Implementation of the History of Mathematics in Catalan Secondary Schools

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Introduction

The history of mathematics in the classroom enables improvements to be made in the learning of mathematics. Through the analysis of significant texts from the historical evolution of mathematical concepts, the History of Mathematics Group of the Barcelona Association for the Study and Learning of Mathematics (ABEAM) have prepared and tested some historical materials to be used in the classroom (Massa *et al.* 2011). The aim of this article is to discuss, by means of analysis of some examples implemented in the mathematics classroom, the criteria for preparing activities with historical texts and the conditions for transforming these historical activities into a powerful tool for learning mathematics.
First we present a survey of the evolution of this implementation over the last few years in Catalonia. In Spain, every autonomous community is in charge of its own secondary and graduate education, so we focus only on the implementation of history in the mathematics classroom in Catalonia.

Background of the implementation of the history of mathematics

In Catalonia, the implementation of the history of mathematics has for twenty years been inspiring some individual actions among teachers. Thus, since the academic year 1990-91, financial aid for research work, granted every year to teachers by the Department of Education of the government of Catalonia, has been devoted to research into the relations between the history of science (including mathematics) and its teaching. This research work has resulted in the drawing up of reports that are now available to other teachers. In addition, workshops, centenary celebrations and conferences by teachers in high schools constitute further examples of activities in which history can be used to achieve a more comprehensive learning experience for students. For example, the workshop devoted to the study of the life and work of René Descartes (1596-1650), held in 1996 at the INS Carles Riba (a Catalan high school), provided students with additional background from a mathematical, philosophical, physical and historical perspective.
As a collective action, we may mention that since 2003 to the present, every two years Pere Grapi and Mª Rosa Massa have coordinated a workshop on the History of Science and Teaching organized by the Catalan Society of History of Science and Technology (SCHCT), and subsequently they have coordinated the publication of the proceedings. The aim of these

workshops is to enable teachers to show their experiences in the classroom as well as to discuss the criteria and conditions for these implementations.

In the academic year 2007-2008, the Department of Education of the Government of Catalonia introduced some contents of the history of science into the curriculum for secondary education, namely, the new Catalan mathematics curriculum for secondary schools, published in June 2007, which contains notions of the historical genesis of relevant subjects in the syllabus.[1]

In the academic year 2009-2010 a Master's degree was introduced for training future teachers of mathematics. The syllabus of this new course launched at the University includes a part on the history of mathematics and its use in the classroom. For example, at the Polytechnic University of Catalonia, the title of the historical part is: "Elements of the history of mathematics for the classroom", while at the Pompeu Fabra University the title is: "The history of mathematics and its use for teaching math". Furthermore, in the academic year 2009-2010, an online pilot course on the history of science for science teacher training was put into practice. This course was drawn up by historians of science belonging to the Catalan Society for the History of Science and Technology (SCHCT) under the name "Science and Technology through History"

The setting up of the Group of History of Mathematics of Barcelona (ABEAM) in 1998[2] was also a significant step. The aim of this group of teachers of Mathematics is to create History of Mathematics materials to be used in the classroom. The list of the texts implemented includes: *On the sizes and distances of the Sun and Moon* by Aristarchus of Samos (ca. 310-230 BC) (Massa Esteve 2005b; Aristarco 2007); Euclid's *Elements* (300 BC) (Romero, Guevara and Massa 2007); Menelaus' *Spheriques* (ca. 100) (Guevara, Massa and Romero 2008a-2008b); *Almagest* by Ptolemy (85-165) (Romero and Massa 2003); *Nine Chapters on the Mathematical Art* (s. I. AC) (Romero *et al.* 2009); *Traité du quadrilatère* by Nassir-al-Tusi (1201-1274) (Romero, Massa and Casals 2006) and *Triangulis Omnimodis* by Regiomontanus (1436-1476) (Guevara and Massa 2005). The essential ideas on the implementation of history in the mathematics classroom of this group are reflected in the following sections.

Teaching mathematics using its history.

The history of mathematics in the mathematics classroom can be used in two ways: as an integral educational resource and as a didactic resource for understanding mathematics (Jahnke *et al.* 1996; Barbin 2000; Massa Esteve 2003).

In the first instance, history in the mathematics classroom can provide students with a conception of mathematics as a useful, dynamic, human, interdisciplinary and heuristic science.

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[1] The list of these historical contexts includes: The origins of the numeration system; the introduction of zero and the systems of positional numeration; geometry in ancient civilizations (Egypt, Babylonia); the first approaches to the number π (Egypt, China and Greece); Pythagoras' theorem in Euclid's *Elements* and in China; the origins of symbolic algebra (Arab world, Renaissance); the relationship between geometry and algebra and the introduction of Cartesian coordinates; the geometric resolution of equations (Greece, India, Arab World); the use of geometry to measure the distance Earth - Sun and Earth - Moon (Greece).

[2] The coordinator of the group is Mª Rosa Massa Esteve and the other members of group are: Mª Àngels Casals Puit (INS Joan Corominas), Iolanda Guevara Casanova (INS Badalona VII), Paco Moreno Rigall (INS XXV Olimpíada), Carles Puig Pla (UPC) and Fàtima Romero Vallhonesta (Inspecció d'Educació). The group is subsidized by the *Institute of Science of Education* (ICE) of the University of Barcelona.

- A useful science. Teachers should explain to students that mathematics has always been an essential tool in the development of different civilizations. It has been used since antiquity for solving problems of counting, for understanding the movements of the stars and for establishing a calendar. In this regard, there are many examples right down to the present day in which mathematics has proved to be fundamental in spheres as diverse as computer science, economics, biology, and in the building of models for explaining physical phenomena in the field of applied science, to mention just a few of the applications.
- A dynamic science. It will also be necessary to teach students whenever appropriate about problems that remained open in a particular period, how they have evolved and the situation they are in now, as well as showing that research is still being carried out and that changes are constantly taking place. History shows that societies progress as a result of the scientific activity undertaken by successive generations, and that mathematics is a fundamental part of this process.
- A human science. Teachers should reveal to students that behind the theorems and results there are remarkable people. It is not merely a question of recounting anecdotes, but rather that students should know something about the mathematical community; human beings whose work consisted in providing them with the theorems they use so frequently. Mathematics is a science that arises from human activity, and if students are able to see it in this way they will probably perceive it as something more accessible and closer to themselves.
- An interdisciplinary science. Wherever possible, teachers should show the historical connections of mathematics with other sciences (physics, biology, medicine, architecture, etc.) and other human activities (trade, politics, art, religion, etc.). It is also necessary to remember that a great number of important ideas in the development of science and mathematics itself have grown out of this interactive process.
- A heuristic science. Teachers should analyze with students the historical problems that have been solved by different methods, and thereby show them that the effort involved in solving problems has always been an exciting and enriching activity at a personal level. These methods can be used in teaching to encourage students to take an interest in research and to become budding researchers themselves.

In the second instance, the history of mathematics as a didactic resource can provide tools to enable students to grasp mathematical concepts successfully. The History of mathematics can be employed in the mathematics classroom as an implicit and explicit didactic resource. The history of mathematics as an implicit resource can be employed in the design phase, by choosing contexts, by preparing activities (problems and auxiliary sources) and also by drawing up the teaching syllabus for a concept or an idea.

Nevertheless, it is necessary to bear in mind that the historical process of building up a body of knowledge is a collective task that depends on social factors. In the past, many mathematicians adopted the solution of particular problems as the aim of their research and were able to devote many years to their objectives. It is worth remembering that our students, while having the ground before them well-prepared, are addressing these notions for the very first time and often lack the motivation for solving mathematical problems.

Indeed, it is not history itself that is relevant for teaching, but rather the genesis of problems, the proofs that favored the development of an idea or a concept. The clarification of this development of ideas and notions can also act as a motivation for solving current problems. The evolution of a mathematical concept can thereby reveal the learning difficulties encountered by students, as well as pointing the way towards how the concept can be taught (Massa Esteve 2005a).

In addition to its importance as an implicit tool for improving the learning of mathematics, the history of mathematics can also be used explicitly in the classroom for the teaching of mathematics. Although by no means an exhaustive list, we may mention four areas where the history of mathematics can be employed explicitly: 1) for proposing and directing research work at baccalaureate level using historical material[3]; 2) for designing and imparting elective subjects involving the history of mathematics; 3) for holding workshops, centenary celebrations and conferences (Massa Esteve, Comas and Granados1996), and 4) for implementing significant historical texts in order to improve understanding of mathematical concepts (Massa Esteve and Romero 2009). In this article we analyze the last point.

Implementing historical texts in the mathematics classroom

The use of significant historical texts in the classroom to facilitate the understanding of mathematical concepts is an activity that can provide students with more valuable means for learning mathematics.

The main aims for its implementation in the mathematics classroom are to enable students to; a) learn the original source on which the knowledge of mathematics in the past is based; b) recognize the socio-cultural relations of mathematics with the politics, religion, philosophy or culture in a certain period and, last but not least; c) improve mathematical thinking through reflections on the development of mathematical thought and the transformations of natural philosophy.

What historical texts are suitable for use in the mathematics classroom? Not all historical texts are useful for the mathematics classroom. The initial selection could be based on historical texts related to the historical contexts in the new Catalan curriculum. Historical texts (e.g. proof or problems) should in some way be anchored in the mathematical topic to be addressed. Different types of historical texts should be used, depending on the stage in the didactical sequence.

At what stage in the teaching process should we use historical texts in the mathematics classroom? Historical texts could be used to introduce a subject or a concept, to explore it more deeply, to explain the differences between two contexts, to motivate study of a particular type of problem or to clarify a process of reasoning.

How do we use historical texts in the mathematics classroom? It is necessary to bear some points in mind: The relation between the historical text and the mathematical concept under study should be clarified in order for the analysis of the text or significant proof to be integrated into the mathematical ideas one wishes to convey. The mathematical reasoning behind the proofs should be analyzed. Indeed, addressing the same result from different mathematical perspectives enriches students' knowledge of mathematical understanding. The proof should be contextualized within the mathematical syllabus by associating it with the mathematical ideas studied on the course, so that students may see clearly that it forms an integral part of a body of knowledge, and it should also be situated within the history of mathematics to enable students to evaluate the historical development of the concept.

[3] The list of titles of these research works can be very long, for instance: Pythagoras and music, On Fermat's theorem, On Pascal's Arithmetic Triangle, On the beginning of algebraic language, Women and science, On the incommensurability problem, Scientific Revolution,...

In order to use historical texts properly, teachers are required to present some features of the historical period and also to describe historical figures in context, both in terms of their own objectives and the concerns of their period. Situating authors chronologically enables us to enrich the training of students by showing them the different aspects of the science and culture of the period in question in an interdisciplinary way. It is important not to fall into the trap of the amusing anecdote or the biographical detail without any historical relevance. It is also a good idea to have a map available in the classroom to situate the text both geographically and historically.
We have implemented several of these activities in the classroom with satisfactory results. One such activity is about Aristarchus of Samos, which we describe briefly below.
This activity deals with the work *On the Sizes and Distances of the Sun and Moon* (ca. 287 BC) by Aristarchus of Samos (ca. 310-230 BC). In order to implement the activity, we begin with a brief presentation of the context, Greek astronomy, and the person of Aristarchus himself. We then situate his work in the history of trigonometry, analyze the aims of the author as well as the features of the work, and finally encourage students to follow the reasoning of this work, Proposition 7 (Figure 1), in order to arrive at new mathematical ideas and perspectives. This classroom activity was implemented in the last cycle of compulsory education (14-16 year-old students).

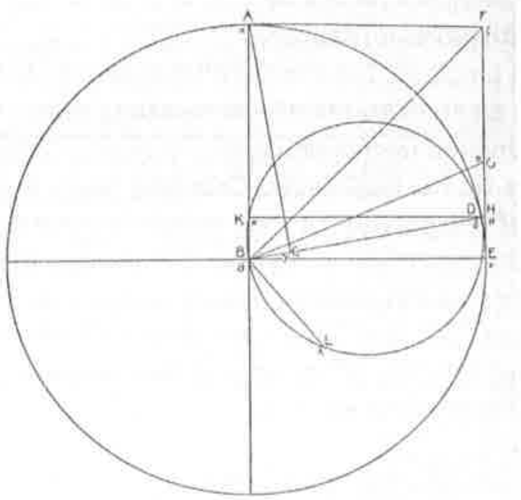

Figure 1: *Aristarchus' Proposition 7*

Another example of an activity implemented in the classroom concerns solving quadratic equations by completing squares in the style of al-Khwarizmi, which we now describe briefly. This activity deals with the work *Hisâb al-jabrwal-muqqabala* by Abu Ja'far Mohamed Ben-Musa al-Khwârizmî (780- 850). In order to implement the activity, we begin with a brief presentation of the context, Arabian mathematics, and the person of al-Khwârizmî himself. We then situate his work in the history of algebra and provide the explanation that al-Khwârizmî gave about solving quadratic equations at the beginning of his work. We present the original text with the drawings of the squares used to develop the geometrical reasoning and invite students to solve some quadratic equations with this procedure, completing squares, without algebraic calculations and using only a visual reasoning connecting algebra with geometry (see figure 2). This classroom activity was implemented in the last cycle of compulsory education.

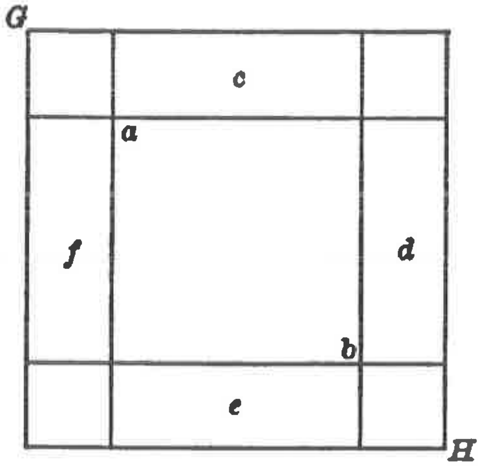

Figure 2: *Al-Khwârizmî's geometrical reasoning*

A third implemented example of an activity in the classroom consists in solving problems about right-angled triangles using the procedure that in Chinese mathematics is called the *base and high* procedure (Pythagorean Theorem).

This activity deals with some problems from the ninth chapter of the book of *Nine chapters on the Mathematical Art,* an anonymous Chinese work dating from the first century. As was the custom in Chinese and Oriental mathematics, this work was commented on by some later mathematicians such as Liu Hui (263) and Li Chunfeng (656). In order to implement this activity, we begin with a brief presentation of the context, Chinese mathematics, and the use of *Nine chapters on the Mathematical Art* in his time. We present the original text and use the later remarks and drawings that the commentators added in order to encourage students to prove Pythagorean Theorem in the same way as Chinese mathematicians, as well as solving some problems from the 9th chapter of *Nine chapters on the Mathematical Art* by using the Chinese procedure (see Figure 3).

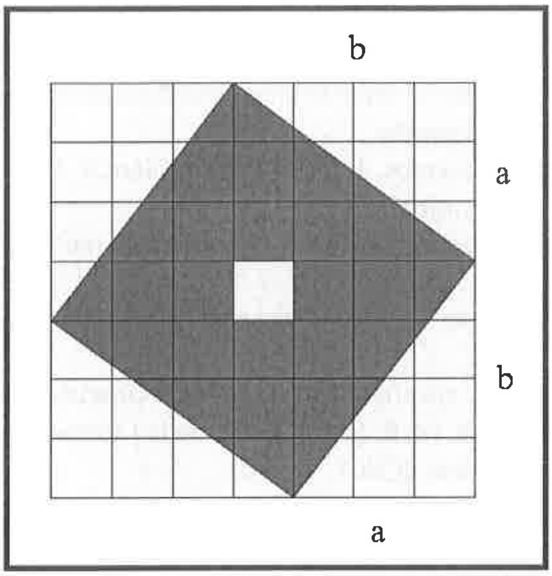

Figure 3: *First main figure*

Some remarks

By designing activities related to topics such as geometry, trigonometry and algebra, we will improve the students' mathematical education.

To this end, we have created a new teaching approach containing explanations introduced via an autonomous learning method. It has been satisfactorily tested with students who build their own reasoning in the same way as that employed by ancient mathematics. The analyses of significant historical texts improve the students' overall education by providing them with additional knowledge about the social and scientific contexts of these periods. The analyses of significant proofs reveal to students the different ways of approaching and addressing problems, thereby enabling them to tackle new problems and to develop their mathematical thinking.

We therefore believe that the history of mathematics should form part of teacher training courses, both in the initial stages and on a permanent basis. With such training, teachers will be able to extend their own knowledge, student learning will be enriched, and the quality of mathematical teaching as a whole considerably improved.

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