
	10	5	118,065.4	116,740.4	217	104,075.9	102,689.1	187	
		50	162,080.6	157,048.4	220	141,550.5	140,000.5	190	

Moreover, the ratio between the average shipment volume and truck capacity plays a significant role in the optimization range of the *P-LTL* algorithm. When the fleet of 33.5 m³ capacity is selected, the number of full truckload shipments rises as the probability of *stopover* and *hub and spoke* strategies drops significantly. In those problem instances, the average cost saving of *P-LTL* algorithm in the final solution is 11.4 % with regard to the *L-LTL* algorithm. However, when the vehicle capacity parameter is increased (C=80 m³ or C=52.5 m³), the *P-LTL* algorithm presents a higher performance with an average cost reduction of 15.6% in comparison with *L-LTL* algorithm. Finally, the variation of the transfer cost parameter c_t does not have any clear effect in the comparative efficiency of *P-LTL* since it uniquely affects the part of the algorithm shared by *P-LTL* and *L-LTL*.

Eventually, the results provided by the implementation of the solving procedures in the real network of the largest Iberia carrier are summarized in Table 3. The total transportation cost estimated by the *L-LTL* greedy algorithm was 169,958 euros/day. The subsequent fine-tuning process was able to improve this solution by 15%. On the other hand, the probabilistic criterion (*P-LTL* algorithm) in its standard definition ($k_r=1$) overestimates the solution of the problem. This is due to the non-uniform distribution of flows among centers and the relevant concentration of distribution centers in the

Spanish periphery (Figure 7a). However, we slightly modified the probabilistic criterion allowing that the expected number of candidate centers in the critical ellipse would be higher than 1. The aim of this modification is to increase the searching circular area with a radius greater than $|R_s|^{1/2}$. The best initial solution obtained with *P-LTL* algorithm corresponds to the case when $k_r=2$ and presents an objective function value of 165,305 euros per day. The later fine-tuning process with Tabu Search algorithm reduces the final distribution cost to 141,908 euros per day (14.1% reduction). All final solutions presented in Table 3 with Tabu Search heuristic are significantly optimized considering the real distribution cost reported by the Spanish carrier for the day of study (151,804 Euros/day). So, the daily long-haul cost saving for the carrier due to the optimization methodology presented in this paper would be roughly 6.5%.

Table 3. Results of optimization in a real long-haul carrier network

Algorithm	k_r	Initial Solution (€)	Final Solution with TS (€)
L-LTL	--	169,958	143,326
	1	195,432	147,981
P-LTL	1.5	167,871	142,341
	2	165,305	141,908
	3	174,304	145,272
Real carrier	--		151,804

6. Conclusions

This paper has introduced the *probabilistic* consolidation strategy to solve the long-haul routing problem for *LTL* carriers. This strategy selects a distribution center to be included in an intermediate position of an existing long-haul route, or as a hub, by means of both cost reduction and proximity criteria. The proximity criterion is based on the likelihood of finding two complementary distribution centers closer than r_c units of distance, and it provides a useful look-ahead vision to avoid the adoption of local optimal solutions.

This strategy has been complementarily incorporated with *direct* shipment strategies to build a heuristic algorithm (*P-LTL*). Experimental results indicate that the *P-LTL* algorithm gives high quality solutions compared to a greedy heuristic (*L-LTL*) where consolidation strategy is selected uniquely by cost reduction criterion. Hence, the cost of the solutions obtained with *consolidation* strategies considering proximity criterion (*P-LTL* algorithm) is, on average, 10% less than the value of the *L-LTL* algorithm. The consideration of a *consolidation* strategy is more efficient when the average shipment load factor is low. In these scenarios, the high number of combinations to consolidate goods affects seriously the efficiency of existent fine-tuning techniques (Tabu Search). Hence, the *consolidation* strategies are extremely useful when the average shipment load factor is less than 0.3. Its inclusion in the *P-LTL* algorithm produces 20% of transportation cost savings with regard to heuristic techniques in typical less-than-truckload problems. Therefore, the *P-LTL* algorithm presented in this paper has been demonstrated to be crucial at first stages of sequential route construction heuristics.

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