



## Contents and Sources of Practical Geometry in Pedro Luce's Course at the Barcelona Royal Military Academy of Mathematics

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### Introduction

Established in 1720, the Royal Military Academy of Mathematics of Barcelona seems particularly interesting as an example of organizing training for engineering, insofar as its main goal was to provide knowledge in mathematics to both young and experienced officers in military fields.

There are many historical studies of the Academy that describe the running of the institution, the type of education and the books most frequently employed there, etc..<sup>1</sup> However, there has yet to appear an in-depth study of the content of the texts used in teaching at the Academy, and its comparison with other institutions in Europe during that period. Our studies have therefore focused on the contents of this education,<sup>2</sup> and in this communication we analyse the treatment of pure geometry in a Mathematical Course used at this Academy for 40 years. In fact, pure geometry is a version of Euclid's *Elements*. We study what the contents of this geometry were, and also the significance of the treatment of Euclid's *Elements* when this course is compared with other contemporary textbooks.

### Teaching mathematics at the Royal Military Academy of Mathematics (1720-1803)

In 1712, the General Engineer Jorge Próspero de Verboom presented the project for a military academy that was modeled on the Royal Military Academy in Brussels, set up by his teacher, Sebastián Fernández Medrano (Navarro Loidi 2004). The lack of centers for military education gave rise to a number of debates on how to develop the training of Military Officers. On the basis of his experience at the Brussels Academy, Verboom was convinced that Officers should be educated in pure and mixed Mathematics and, after some negotiation, the Barcelona Academy opened in October 1720.

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<sup>1</sup> See Riera 1975, Capel 1988 and Muñoz Corbalán 2004. There are also many documents in the AGS (Archivo General de Simancas) and in the Archive of Military Region in Barcelona containing copies of exams, lists of students, letters, etc.

<sup>2</sup> In fact, we have recently published an article in which we show how training of the military was centred on the teaching of several subjects (multidisciplinary) under the single heading of Mathematics, presented as a Mathematical Course in accordance with the tradition of the mathematical courses initiated in the seventeenth century (Massa et al. 2011).



The first directors of the Barcelona Royal Military were Mateo Calabro (1720-1738) and Pedro Lucuce y Ponce (1738-1779).<sup>3</sup> We focus our research on the time when Lucuce was the director, which corresponds to the period regarded as the most brilliant at this Academy. In 1739, a Royal Ordinance set out the contents of the course in mathematics that was to be taught in the academies. The Ordinance followed the reports made by Verboom in 1730 and also by Lucuce in 1737. The director was required to prepare a course of mathematics according to the guidelines of this Ordinance, Article 9 of which states that: "In order to achieve education according to this idea, the Director General should select the most useful mathematical Treatises, arranging them methodically so as to be of greatest benefit to the Academics, writing the subjects that must be taught as his own doctrine, which should include everything to be imparted in the Academy, extending explanation as far as should be deemed necessary."

What were these Mathematical Treatises referred to in the Ordinance? We have no direct information about this in the Ordinance, although some Mathematical Courses are mentioned in the previous reports by Verboom and Lucuce. It should be remembered that the tradition of Mathematical Courses as textbooks began in seventeenth century, and the courses by Dechaies, Ozanam, Tosca and Belidor constitute those most relevant for our research. The three volumes on pure and mixed mathematics by Claude François Milliet Dechaies (Chambéry, 1621-Turín, 1678), which form the *Cursus seu mundus mathematicus* (1674), included the first eight books of Euclid, Arithmetic, the Spheres of Theodosius, Trigonometry, Practical Geometry, Mechanics, Statics, Universal Geography, Civil Architecture, etc. This course was widely read and exerted much influence in Europe; for example, it influenced Mathematical Courses such as Tosca's *Compendio* and possibly Lucuce's *Course*. It is also noteworthy that in the XVII century Jacques Ozanam (1640-1717) wrote a *Cours de Mathematique qui comprend toutes les parties de Cette Science les plus utiles et les plus necessaires à un homme de Guerre et à tous ceux qui se veulent perfectionner dans les Mathematiques* (Paris, Jombert, 1693), dedicated to men of War. It consists of five volumes: 1) Introduction to Mathematics and to Euclid's Elements; 2) Arithmetic and Trigonometry; 3) Geometry and Fortifications; 4) Mechanics and Perspectives; 5) Geography and Gnomics. In the eighteenth century, and following the tradition of these courses, Tomàs Vicent Tosca (1651-1723) wrote a *Compendio Mathematico* (1707-1715) consisting of 28 treatises in 9 volumes (Navarro Brotons 1985). The first volume deals with Elementary Geometry, lower Arithmetic and Practical Geometry; the second addresses high Arithmetic, Algebra and Music; the third, Trigonometry, Conic Sections and Machinery; the fourth, Statics, Hydrostatics, Hydrotechnics and Hydrometry; the fifth, Civil Architecture, Military Architecture, Pyrotechnics and Artillery; the sixth, Optics, Perspective, Catoptrics, Dioptrics and Meteors; the seventh, Astronomy; the eighth, Practical

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<sup>3</sup> Pedro Lucuce studied Canon Law at the University of Oviedo, but he left this career to join the Army in 1711, during the War of Spanish Succession (1705-1714). After the War he joined a regiment in Madrid where he had the opportunity to study Mathematics on his own initiative. In 1730, he was elected simultaneously member of the Corps of Military Engineers and of the Corps of Artillery. He chose the engineering corps and in 1736 joined the Academy of Mathematics of Barcelona.



Astronomy, Geography and Seamanship; the ninth, Gnomics, the Ordering of Time and Astrology. Tosca's work went into a further three editions during the 18th century, and some volumes were published separately. This is a clear indication of the reception and influence enjoyed by this work. Also in the 18th century, and as an antecedent to Lucuce's course, we may mention the work of Bernard Forest de Bélidor (1698-1761), a French military engineer born in Catalonia, entitled *Nouveau Cours de Mathematique, a l'usage de l'artillerie et d'ingenie ou l'on applique les Parties les plus utiles de cette science à la Théorie & à la Pratiques des différents sujets qui peuvent avoir rapport à la Guerre* (Paris, Chez Nyon, 1725) consisting of ten parts with many chapters including geometry, trigonometry, geodesy, mechanics and hydraulics. As the title indicates, it was an eminently practical course to be used as part of military training, in which Bélidor was involved. This book was used in the new schools of artillery established by royal decree in France (Hahn 1986). Bélidor is considered to be the modern creator of engineering science, and his works were enormously influential during the eighteenth century in France as well as in many other countries.

In fact, in 1730 Verboom quotes Tosca and Belidor as examples to follow in some parts of course. In this sense, the promoters of the Royal Military Academy of Mathematics of Barcelona shared the general approach adopted by these authors, and emphasized that engineering training should be conducted through a Mathematical Course of this kind.

We therefore present the Mathematical Course drawn up by Lucuce according to a broad and modern vision of mathematics, which consists of eight treatises dealing with the main fields of mathematics, including "pure" mathematics (arithmetic and geometry), and "mixed" mathematics (the rest): 1) On Arithmetic; 2) On Elementary Geometry; 3) On Practical Geometry; 4) On Fortifications; 5) On Artillery; 6) On Cosmography; 7) On Statics and 8) On Civil Architecture (Massa et al. 2011, 243-244). Although this course was never published, it has been preserved in several manuscripts written by different students from the Barcelona Academy, and also from two other centres in Oran and Ceuta.<sup>4</sup> We wish to point out that all the manuscripts describing the course were virtually identical, in spite of being imparted by different professors, in different locations, and in different years.

### **Euclid's *Elements* in Lucuce's Course**

Here we focus our research on the treatment of Euclid's *Elements* in Lucuce's course, of which it forms an original part. In 1730, in his "Proyecto", Verboom had already signaled the sources on which this course should be based (Verboom 1730, 44). For the first six books of Euclid, Verboom suggested the "Elements" of Port Royal (by Antoine Arnaud (1612-1694) in 1670) and also the course prepared by Jean-Pierre de Crousaz (1663-1750), professor at the Academy of Lausanne. In fact, after the publication and diffusion of Descartes' *Géométrie* (1637), the relations between algebra and geometry changed and geometry was presented from another

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<sup>4</sup>For the purposes of our analysis, we use the copy made by Antonio Remon Zarco Torralbo, which includes all the figures, and is conserved in the Central Military Library of Madrid.



perspective. So the *Nouveaux éléments de Géométrie* (1667) by Arnauld constitute a new order and present new ideas.<sup>5</sup>

Nevertheless, later, in 1739, the only reference in the Ordinance of the Royal Academy is that Euclid's *Elements* should be part of the syllabus.

So, in his introduction to Treatise II entitled "On Elementary Geometry", which deals with Euclid's *Elements* and Conic sections, Lucuce clarifies which books of the *Elements* he wishes to work with and which in part are explained:

"Since the work is extensive and diffuse, in this treatise we explain Books 1, 2, 3, 6, 11 and 12, with respect to which Book 4 is addressed to Practical Geometry and Book 5 to Arithmetic, while the others, being of little use, are omitted. The order I follow in the propositions is the same as that given by Euclid, so that they may be cited whenever necessary, the most useful being demonstrated with all possible brevity and clarity in order to save time for the explanation of other subjects that are of concern for the instruction of military personnel." (Lucuce 1739-1744, introduction).

The choice of explaining only Books 1 to 6 and 11 and 12 was usual at that time, because these books were considered to be the most useful. In fact, Tosca in his *Compendio* and Dechaes in his *Cursus* explain eight books, specifying that Books seven and eight correspond to 11 and 12, respectively. Unlike Béliidor in his *Nouveau Cours*, for example, Lucuce did not base his geometry directly on the "Nouvelle Géométrie de Port Royal" but rather explained some books of Euclid's *Elements* in an original way.

Lucuce explains the books of the *Elements* making proofs by figures, by letters and by numbers. Thus, in the introduction to Book II in Treatise II, he emphasizes: "In this book we consider the rectangles and squares which are formed by dividing a rectilinear line into parts; its comprehension is of great usefulness in Mathematics and specially for Algebra; and although its theorems are obscure, its proofs will be facilitated by lines as well as by literal calculus and also by numbers." (Lucuce 1739-1744, 42).

Ozanam made the proofs in a similar way, but without referring to it, and using a rhetorical style.

Let us now consider the treatment of Book V on the theory of proportions.<sup>6</sup> In an original and modern way, Lucuce moves the book on proportions from the second treatise dealing with geometry to the first treatise dealing with arithmetic. In Dechaes' and Ozanam's courses, all the books in Euclid's *Elements* remain together, while Tosca's course, like Lucuce's, only moves Book IV to Practical Geometry. In a certain sense, Lucuce performs an arithmetization of the theory of proportions. We wish to characterize the arithmetization of theory of proportions that began in

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<sup>5</sup> In relation to the 11<sup>th</sup> and 12<sup>th</sup> Books of Euclid's *Elements*, "the Course will follow the method of Father Tosca, avoiding [detailed demonstrations of] the curious and the abstract". (He refers to *Compendio Mathematico* by Tosca). For Practical Geometry, the practical instructions for Engineers and Artillery Officers, Béliidor, in the *Nouveau Cours de Mathématique*, was identified by Verboom as being taught at "one of the five academies established in France".

<sup>6</sup> On the treatment of the theory of proportions in the treatises of the late seventeenth century, see Lamandé 2013.



the sixteenth century. In the simplest stage, authors illustrate Euclid's definitions with numbers or/and algebraic symbols, while in a second stage proportions apply to all kinds of quantities, both discrete and continuous. The third stage can be represented by the identification of a ratio with a numerical value, and finally the proof of propositions on theory of proportions is performed by arithmetical lemmas. Lucuce argued that the theory of proportions can be applied to numerical ratios as well as to magnitudes, and that this is necessary for comprehending mathematics as a whole: both pure and mixed. So, at the beginning of Book III in *Treatise I* Lucuce explains as follows: "Book 3. On ratio and proportion in common. This book, which is the 5<sup>th</sup> in Euclid and deals with the ratio and proportion in common, whose doctrine is suitable for *all kinds of quantities* both discrete and continuous, that is, it serves for numbers, lines, surfaces and solids; being the universal key for acquiring the knowledge of as many parts as of which the mathematics are composed. Its propositions preserve Euclid's order, so they may be cited where appropriate, the least important being omitted and only the most helpful remaining, to be demonstrated by *letters* and explained by numbers to facilitate their understanding." (Lucuce 1739-1744, 58). At the end of the seventeenth century and at the beginning of the eighteenth century, the arithmetization of Euclid's *Elements* gave rise to considerable debate; several authors attempted to move the foundation of mathematical proportions from geometrical magnitudes to numerical ratios, thereby avoiding incomprehensible definitions and adapting it to modern ideas in algebra. It is noteworthy that this process is linked to the contemporary process of the algebrization of mathematics from the diffusion of works by Viète and Descartes. As regards the contents of the book, Lucuce tries to adapt it for a modern readership, although he preserves Euclid's order, which makes this a pioneering choice that did not clash with the conservative approach. Indeed, Dechaies in the definitions also tries to deal with all kinds of quantities, but remains Euclidian in his procedure for the demonstration of the propositions. Tosca in his *Compendio* presents a conservative edition of Euclid's *Elements*, only illustrated with numbers, while in the definitions Ozanam also deals with quantities. In fact, Ozanam's course is the most similar in terms of ideas, although in his presentation of the demonstrations he follows the rhetorical approach.

Definition 5 presented by Lucuce is also original: "Definition 5. The exponent of the ratio is the quotient obtained by dividing the antecedent by the consequent. For example, if in the ratio of 6 to 2, 6 is divided by 2, the quotient 3 is the exponent which states the number of times that 6 contains 2 and type  $6/2$ ." (Lucuce 1739-1744, 59-60). Lucuce uses this definition to show the relationship between ratios through the exponent of the ratio, that is to say, the numerical value of the ratio. After the definitions and before the propositions, Lucuce presents two arithmetical lemmas that facilitate the demonstrations. "Lemma 1. If four quantities be proportional, the product of the two extremes is equal to the product of the two means; and if the product of the extremes is equal to the means, the quantities are proportional." (Lucuce 1739-1744, 66). Then he gives the demonstrations of the propositions used as well as the idea of the exponent or the arithmetical lemmas. Ozanam also proves some propositions according to Euclid, and from the proposition



XV he enunciates the same arithmetical lemmas and demonstrates the rest of the propositions by these arithmetical lemmas.

### **Some concluding remarks**

After a preliminary analysis of this Treatise II, we surmise the first answers to the questions. We put forward the hypothesis that, although Lucuce knows Tosca's *Compendio Matematico*, Belidor's *Nouveau Cours Mathématique* and Ozanam's *Course*, to a great extent he designed his own course. Now we provide new evidence to justify this hypothesis. Lucuce made no copies, nor did he rewrite a course that had already been published; rather he chose the appropriate subjects according to didactical criteria, without omitting the latest inventions and always mindful of the most suitable and comprehensible mathematics to provide good theoretical and technical training.

This analysis shows Lucuce to be an original mathematician with his own ideas about teaching mathematics for engineering education. Its didactic function makes it a set of choices for subjects designed to be useful in the training of building engineers and artillery personnel.

Lucuce's version of Euclid's Elements consciously sought to adapt the Elements for a contemporary readership (military and engineers) in order to facilitate their comprehension and usefulness.

Finally, it should be remembered that the readership for mathematics in the 18<sup>th</sup> century began to spread beyond the universities, while the newly-created academies instilled a mathematical worldview that emphasized the usefulness of results, maintaining the reference to classical authorities. In fact, to most 18<sup>th</sup> century practitioners, results mattered more than classical authority, and Lucuce's early attempt provides an example of multidisciplinary engineering education that contributed in some way to defining engineering as a scientific profession in Spain.

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