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SUMMARY

Drying soil samples in microwave ovens presents serious difficulties from an experimental and from a mathematical modeling point of view. Here, starting from the conservation of mass and energy in an unsaturated porous medium, a mathematical model is developed. The results of the numerical simulations are compared with the available experimental data. A complete study of the different drying procedures using microwave ovens is also presented.

Keywords: soil drying, microwave, suction, mathematical model, numerical simulation.

1. INTRODUCTION

Water content is one of the most useful geotechnical properties of soils. Humidity and porosity in saturated soils, or liquid saturation, in unsaturated soils, are determined from the water content. These characteristics are not only important by themselves; they are also essential to classify the soil and to establish approximate values of other properties using empirical relations.

Several tests are frequently used to find the water content in soils. These tests can be classified as direct and indirect tests. With the direct tests the water content is determined comparing the weights before and after the water extraction (stove drying, microwave drying and alcohol procedure). With the indirect tests the amount of water is determined measuring another magnitude (calciun carbide gas pressure tester method and nuclear method). The most used tests are stove drying and the nuclear method. Stove drying is the standard one and is used as a benchmark for all other tests. It is based on keeping a soil sample in a stove, with an interior temperature of 110±5°C until there is no change in weight. The time needed depends heavily on the type of soil, the sample size and the oven used; it can last from four hours (with sand samples in forced-draft drying oven) to sixteen (in the case of very plastic clay). The nuclear method is based on the determination of water content by the slowing of fast neutrons emitted through the soil. From the measures obtained, the presence of hydrogen atoms in the ground is computed through a calibration process. This test is performed in situ, is immediate and non-destructive. However, the great disadvantage of the nuclear method is that all hydrogen atoms in the test sample are detected, including those in bound water and organic matter. Additionally some metals and chemical elements will cause measurements lower than the true value.

In some circumstances, especially in engineering field work, it is desirable to have faster alternatives to the stove drying, which are less expensive and difficult than the determination using nuclear sensors. With this goal, in the late sixties the feasibility of sample soil drying using microwave ovens began to be studied.

The first experiments showed that too high temperatures are frequently reached using a microwave oven at maximum power without control, for instance due to the dehydration of the clay present in the soil. In that cases, misleading results are obtained. Therefore a simple procedure that yields the same results as the stove drying has been searched since then. Nowadays the
following microwave soil drying tests are available: temperature controlled, computer weight controlled, and manual weight controlled (ASTM).

The temperature controlled microwave drying involves keeping the sample temperature fluctuating between 90°C and 100°C, until a constant weight is reached. This is achieved turning on and off the microwave oven, according to data provided by a temperature probe inserted in the soil sample. This procedure ensures test conditions similar to those of the standard oven drying, but in some cases, for instance with clay samples, the drying time is not reduced enough (since it is between two and seven times faster than the standard oven drying, it still can last several hours). The ASTM test procedure consists in a three-minute initial drying interval and an iterative process of removing the sample from the oven, cooling to allow handling, recording soil weight and re-heating in the oven for one more minute, until the change between two consecutive mass determinations has an insignificant effect on the calculated moisture content. This test is quite dependent on the person who makes the test. The test specifications advise that power settings and drying times should be established for each oven, and that standard oven drying results must be used as a reference. The computer weight controlled microwave drying needs a constant power output microwave oven, an electronic balance, a small microcomputer and some other small components. The soil sample is weighed continuously while it is microwave heated. The computer monitors the evolution of weight, and decides to start the process of turning off and on the microwave emission. Moreover, the computer stops the test when constant weight is reached. With this test the same results as the standard test are obtained; however, in some cases the sample temperature rises up to 180°C.

Due to the drawbacks associated to each of these procedures, they are now seldom used, even in situations where their theoretical advantage is notorious. The studies carried out until now (mainly experimental) have not satisfactorily defined a procedure which could be a valid alternative to the stove oven drying or the nuclear method. Therefore, a mathematical model that simulates the main processes taking place in the microwave oven drying is developed in this paper. A complete description of these processes form the porous media flow theory and electromagnetism theory is far too complex: the equations to be solved are very complex, a lot of information on the physical parameters of the problem would be needed and the numerical computation would be very expensive. We have therefore made several assumptions that simplify the mathematical problem. Among the different possible simplification levels, we have chosen to have a mathematical model that is capable of predicting the evolution of temperature and water content with the drying time, along with a simplified description of the test characteristics and soil sample.

In Figure 1 the typical pattern of these evolution curves for the microwave oven are shown. These curves present three distinct stages: in the first, the temperature rises up to 100°C; in the second stage the temperature remains constant and the maximum rate of water loss is reached; and in the third, the rate of water loss decreases and the temperature rises again, except for some sand samples. The test control in this third stage is what allows to reach the same result as the stove drying.

In Section 2, the mathematical model is developed: balance equations for mass and energy and necessary physical relationships are presented, and the rate of internal energy generation by microwave heating is expressed as a function of
the type of oven and the soil characteristics. In Section 3 different drying tests are simulated; stove drying with forced air renovation (although the model is not specially designed for it, good results are found) and microwave oven drying, studying the different procedures mentioned before. Finally, the parameters that define a microwave oven are calibrated for a particular case and experimental results presented by Charlie et al., are predicted.

2. THE MODEL

2.1. Conservation equations

An unsaturated soil is modelled as a porous medium formed by three phases (solid, liquid and gas) and three components (solid skeleton, water and dry air). The solid phase is assumed to remain constant over the problem temperature range, 20°C to 150°C. Thus, gypsum or organic soils are not considered. The liquid phase is considered to be pure water, neglecting dissolved air and other solutes, and the gas phase is assumed to be a binary mixture of water vapor and dry air.

Energy, water mass and dry air mass conservation equations are considered over the sample volume, Ω. Neglecting viscous dissipation and internal energy changes due to pressure changes of the fluid phases, these equations are:

\[
\frac{\partial W}{\partial t} + \nabla \cdot (\rho_c \mathbf{v}_g + \mathbf{j}_h) + \nabla \cdot (\rho_l \mathbf{v}_l) = 0
\]

\[
\frac{\partial A}{\partial t} + \nabla \cdot (\rho_c \mathbf{v}_g + \mathbf{j}_h) = 0
\]

\[
\frac{\partial U}{\partial t} + \nabla \cdot (\epsilon U \mathbf{v}_l + \epsilon_l (\rho_c \mathbf{v}_g + \mathbf{j}_h) + \epsilon_a (\rho_a \mathbf{v}_g + \mathbf{j}_a)) + \nabla \cdot \mathbf{q} = Q,
\]

with

\[
W = \phi S_l \rho_l + \phi (1 - S_l) \rho_c,
\]

\[
A = \phi (1 - S_l) \rho_c,
\]

\[
U = \phi \epsilon_l S_l \rho_l + \phi (1 - S_l) (\epsilon_l \rho_c + \epsilon_a \rho_a) + (1 - \phi) \epsilon_a \rho_a,
\]

where \(W\), \(A\) and \(U\) are respectively water mass, dry air mass and internal energy per unit volume, \(\rho\) is density, \(\mathbf{v}\) is convection velocity, \(\mathbf{j}\) is diffusion flux, \(\epsilon\) is specific enthalpy, \(\mathbf{q}\) is conductive energy flux, \(\phi\) is porosity, \(S_l\) is liquid saturation and \(Q\) is the rate of internal energy generation per unit volume. The subscripts \(g\), \(v\), \(a\) and \(l\) refer to properties of gas phase, water vapor, dry air and liquid water respectively.

We consider as state variables the temperature, \(T\), the liquid saturation, \(S_l\), and the dry air partial pressure, \(P_a\). Thus, all magnitudes present in Equations (1) and (2) are expressed as functions of \(T\), \(S_l\) and \(P_a\). The principal relationships used are: the law of ideal gases (that relates density with temperature and partial pressure), the psychometric law (that relates water vapor partial pressure with temperature and suction) and Van Genuchten’s relationship between suction and liquid saturation. This last relationship depends on four parameters (\(\lambda\), \(P_{in}, S_{min}\) and \(S_{max}\)) that are characteristics of each type of soil (see Figure 2 for a brief description).
The rate of internal energy generation, $Q$, represents the heat generation due to microwave radiation. Microwave radiation is an electromagnetic radiation. The interaction of electromagnetic waves with non-magnetic matter is characterized by the dielectric permittivity, $\varepsilon$, usually expressed as a factor of the permittivity of vacuum, $\varepsilon_0 = \varepsilon_0/\varepsilon$. Relative permittivity is a complex quantity, written as $\varepsilon = \varepsilon'_r - i\varepsilon'_i$, where $i$ is the imaginary unit. The real part, $\varepsilon'_r$, affects the electric field of a propagating wave. The imaginary part, $\varepsilon'_i$, is a measure of how dissipative a medium is. In a lossy medium, the electromagnetic energy is gradually turned into heat by the friction due to displacing the internal charges when the material is polarized. This phenomenon is strongly frequency-dependent. In Figure 3 a schematic relationship between dielectric permittivity and frequency is depicted, and the main physical factors are indicated.$^{11}$

In conventional microwave ovens the electromagnetic radiation is emitted at a frequency, $f$, of 2450 MHz because the imaginary part of liquid water permittivity presents a maximum near this value. Nevertheless, the maximum value depends on many different factors, including temperature, salinity and, in the case of aggregate samples, suction and microstructure.$^{11,17}$ The dependencies are established through experimental calibration. We apply the formulation developed by Wang and Schmugge,$^{17}$ that takes into account liquid content and soil type differences at constant temperature. Moreover, an experimental relationship between water permittivity and temperature has been considered.$^{11}$ Wang and Schmugge's formulation depends on two parameters that are related to the wilting point. Thus, the soil dielectric permittivity can be expressed as a function of the four Van Genuchten parameters. This approach allows us to describe the differences observed between the different soil types without having to add more parameters to the model.

The rate of internal heat generation per unit volume, $Q$, depends on the dielectric permittivity of matter and the intensity and the frequency of the electric field. The characteristics of the electric field depend on the dimensions of the oven cavity because a resonating electromagnetic field is developed inside it. The resonant frequency can be approximated by the emitted frequency, but the electric field intensity depends on the dielectric properties and spatial distribution of the filling substance. A rigorous approach to this phenomenon leads to solving Maxwell equations over the oven cavity, but this is beyond the scope of this paper. Thus, in Subsection 2.3 the perturbation theory and a mean value of the electric field intensity are used to approximate $Q$.

2.2. Initial and boundary conditions

Initial and boundary conditions are needed to solve the mathematical problem defined by Equations (1) and (2). A general set of initial conditions is given by

$$\begin{align*}
T &= T^{ini} \quad S_i = S_i^{ini} \quad P_o = P_o^{ini}.
\end{align*}$$

The liquid water flux through the boundary is null

$$\mathbf{v}_{liq} \cdot \mathbf{n} = 0,$$

and radiation boundary conditions are imposed to water vapor, dry air and
conductive energy fluxes:

\[
(\rho_v \mathbf{v}_g + \mathbf{j}_i) \cdot \mathbf{n} = h^W \rho_g (\omega_v - \omega_v^{ext}) \\
(\rho_i \mathbf{v}_g + \mathbf{j}_a) \cdot \mathbf{n} = h^A \rho_a (\omega_a - \omega_a^{ext}) \\
\mathbf{q} \cdot \mathbf{n} = h^T (T - T^{ext}) .
\]

In Equations (4) and (5), \( \mathbf{n} \) is the outward unit normal, \( \omega_v \) and \( \omega_a \) the mass fractions of vapor and dry air respectively, and \( h^W, h^A \) and \( h^T \) the vapor, dry air and energy transfer coefficients. These coefficients are related to the sample geometry and some dimensionless numbers that characterize the exterior fluid and the flow type around the sample. These relationships are established through experimentation and dimensional analysis.\(^7\)

Since microwave ovens have a strong forced ventilation, it is considered that the air flow around the sample is controlled by its inertial forces, and not by the density changes due to temperature changes. This flow type is called forced connection with separated flow.\(^7\) Moreover, the geometry sample is approximated as a vertical cylinder with its axis perpendicular to air flow. Thus, the model is restricted to wells that do not need to be contained in a vessel. Moreover, we have simplified the geometric description considering the cylinder height equal to three times its radius, \( R \).

2.3. Spatial integration of the balance equations

Equations (1) to (5) and the constitutive relationships explained in Subsection 2.1 form a well defined system of nonlinear partial differential equations. By integrating these PDEs, the evolution of the state variables as spatial functions defined over the sample can be found. However, in order to have a simple mathematical model, the balance equations will be analytically integrated over their spatial domain. Therefore the system of nonlinear partial differential equations will become a system of nonlinear ordinary differential equations. Moreover, the state variables will become time functions and they will represent a mean value of their actual spatial distribution. This new definition of state variables will be representative enough as long as the actual spatial distribution is sufficiently uniform. In microwave drying, the heat is generated over all the sample and the sample size is quite small. Thus this procedure seems specially adequate.

Integrating Equations (1) over \( \Omega \) and applying the divergence theorem yields

\[
\frac{\partial}{\partial t} \int_{\Omega} W d\Omega = - \int_{\Gamma} (h^W \rho_g (\omega_v - \omega_v^{ext})) d\Gamma \\
\frac{\partial}{\partial t} \int_{\Omega} A d\Omega = - \int_{\Gamma} (h^A \rho_a (\omega_a - \omega_a^{ext})) d\Gamma \\
\frac{\partial}{\partial t} \int_{\Omega} u d\Omega = - \int_{\Gamma} (e_v h^W \rho_g (\omega_v - \omega_v^{ext}) + e_a h^A \rho_a (\omega_a - \omega_a^{ext})) d\Gamma - \int_{\Omega} Q d\Omega \\
\int_{\Omega} dT (T - T^{ext}) d\Gamma + \int_{\Omega} Q d\Omega .
\]

After that, the integral form of the mean value theorem is applied to each
spatial integral. Thus the system of equations becomes

$$
\begin{align*}
V_\Omega \frac{dW}{dt} &= \bar{\bar{W}} \rho_2 (\omega_\text{ext} - \omega) \\
V_\Omega \frac{dA}{dt} &= \bar{\bar{A}} \rho_2 (\omega_\text{ext} - \omega) \\
V_\Omega \frac{dU}{dt} &= \varepsilon_c \bar{\bar{W}} \rho_2 (\omega_\text{ext} - \omega) + \varepsilon_c \bar{\bar{A}} \rho_2 (\omega_\text{ext} - \omega) + \\
&\quad \bar{\bar{T}} (T_\text{ext} - T) + V_\Omega \bar{Q},
\end{align*}
$$

(7)

where $V_\Omega$ is the sample volume, $\bar{Q}$ is the mean value of the internal heat generation rate, and $\bar{\bar{W}}$, $\bar{\bar{A}}$ and $\bar{\bar{T}}$ are the integrals of $\dot{\bar{W}}$, $\dot{\bar{A}}$ and $\dot{\bar{T}}$ over the part of the boundary that takes part in the fluxes. Equations (7) can be easily reinterpreted as a global balance of water, dry air and energy over the sample. In the following, the expressions of $\bar{\bar{M}}$, $\bar{\bar{F}}$ and $\bar{Q}$ are developed. The term $\bar{\bar{A}}$ is not presented because it is not used in the final version of the model.

**Transfer coefficients**

We will consider that energy transfer takes place through the cylinder side surface, $S_\Omega$, and water vapor transfer through the part of $S_\Omega$ that connects pores with the outside (we consider it equal to $S_\Omega$ multiplied by porosity, $\phi$). Therefore, transfer coefficients are

$$
\begin{align*}
\bar{\bar{W}} &= \phi S_\Omega \frac{D_v \text{Nu}_M}{2 R} \\
\bar{\bar{T}} &= S_\Omega \frac{k_g \text{Nu}}{2 R},
\end{align*}
$$

(8)

where $D_v$ is water vapor diffusion in air, $k_g$ is air thermal conductivity, Nu is the Nusselt number for energy flux (ratio between energy flux and thermal conductivity) and $\text{Nu}_M$ is the Nusselt number for water vapor mass flux (ratio between mass flux and mass diffusion). $D_v$ and $k_g$ are functions of the temperature determined experimentally$^{12}$.

Moreover, in the case of forced convection with separated flow$^7$

$$
\text{Nu} = 0.43 + 0.5328 (\text{Pr})^{0.21} \sqrt{\text{Re}},
$$

(9)

where Pr is the Prandtl number (ratio between viscosity and thermal diffusion) and Re is the Reynolds number (ratio between inertial and viscous forces). Considering the characteristic length of 2 $R$, the Reynolds number is equal to

$$
\text{Re} = \frac{2 R \bar{u}_s}{\nu},
$$

(10)

where $\nu$ is the kinematic air viscosity and $\bar{u}_s$ is the mean air velocity around the sample. The Prandtl number is also a function of temperature$^{12}$. Thus $\bar{\bar{T}}$ is fully determined.

Drake$^7$ presents an expression for $\text{Nu}_M$ analogous to Equation (9) for Nu, just substituting the Prandtl number by the Schmidt number, Sc (ratio between viscosity and diffusion of water vapor). Nevertheless, Drake's expression for $\text{Nu}_M$ does not take into account that in some cases water vapor escapes from
the sample due to pressure differences. In the cases with a high moisture content and the temperature close to 100°C, the vapor pressure, \( P_v \), tends to the total air pressure, \( P_a \). At the same time, evaporation in the pores increases, and this causes the total air pressure also to increase. This over-pressure is dissipated through small cracks that allow vapor to escape from the sample. Usually this process occurs in a quasistatic way, but in some cases (for instance, high impermeable cohesive clay and fast heating) the over-pressure can cause the specimen explosion. In this model only quasistatic dissipation is considered.

To take into account the quasistatic dissipation of over-pressure, we have defined a corrective factor that multiplies Drake’s expression. The correction has been defined through the pressure ratio term of the following equation:

\[
N_a^M = \left( 0.43 + 0.5328 (Sc)^{0.31} \sqrt{Re} \right) \left( \frac{\sigma_v}{\sigma_a - \sigma_v} \right)^\beta,
\]

where \( \beta \) is a parameter that must be calibrated, and that after several analysis we have fixed equal to 1/3. The correction term is equal to one when the difference between the vapor pressure and the total air pressure is equal inside and outside the sample, and it tends to infinity when the vapor pressure tends to the total air pressure inside the sample. Thus, it does not allow the over-presures mentioned before and it does not have influence if the situation inside the sample is equal to the situation outside. In other contexts, similar corrective factors have been defined in experimental relationships between \( h^W \) and pressure magnitudes.

**Heating source**

The mean value of internal heat generation rate, \( \bar{Q} \), can be calculated as

\[
\bar{Q} = \frac{2\pi f_r \mathcal{E}}{V \eta},
\]

where \( \eta \) is the quality factor of the microwave oven cavity, \( f_r \) is the resonant frequency and \( \mathcal{E} \) the electromagnetic energy stored in the cavity. For some cavity shapes and using the small perturbation theory, the quality factor can be approximated. Although we cannot ensure that the hypotheses of small perturbation theory are fulfilled, the quality factor is approximated by the formula given by Tiuri et al. (because they used it for different material permittivity measurements with cylinder samples of 10 to 100 cm³)

\[
\frac{1}{\eta} = \frac{e_r}{(e'_{rs})^{\beta}} S,
\]

where \( \zeta \) is a parameter related with the sample geometry and location, and \( S \) is the filling factor of the cavity. The parameter \( \zeta \) is unknown, and it must be calibrated.

The filling factor of the cavity is the ratio between the energy stored in the sample and the energy stored in the oven,

\[
S = \frac{\epsilon_0 f_s |\mathcal{E}|^2 dV}{\mathcal{E}},
\]

7
with $E$ equal to the electric field vector.

From Equations (12), (13) and (14), and approximating the resonant frequency, $f_r$, by the emitted frequency, $f$, the mean value of internal heat generation rate becomes

$$
\bar{Q} = 2\pi f \epsilon_0 \frac{\epsilon_r' \epsilon_r'}{\epsilon_r''} \int_{\Omega} |E|^2 dV = 2\pi f \epsilon_0 \frac{\epsilon_r' \epsilon_r'}{\epsilon_r''} E^2,
$$

(15)

where $E$ represents the mean electric field strength over the sample.

Numerical simulations of microwave drying of samples with different radii have shown that if $\zeta$ and $E$ are fixed, the time needed to dry the sample is independent of the sample radius. This situation does not agree with experimental results, and hence we are led to think that the mean electric field strength over the sample is a function of the sample size. Thus, we will consider the following expression for the mean value of internal heat generation rate

$$
\bar{Q} = 2\pi f \epsilon_0 \frac{\epsilon_r' \epsilon_r'}{\epsilon_r''} E^2 \exp(-bR),
$$

(16)

where $E$ and $b$ are two parameters related with the electric field strength over a sample of radius $R$. These two parameters depend on the characteristics of the microwave oven used, and they must be calibrated.

2.4. Dimensionless equations and simplified mathematical model

In the following, the Equations (7) are presented in dimensionless form and, after some reasonable hypotheses, they are simplified to a system of just two ordinary differential equations.

First, the state variables and the time are substituted by their dimensionless form

$$
x = \tilde{S}, \quad y = \frac{T - T^0}{T^a - T^0}, \quad z = \frac{P_a}{\bar{P}_a}, \quad \tau = \frac{t}{t^*},
$$

(17)

where $T^a \neq T^0$ are two reference temperatures, $P_a$ is the atmospheric pressure and $t^*$ a generic reference time. Then, the water mass and energy conservation equations are respectively divided by $\rho_0$ and $\rho_0 (\epsilon_r' - \epsilon_r'')$, where $\epsilon_r''$ is the specific enthalpy at $T^0$ and $\epsilon_r'$ at $T^*$, obtaining

$$
\left[ 1 - \frac{\rho_0}{\rho_0} \frac{1}{\rho_0} \frac{\partial \rho}{\partial x} \right] \frac{dx}{d\tau} + \left[ 1 - \frac{\rho_0}{\rho_0} \frac{1}{\rho_0} \frac{\partial y}{\partial x} \right] \frac{dy}{d\tau} = \frac{1}{V_0 \phi_0} \rho_0 (\epsilon''_r - \omega_0)
$$

$$
\left[ \frac{\phi}{\epsilon_r' - \epsilon_r''} \right] \frac{dx}{d\tau} + \left[ 1 - \phi \right] \frac{\rho_0 c_p}{\rho_0} \frac{dx}{\phi (1 - x)} + \phi \left( 1 - \frac{\rho_0 c_p}{\rho_0} \frac{dx}{\phi (1 - x)} \right) \frac{dy}{d\tau} = \frac{\tau^* \tilde{Q} (T^{a_{\text{ref}}} - T^a - \gamma (T^b - T^a))}{V_0 \phi_0 (\epsilon''_r - \epsilon'_r)} + \frac{1}{\rho_0 (\epsilon''_r - \epsilon'_r)}
$$

(18)

where $c_0$ and $c_a$ are the specific heats at constant pressure of water vapor and dry air.
Dependence of first part of Equation (18) on $z$ (dimensionless $Pa$) can be dropped because we will assume that $P_t$ and $P_a$ are never larger than $P_{atm}$, thus
\[
\frac{P_t}{\rho_t} \ll 1, \quad \text{and} \quad \frac{\rho_t c_v}{\rho c_t} \text{ and } \frac{\rho_t c_a}{\rho c_t} \ll \left( \frac{1}{\phi} - 1 \right) \frac{\rho_t c_s}{\rho c_t}.
\] (19)
Moreover, in all the numerical simulations the following inequalities are verified
\[
\frac{(1-x)}{\rho_t} \frac{\partial p_t}{\partial x} \ll 1, \quad \frac{(1-x)}{\rho_t} \frac{\partial p_t}{\partial y} \ll 1
\]
and
\[
\frac{(1-x)}{\rho_t} \frac{\partial p_t}{\partial y} \ll \frac{q_t W}{h} \rho_t (\omega^W - \omega_t) \frac{1}{h T(1-y) + V_t Q}.
\] (20)
Therefore, if we assume that
\[
P_t \approx P^e_t,
\] (21)
dependence of first part of Equation (18) on $z$ can also be eliminated. Consequently, the dry air mass conservation equation is not needed in the mathematical model. The hypothesis expressed in Equation (21) is equivalent to not considering high over-pressure inside the sample (see Subsection 2.3).

Finally, for any relative humidity it is verified that
\[
y \frac{p^e}{\rho_t} \ll \rho_t.
\] (22)
Thus, substituting
\[
T^a = 0^\circ C \quad \text{and} \quad T^b = T^ext
\] (23)
in Equation (18) the following simplified version of the mathematical model is obtained
\[
\frac{d x}{d \tau} = - \frac{t^* D_t N_t M}{\rho_t h^2} \rho_t
\]
\[
\left[ \left( 1 - \frac{c_v}{c_t} \right) \phi y - \frac{c_v}{c_t} T^ext \right] \frac{d x}{d \tau} + \frac{t^* k_t N_t}{\rho_t c_t h^2} (1-y) + \frac{t^* Q}{\rho_t c_t T^ext}.
\] (24)

To solve this system of equations it is necessary to fix the value of the different constants, properties and parameters. The values of constants and general properties involved in Equations (24) are shown in Table 1.\textsuperscript{11-14} The parameters that define the test ($T^test$, $T^ini$, $u_s$, $z$, $E$ and $\theta$) will be defined for each particular test. The parameters that characterize the sample ($R$, $S^{fin}$) will be approximated for each sample. And finally, the parameters that characterize the soil type ($\phi$, $\lambda$, $P_{atm}$, $S_{min}$ y $S_{max}$) will be approximated by the relationships presented by Rawls and Brakensiek.\textsuperscript{14} that relate these parameters with the percentile content of clay and sand.

It is important to note that the nonlinear system of three partial differential equations defined along a tri-dimensional domain, Equations (1) to (5), has been simplified to a system of two ordinary differential equations, Equations (24).
This simplification in the mathematical description of the problem not only highly reduces the computational cost of each numerical simulation but, more importantly, it allows us to describe the physics of the problem with relatively few parameters.

3. SIMULATIONS OF THE DIFFERENT DRYING TEST

The first requirement that microwave drying must verify is that it has to extract the same amount of water as stove drying. Considering the soil description explained before, this requirement is obtained if the soil skeleton remains unaltered during the microwave drying, and if the degree of saturation reached at the end of the test is the same in both cases. The first condition is fulfilled limiting the maximum temperature that the sample can undergo. Although this limit depends on the soil type studied, we will consider a top temperature of 150°C. To verify the second condition, the degree of saturation reached at the end of stove drying is required. With this aim, forced-draft oven drying is simulated in Subsection 3.1. Even though the model has not been designed to reproduce oven drying, the forced-draft drying is easy to simulate and the results are accurate enough for our purposes. In the Subsection 3.2, microwave drying is simulated and the different procedures are analyzed.

3.1. Forced-draft oven drying

With the mathematical model developed in Section 2, we can simulate forced-draft oven drying just imposing $T^{in} = 110\, ^\circ\mathrm{C}$, $T^{ini} = 20\, ^\circ\mathrm{C}$, $Q = 0$ and the mean air velocity around the soil sample, $u_a$. In what follows, numerical results for two different samples are shown, influence of $u_a$ and $R$ in the results is analyzed and finally the degree of saturation reached at the end of the test is characterized.

In Table 2 the main results of two sample drying simulations are shown. Sample A has $R = 1.5\, \text{cm}$, $S^m_i = 0.95$ and is a clay loam (40% of weight from particles less than 0.002 mm, clay, and 25% between 0.002 and 2 mm, sand). Sample B has $R = 2\, \text{cm}$, $S^m_i = 0.8$ and is a quite pure sand (3% clay and 90% sand). In Figure 4 the evolution of temperature and the evolution of determined humidity are depicted. Note that the results agree with indications of ASTM for the two samples. Thus, even in this case where the heating source is outside the sample, the hypothesis of uniformity of the temperature distribution over the sample is adequate enough for a general description.

On the other hand, the mean air velocity around the sample and the sample size affect the time scale, but they do not affect the final degree of saturation: $R$ and $u_a$ disappear from Equations (24) considering $Re^{ini} > 100$ and

$$t^* = \frac{R^2}{0.5328\, D_i\, (S_x^{ini})^{0.23} \sqrt{Re^{ini}}} \quad (25)$$

where the superscript $^{ini}$ refers to the initial temperature. This simplification is easily verified, i.e., $R = 2\, \text{cm}$ and $u_a > 4\, \text{cm/s}$, or $R = 1\, \text{cm}$ and $u_a > 7\, \text{cm/s}$.

The final degree of saturation, $S_{res}$, is different for each soil type. However, we have verified that it is related to the maximum level of suction reached in the test, $\Psi_{res}$, that does not depend on the soil type (see numerical results in
Figure 5). When the water of the sample reaches this limit, the velocity of water less is so slow that no change is perceptible during a real drying time. The relationship between $S_{\text{re}}$ and $\Psi_{\text{re}}$ depends on two of the parameters that define soil type: $\lambda$ and $P_b$. This relationship and the actual values of $\lambda$ and $P_b$ are depicted in Figure 6. Combining Figure 6 with the relationships presented by Rawls and Brakensiek,\textsuperscript{14} we can conclude that the samples with less than 25% of clay are completely dried by standard test. Moreover, $\phi$, $S_{\text{re}}^\text{TP}$, $S_{\text{min}}$ and $S_{\text{max}}$ do not affect $\Psi_{\text{re}}$.

Hence, humidity determined by oven drying is directly related with hydraulic parameters that define soil type. This conclusion allows us to compare microwave drying results with standard ones even though experimental oven results are not available.

3.2. Microwave oven drying: different procedures

Two sets of parameters characterize microwave drying simulations: environmental ones ($T^\text{env}$ and $T^\text{env}$) and energy ones ($\zeta$, $E$ and $b$). The first ones have been fixed to 20°C. The second ones depend on the microwave oven, the position of the sample in its cavity and the geometry of the samples. Although the three energy parameters are needed, in order to have exact predictions for different samples and different microwave drying procedures, for a qualitative approach and a fixed sample size it suffices to consider a reasonable combination of their values. Thus, because of the aim of this Subsection, $R = 2$ cm, $b = 1$ cm$^{-1}$, $\zeta = 0.75$, and $E = 740$ V/cm are considered in the next simulations.

Humidity evolution curves corresponding to drying samples of clay, clay loam and sand are depicted in Figure 7; in Figure 8 temperature evolution is depicted just for clay. In these figures the most important characteristics of each microwave drying procedure explained in the introduction\textsuperscript{1,6,8} stand out. Note that the drying procedures that are based on turning on and off the microwave oven present a non-smooth temperature evolution because the sample cools by the surrounding air.

First, comparing Figures 7a, 7b and 7c it is clear that drying sand samples the final humidity and the time needed to reach it are independent of the procedure used. Because of the low capillarity and adsorption of sand, when the sample reaches 100°C, it dries completely. This situation occurs in the three procedures, and it agrees with observations of Charlie et al.\textsuperscript{5} Moreover, as indicated before, this final humidity is equal to that obtained with the standard test. On the other side, to reach the same result as that of the standard test with clay and silt samples, either quite high temperatures or long drying time are needed. This is because the energy needed to evaporate water is proportional to the degree of suction that the sample presents. Another phenomenon remarked by Charlie et al.\textsuperscript{5} is reproduced by the model: when sand samples are almost dried, the temperature falls off. The electromagnetic heating energy is mainly proportional to the water present in the sample. Thus, if the sample does not have any adsorbed water and it is almost dried, then no heating energy is produced and the sample cools by the surrounding air.

In the following, the results of the simulations of drying clay and loamy samples are analyzed. In Figure 7a, it is clear that the simulations of full power drying lead water contents higher than which are given by the standard test. Although in real tests it is caused by several factors, with this model it is because
the suction inside the sample reach higher values that with the standard test. Thus, the amount of water extracted from the sample is also higher. In the other hand, at the final stage of the test the sample is heated up to 300°C (see Figure 8a). We have remarked in previous sections that so high temperatures are not admissible. Therefore, as Charlie et al. and Gilbert conclude, full power drying is not recommended.

The simulations of computer weight controlled microwave drying lead results close to the standard ones. Nevertheless, the simulations also show that when clay samples are almost dried, the temperature can reach 200°C (see Figure 8a). This last result is in agreement with the comments of Gilbert (Gilbert remarks that the high temperatures are reached only during a few minutes and that it does not have influence in the results, but, although in some cases this observation is true, in general we cannot be sure. The other weight controlled microwave drying, exposed in ASTM, can also lead to correct results, but they depend mainly on how the test is performed and it neither has any control of the temperature. Thus, the sample can also reach too high temperatures. Therefore, the tests with control of the weight and without control of the temperature are not fully recommended. Nevertheless, the drying with temperature control exposed by Charlie et al., has the disadvantage that in clay samples and during the final stage of the test, the rate of drying is so slow that the test takes some hours to finish (see Figure 7b).

The main conclusion with respect to microwave drying of clay and loamy samples is that to obtain the same result that in the standard test it is necessary to get an equilibrium between the time of drying and the maximum temperature reached. To show the effectiveness of the model, the drying of a sample holding the maximum temperature in 150°C has been simulated (see Figure 8b). With this procedure the results are similar to the standard test (Figure 7c), with a maximum time of one hour (for the oven characteristics defined before) and without any additional facilities.

3.3. Calibration of the microwave oven

In the Subsection 3.2, the different microwave drying procedures have been analyzed considering the energy parameters (C, E and b) fixed. Their exact values are not needed for qualitative analysis; however, to simulate a real test and to make quantitative predictions accurate enough values are needed. Through a theoretical analysis, only the limit values of the parameters can be established. Thus, their value must be calibrated with experimental results.

The calibration of a numerical model can be done in a direct or an indirect form. The direct calibration consists on measuring the evolution of $Q$ in samples of different soils and different sizes. This term contains all the influence of the three parameters in the mathematical model; see Equation (16). The measurement of the evolution of $Q$ is difficult and very specific. It requires the evolution of the electromagnetic potency emitted by the microwave oven, the potency taken by the receiving antenna and an estimate of the energy loss by the reflecting and absorbing phenomena inside the cavity. We cannot use the direct calibration because those measurements are not available in the literature.

The indirect calibration consists on solving an inverse problem to identify the value of the desired parameters. The evolution of some characteristic magnitude of the sample (for instance, the temperature, the weight or the humidity) is
needed. The main advantage of indirect calibration is that the experimental measures are easy to obtain. Nevertheless, the main disadvantage is that the calibration is made using the model. Thus, the influence of other physical phenomena not considered in the formulation can be hidden.

Evolution curves of humidiﬁcation of six samples during the maximum power drying are presented by Charlie et al.6 Three samples are of Bear Paw shale, BPS, and the other three of Wyoming bentonite, WB. For each soil, the initial weights of the samples are 25, 50 and 75 g. To calibrate the model with this information, it is necessary to know the values of the parameters that characterize both soils: \( \phi \), \( S_{min} \), \( S_{max} \), \( \lambda \), and \( P_{0} \), and the characteristics of the samples: \( R \) and \( S_{l}^{min} \). Some of the soil and sample parameters can be approximated easily. Both soils are beyond their corresponding liquid limit; thus, \( S_{max} \) and \( S_{l}^{min} \) have been fixed equal to 1 and 0.99 respectively. Moreover, the radius of each sample has been related with the weight and the porosity of each soil. However, due to the special characteristics of both soils, \( \phi \), \( S_{min} \), \( \lambda \) and \( P_{0} \) cannot be approximated a priori. Thus, there are eleven parameters to determine: four for each soil and the three of the microwave oven.

First, the influence of the different parameters in the humidity curves has been analyzed. The soil parameters, \( \phi \), \( S_{min} \), \( \lambda \) and \( P_{0} \), determine the shape of the last part of the curves and the final value of the humidity. The constant \( b \) establishes the relationship between curves corresponding to different sample sizes, \( \zeta \) the shape of the curves and the range of variation of \( E \), and \( E \) fixes the shape of the curves and the total drying time.

The calibration has been performed in three steps: 1) BPS soil parameters, 2) energy parameters and 3) WB soil parameters. The soil parameters have been calibrated separately from the energy ones because their influence is almost restricted to the final value of the humidity, and they are not related with the differences due to sample size. Incorrect values for soil parameters would lead to final humidities very different from desired ones, and it would disturb the calibration of the energy parameters and the comparison between predicted results and experimental data.

The BPS soil parameters have been calibrated with the final four points of the curve of the 75 g sample (see Figure 9), considering a ﬁrst approximation of the energy parameters. The results are showed in Table 3. After that, the energy parameters have been calibrated using the curves of 25 g and 75 g of BPS soil. The calibrated curves and the experimental data are shown in Figure 9. The results are:

\[
\zeta = 0.885, \quad E = 568 \text{ V/cm} \quad \text{and} \quad b = 1.285 \text{ cm}^{-1}.
\] (26)

Finally, using the energy parameters calibrated in the second step, the WB soil parameters have been calibrated with the ﬁnal three points of the curve of the 75 g sample. The results are showed in Table 3. Note that three sets of experimental data have not been used: 50 g of BPS and 25 g and 50 g of WB; and that except the last points, the curve of 75 g of WB has neither been used.

Predictions of the four unused curves have been made with the calibrated parameters. The results are depicted in Figure 10. Good agreement between the experimental data and the simulated curves is found, despite the special characteristics of these soils. It is important to remark that the main aim of the calibration is to determine the parameters that characterize the microwave
heating, and it is not the soil parameters. Thus, the principal aspect of Figure 10 is the good agreement in the relationship between samples of the same soil type and different size (see predictions for WB in Figure 10b). Of course, better indirect calibrations can be done if the hydraulic parameters are known through specific tests.

4. CONCLUSIONS

A mathematical model of the evolution of the water content and the mean temperature of soil samples under microwave drying has been presented. The soil type is defined through the porosity and the relationship between the degree of saturation and suction, or if it is not possible, through percentile content of clay and sand. The sample is defined through its size and its initial degree of saturation. Finally, the microwave is characterized through three parameters that can be calibrated.

The results obtained in the simulation of different microwave drying procedures are in agreement with the references. Moreover, the main advantages and disadvantages of each of them have been assessed. The main conclusion of this analysis is that for clay soils it is necessary to choose an intermediate option between a fast test that reaches quite high temperatures, and drying the sample at 100°C during some hours. Moreover, the temperature controlled tests are the only ones that can guarantee that not too high temperatures are reached. Considering these conclusions and with the model simulations new procedures can be defined. For instance, simulations of a temperature controlled test with a maximum of 150°C gives us correct results in a maximum time of one hour.

REFERENCES


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Table 1. Constants and properties used in the model.

Table 2. Simulations of standard oven drying of two samples: clay loam (A) and pure sand (B).

Table 3. Calibrated parameters that define the two soils used in microwave oven calibration.

FIGURE CAPTIONS

Figure 1. Typical evolution of amount of water lost during microwave drying (from Gilbert[6]).

Figure 2. Van Genuchten’s relationship[16] between degree of saturation, $S_s$, and suction.

Figure 3. Qualitative representation of $\varepsilon'_m$ and $\varepsilon''_m$ as functions of frequency. The main polarization effects are indicated: ionic conductivity (radio waves), orientation of polar molecules (microwaves), vibration of atoms (infrared waves) and electronic polarization (visible waves), from Tiuri et al.[15].

Figure 4. Evolution of humidity (a) and temperature (b) of the two samples defined in Table 2 during standard forced-draft oven drying.

Figure 5. Evolution of effective degree of saturation and suction during standard oven drying of five different soils.

Figure 6. Relationship between $R_h$, $\lambda$ and $S_{eq}$, considering $\Psi_{eq} = 10^{0.2}$. The usual values of $\lambda$ and $R_h$ are limited by the discontinuous line.

Figure 7. Evolution of humidity during full power drying (a), during temperature controlled drying at 100°C (b) and at 150°C (c).

Figure 8. Evolution of temperature during full power drying and computer weight controlled microwave drying (a) and temperature controlled drying at 100°C and 150°C (b).

Figure 9. Calibration of the microwave oven. Humidity evolution during drying of the Bear Paw shale samples of 25 and 75 g.

Figure 10. Prediction with the mathematical model. Humidity evolution during drying the Bear Paw shale sample of 50 g (a), and Wyoming bentonite samples of 25, 50 and 75 g (b).
\[
\begin{align*}
\rho_l &= 1.6 \times 10^3 \text{ Kg/m}^3 \\
\rho_s &= 8.37 \times 10^3 \text{ Kg/m}^3 \\
\epsilon_{li} &= 8.37 \times 10^3 \text{ J/(Kg °C)} \\
c_{li} &= 4.184 \times 10^3 \text{ J/(Kg °C)} \\
f &= 2450 \text{ MHz} \\
P_{\text{atm}} &= 1.015 \times 10^5 \text{ Pa} \\
T_{\text{ini}} &= 20 \degree \text{C} \\
m_{\text{w}} &= 0.018 \text{ Kg/mol} \\
R &= 8.3142 \text{ m}^3 \text{ Pa/(mol °K)} \\
\nu[T^*] &= 1.42 \times 10^{-4} \exp \left( -67.5/T \right) \text{ m}^2/\text{s} \\
Pr[T^*] &= 0.643 \exp \left( 26.2/T \right) \\
\kappa[T^*] &= 7.75 \times 10^{-2} \exp \left( -325/T \right) \text{ W/m}^2\text{°K} \\
\sigma[T^*] &= \nu[T^*] / \kappa[T^*]
\end{align*}
\]

Table 1: Constants and properties used in the model.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Initial weight</th>
<th>Final weight</th>
<th>Water content</th>
<th>Drying time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>54.5 g</td>
<td>41.9 g</td>
<td>30.3 %</td>
<td>8.5 h</td>
</tr>
<tr>
<td>B</td>
<td>144 g</td>
<td>123 g</td>
<td>17.1 %</td>
<td>6.5 h</td>
</tr>
</tbody>
</table>

Table 2: Simulations of standard oven drying of two samples: clay loam (A) and pure sand (B).

<table>
<thead>
<tr>
<th>Soil</th>
<th>$\omega_{\text{fin}}$</th>
<th>$\phi$</th>
<th>$S_{\text{min}}$</th>
<th>$\lambda$</th>
<th>$P_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bear Paw shale</td>
<td>13.3%</td>
<td>0.830</td>
<td>0.103</td>
<td>0.30</td>
<td>600</td>
</tr>
<tr>
<td>Wyoming bentonite</td>
<td>85.4%</td>
<td>0.570</td>
<td>0.008</td>
<td>0.25</td>
<td>600</td>
</tr>
</tbody>
</table>

Table 3: Calibrated parameters that define the two soils used in microwave oven calibration.
Figure 1
Figure 2
Figure 3
Figure 4a
Figure 4b
$S_{eff} = (S_l - S_{min})/(S_{max} - S_{min})$

Figure 5
Figure 6
Figure 7a
Figure 7b
Figure 7c
Figure 8a
Temperature controlled at 150 C
Temperature controlled at 100 C

Figure 8b
Figure 9
Figure 10a
Figure 10b