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Non-uniform Sampling in Digital Repetitive Control Systems: An LMI Stability Analysis*

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Abstract

Digital repetitive control is a technique which allows to track periodic references and/or reject periodic disturbances. Repetitive controllers are usually designed assuming a fixed frequency for the signals to be tracked/rejected, its main drawback being a dramatic performance decay when this frequency varies. A usual approach to overcome the problem consists of an adaptive change of the sampling time according to the reference/disturbance period variation. This report presents a stability analysis of a digital repetitive controller working under time-varying sampling period by means of an LMI gridding approach. Theoretical developments are illustrated with experimental results.

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1 Introduction

Repetitive control [1], [2] is an Internal Model Principle-based control technique [3] that allows both the tracking and rejection of periodic signals. Its use has reported successful results in different control areas, such as CD and disk arm actuators [4], robotics [5], electro-hydraulics [6], electronic rectifiers [7], pulse-width modulated inverters [8, 9] and shunt active power filters [10].

Repetitive controllers are usually designed assuming a fixed frequency for the signals to be tracked/rejected. This entails a selection of a fixed sampling period and a further structural embedding of these data in the control algorithm. However, it is also a well known fact that even slight changes in the frequency of the tracked/rejected signals result in a dramatic decay of the controller performance. An example of such situation may be found in the frequency variations experimented by shunt active filters connected to the electric distribution network [11].

Several approaches have been introduced in order to overcome this problem. The proposals may be grouped in two main frameworks, namely, the one that deals with the preservation of the sampling time and the one that changes it adaptively. For the first approach there are also two branches: improving robustness by using large memory elements [12] or introducing a fictitious sampler operating at a variable sampling rate and later using a fixed frequency internal model [13]. Both ideas work well for small frequency variations at the cost of increasing the computational burden. An alternative technique is to adapt the controller sampling rate according to the reference/disturbance period [14, 15, 16]. This allows to preserve the steady-state performance while maintaining a low computational cost but, on the other hand, it implies structural changes in the system behavior which may destabilize the closed-loop system.

The report analyzes the stability of a system containing a digital repetitive controller working under time-varying sampling period by means of a Linear Matrix Inequality (LMI) gridding approach [17, 18]. The theoretical results are experimentally validated through an educational laboratory plant [19].

The structure of the report is as follows. Section 2 contains a brief description of a digital repetitive controller and a study of stability issues in case of constant sampling period. Section 3 studies the stability of the system under a time-varying sampling period using LMI techniques. Experimental results are collected in Section 4, while conclusions and further research lines are presented in Section 5.

2 Digital repetitive control under constant sampling period

Repetitive controllers are composed by to main elements: the internal model, $G_r(z)$, and the stabilizing controller, $G_x(z)$. The internal model is the one in charge of guaranteeing null or small error in steady state, while the stabilizing controller assures closed-loop stability. Several types of internal models are

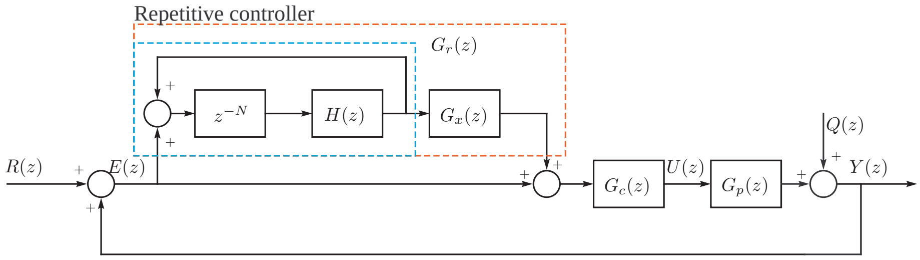

Figure 1: Discrete-time block-diagram of the proposed repetitive transfer function.

used depending on the concrete periodical signal to deal with [19, 20, 21, 22]. In this work the generic internal model will be used, i.e.

$$G_r(z) = \frac{H(z)}{z^N - H(z)},$$

where $N = \frac{T_p}{T_s} \in \mathbb{N}$, T_p being the period of the signal to be tracked/rejected and T_s being the sampling period. $H(z)$ plays the role of a low-pass filter in charge of introducing robustness in the high frequency range [23]. Although the internal model and the stabilizing controller can be arranged in different manners, most repetitive controllers are usually implemented in a "plug-in" fashion, as depicted in Figure 1: the repetitive compensator is used to augment an existing nominal controller, $G_c(z)$. This nominal compensator is designed to stabilize the plant, $G_p(s)$, and provides disturbance attenuation across a broad frequency spectrum.

Assume that either T_p and T_s are constant, which makes N also constant, and let $G_p(z)$ stand for the corresponding z-transform of $G_p(s)$. Sufficient stability criteria are given in the next Proposition:

Proposition 1. *The closed-loop system of Figure 1 is stable if the following conditions are fulfilled [19, 20]:*

1. *The closed-loop system without the repetitive controller $G_o(z)$ is stable, where*
$$G_o(z) = \frac{G_c(z)G_p(z)}{1 + G_c(z)G_p(z)}.$$
2. $\parallel H(z) \parallel_\infty < 1$.
3. $\parallel 1 - G_o(z)G_x(z) \parallel_\infty < 1$, *where $G_x(z)$ is a design filter to be chosen.*

Remark 1. *These conditions hold for a proper design of $G_c(z)$, $H(z)$ and $G_x(z)$. Namely [19, 20]:*

- *It is advisable to design the controller $G_c(z)$ with a high enough robustness margin.*
- *$H(z)$ is designed to have gain close to 1 in the desired bandwidth and attenuate the gain out of it.*

- *A trivial structure which is often used for $G_x(z)$ in case that $G_0(z)$ is minimum-phase is [24]:*

$$G_x(z) = k_r \left[G_o(z)\right]^{-1}.$$

Otherwise, alternative techniques should be applied in order to avoid closed-RHS plane zero-pole cancellations [24]. Moreover, there is no problem with the improperness of $G_x(z)$ because the internal model provides the repetitive controller with a high positive relative degree. Finally, as argued in [25], k_r must be designed looking for a trade-off between robustness and transient response.

3 Digital repetitive control under variable sampling period

The repetitive controller introduced in the previous section contains the ratio $N = \frac{T_p}{T_s}$, which is embedded in the controller implementation. This is not a problem if the reference or disturbance periodic signal has a known constant period. However, the controller performance decays dramatically when a variation of T_p appears. This report propounds to adapt the controller sampling time T_s following the reference/disturbance period $T_p(t)$, with the aim of maintaining a constant value for N. Hence, on the one hand, G_r, G_x and G_o are designed and implemented to provide closed-loop stability for a nominal sampling time $T_s = T_s^N$, in accordance with Proposition 1 and Remark 1; their structure remains always invariant, i.e. it experiments no further structural changes. On the other hand, the period of the sampler device that precedes the plant $G_p(s)$ is accommodated to the variation of T_p; therefore, its discrete-time representation is that of Linear Time Varying (LTV) system.

Regarding the time-varying nature of the sampling time of the plant, the stability analysis is carried out in the state-space formalism. Let $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ represent the continuous-time plant state-space representation, i.e.

$$G_p(s) = \tilde{C}\left(s\mathbb{I} - \tilde{A}\right)^{-1}\tilde{B} + \tilde{D}.$$

Assume that G_p is sampled at $\{t_0, t_1, \ldots, t_k, \ldots\}$, with $t_0 = 0$ and $t_{k+1} > t_k$, the sampling periods being $T_k = t_{k+1} - t_k$. Let also $x_k \triangleq x(t_k)$, $u_k \triangleq u(t_k)$, $y_k \triangleq y(t_k)$, $\bar{A}_k = \bar{A}(T_k)$, $\bar{B}_k = \bar{B}(T_k)$, $\bar{C} = \tilde{C}$ and $\bar{D} = \tilde{D}$, where

$$\bar{A}(T) \triangleq e^{\tilde{A}T}, \quad \bar{B}(T) \triangleq \int_0^T e^{\tilde{A}\tau}\tilde{B}d\tau. \tag{1}$$

Therefore, the plant evolution at the sampling instants is given by the discrete-time LTV system [1]

$$
\begin{array}{rcl}
\bar{x}_{k+1} & = & \bar{A}_k \bar{x}_k + \bar{B}_k \bar{u}_k, \\
\bar{y}_{k+1} & = & \bar{C}\bar{x}_{k+1} + \bar{D}\bar{u}_{k+1}.
\end{array} \tag{2}
$$

Similarly, under varying sampling period, the closed-loop system depicted in Figure 1 can be described in state-space by a quadruple (A_k, B_k, C, D), which may be constructed combining $(\bar{A}_k, \bar{B}_k, \bar{C}, \bar{D})$ and the state-space representations of $G_r(z)$, $G_x(z)$ and $G_c(z)$[2]. Thus, C and D are constant matrices, while A_k and B_k depend continuously on T_k. A methodology for studying the closed-loop system under time-varying sampling conditions will be introduced in the rest of this section.

Let the sampling period, T_k, take values in a known compact subset $\mathcal{T} = [T_0, T_F] \subset \mathbb{R}^+$.

Proposition 2. *The uniform exponential stability of the zero state* $\varepsilon = 0$ *of* $\varepsilon_{k+1} = A_k \varepsilon_k$ *implies the uniform bounded input-bounded output (BIBO) stability of the system* (A_k, B_k, C, D).

Proof. According to [26], the result follows if B_k, C and D are uniformly bounded matrices, $\forall k$, and this is indeed true: B_k depends continuously on T_k, which belongs to a compact set $\mathcal{T}$, while C, D are constant matrices. □

This result allows to reduce the stability analysis of (A_k, B_k, C, D) to that of the zero state of $\varepsilon_{k+1} = A_k \varepsilon_k$.

Let us define

$$
L_{T_k}(P) = A_k^\top P A_k - P. \tag{3}
$$

Proposition 3 ([26])**.** *If there exists a matrix* P *such that*

$$
L_{T_k}(P) \leq -\alpha \mathbb{I}, \quad \forall T_k \in \mathcal{T}, \quad s.t. \;\; P > 0, \; P = P^\top, \tag{4}
$$

where $\alpha \in \mathbb{R}^+$*, then the zero state of* $\varepsilon_{k+1} = A_k \varepsilon_k$ *is uniformly exponentially stable.*

Once at this point, it is immediate to realize that relation (4) in Proposition 3 yields an infinite set of LMIs. The gridding approach introduced in [17, 18] allows a simplified stability analysis that may be performed in two stages, if necessary.

In a first stage, advantage is taken from the fact that (A_N, B_N, C, D), corresponding to (A_k, B_k, C, D) evaluated in $T_k = T_s^N$, is stable by construction.

Proposition 4. *Assume that the stability conditions of Proposition 1 are satisfied for a nominal sampling period* $T_s^N \in \mathcal{T}$. *Then,*

[1]In case that T_k remains constant, i.e. $T_i = T_j$, $\forall i \neq j$, system (2) corresponds to a discrete-time LTI system with z-transform transfer function $G_p(z) = \bar{C}\left(z\mathbb{I} - \bar{A}\right)^{-1}\bar{B} + \bar{D}$. In an aperiodical sampling time framework $\bar{A}_k$ and $\bar{B}_k$ vary with k, and the z-transform representation is no longer valid.

[2]Recall that $G_r(z)$, $G_x(z)$ and $G_c(z)$ are designed for a nominal sampling time $T_s = T_s^N$. Hence, when $T_k = T_s^N$, $\forall k$, the closed-loop system is stable by construction.

1. *The zero state of the LTI system $\varepsilon_{k+1} = A_N \varepsilon_k$ is uniformly exponentially stable.*

2. *The LMI problem*

$$L_{T_s^N}(P) \leq -\alpha \mathbb{I}, \quad s.t. \;\; P > 0, \;\; P = P^\top, \tag{5}$$

   with $\alpha \in \mathbb{R}^+$, is feasible.

3. *Let $P = P_N$ be a solution of the LMI problem (5) for a fixed $\alpha \in \mathbb{R}^+$. Then, there exists an open neighborhood of T_s^N, say $\mathcal{I}_\mathcal{N}$, such that (A_k, B_k, C, D) is BIBO stable in $\mathcal{I}_\mathcal{N}$.*

Proof. It follows from the stability hypothesis that all the eigenvalues of A_N have modulus less that 1, which yields immediately item 1 [26]. Item 2 stems from the fact that the sufficient condition for uniform exponential stability established in Proposition 3 is also necessary for a discrete-time LTI system. Finally, item 3 follows immediately from Propositions 2 and 3 once the continuity of the matrix elements of A_k with respect to T_k is taken into account. □

We are interested in analyzing the stability of system (A_k, B_k, C, D) for all sampling periods $T_k \in \mathcal{T}$. Let then $P = P_N$ be a feasible solution of the LMI problem (5) for a fixed $\alpha \in \mathbb{R}^+$: its existence is guaranteed by Proposition 4. Let also $\{\tau_0, \ldots, \tau_q\}$, with $\tau_{i+1} > \tau_i$, be a sufficiently fine grid of $\mathcal{T}$. If $L_{\tau_i}(P_N) \leq -\alpha \mathbb{I}$, $\forall i = 0, \ldots, q$, stability $\forall T_k \in \mathcal{T}$ may be probably inferred.

Otherwise, in case that there exists at least a single τ_i such that $L_{\tau_i}(P_N) > -\alpha \mathbb{I}$, the gridding procedure proposed in [18] may be carried out as follows. Let $\{\tau_0, \ldots, \tau_r\}$, be a sorted set of candidate sampling periods suitably distributed in $\mathcal{T}$. Then, one may solve the following finite set of LMIs:

$$L_{\tau_i}(P) \leq -\alpha \mathbb{I}, \quad i = 0, \ldots, r, \quad \text{s.t.} \;\; P > 0, \;\; P = P^\top, \tag{6}$$

for a fixed $\alpha \in \mathbb{R}^+$. In case that the problem is feasible and a solution, $P = P_G$, is encountered, the negative-semidefinite character of $L_{T_k}(P_G) + \alpha \mathbb{I}$ is to be checked for intermediate values of T_k in each open subinterval (τ_i, τ_{i+1}). If this fails to be accomplished, (6) has to be solved again for a finer grid of $\mathcal{T}$.

Remark 2. *Comparatively, in the first approach a shorter stability radius may be reasonably expected.*

Remark 3. *The main drawback associated to both analysis paths is associated to the fact that stability is rigorously guaranteed just in open neighborhoods of each sampling time τ_i where either $L_{\tau_i}(P_N) \leq -\alpha \mathbb{I}$ or $L_{\tau_i}(P_G) \leq -\alpha \mathbb{I}$ are satisfied, thus providing not sufficient but necessary stability conditions.*

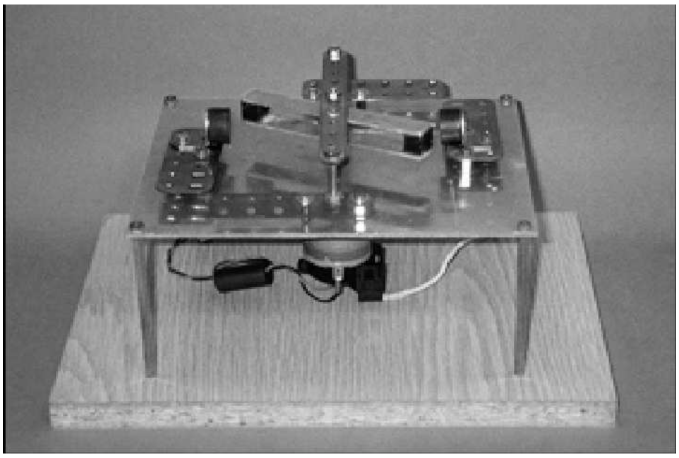

Figure 2: Picture of the main part of the plant: DC motor, optical encoder, magnetic system (load), and supporting structure.

4 Experimental setup and results

4.1 Plant description

Systems with rotary elements are usually affected by periodic disturbances due the movement of these parts (e.g. electrical machines, CD players...). This kind of system is supposed to be moving, in some cases, at a fixed angular speed. Under these working conditions any friction, unbalance or asymmetry appearing on the system generates a periodic disturbance that affects its dynamical behavior. Figure 2 shows the picture of device designed to reproduce this working conditions [19]. This device is composed of a bar holding a permanent magnet in each end, with each magnet magnetically oriented in the opposite way, and attached to a DC motor and two fixed permanent magnets (see a sketch in Figure 3). The rotation of the DC motor causes a pulsating load torque (Γ_p) that depends on the mechanical

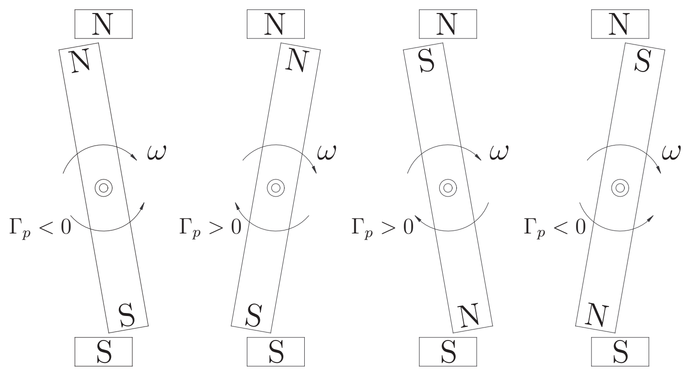

Figure 3: Mechanical load: fixed and moving permanent magnets sketch (ω and Γ_p stand for the angular speed and the disturbance torque, respectively).

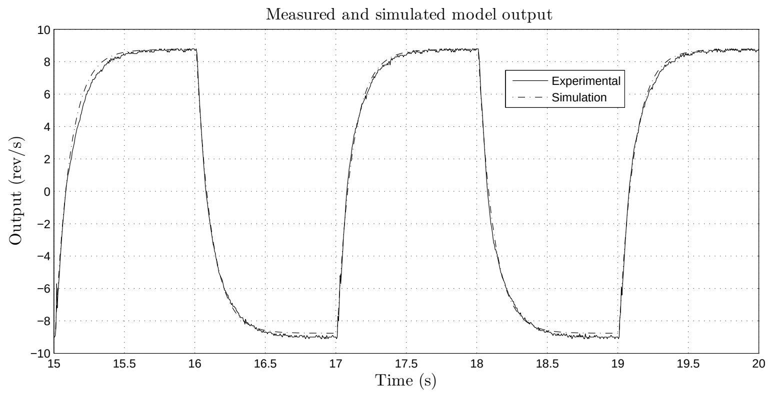

Figure 4: Open-loop time response of the plant without the fixed magnets.

angle θ of the motor axis. When the motor axis angular speed ω is constant ($\ddot{\theta} = \dot{\omega} = 0$), the pulsating torque is a periodic signal with a fundamental period directly related to the axis speed: $T_p = \omega^{-1}$, with ω expressed in rev/s. The control goal for this plant is maintaining the motor axis angular speed constant at a desired value.

Figure 4 shows the open-loop time response of the plant without the fixed magnets. From the observation of this time response and after a parametric identification procedure, the following plant model can be derived:

$$G_p(s) = \frac{8.762}{0.10667s + 1} \frac{\text{rev/s}}{\text{V}} \tag{7}$$

Figure 5 shows the open-loop time response of the plant containing the fixed magnets. It is important to note that the speed describes an almost periodical signal. This type of disturbances may not be rejected by a regular controller, so a repetitive controller is designed in next subsection.

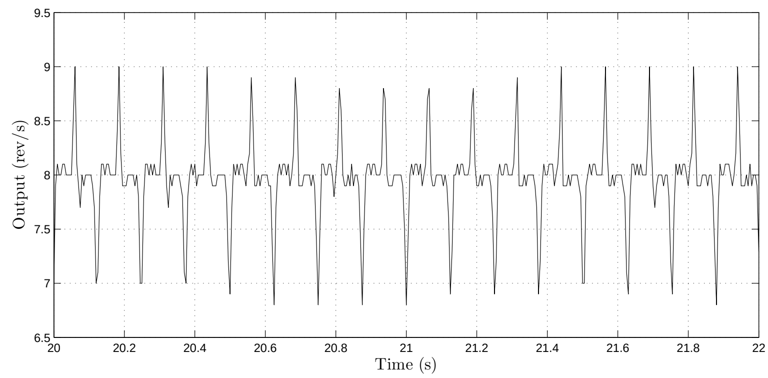

Figure 5: Open-loop time response of the plant with the fixed magnets.

4.2 Control design

The controller is constructed from model (7), for a speed of $\omega = 8$ rev/s and obtaining 25 samples per period, i.e. $N = 25$. These conditions imply a nominal sampling period of $T_s^N = T_p N^{-1} = (\omega N)^{-1} = 5$ ms. Under these assumptions the nominal discrete time plant is:

$$G_p(z) = \frac{0.4012}{z - 0.9542}$$

According to Remark 1, the following design issues have been taken into account:

- $G_c(z) = 0.25$ provides a very robust inner loop.
- The first order linear-phase FIR filter

  $$H(z) = 0.02z + 0.96 + 0.02z^{-1}$$

  provides good performance in the present case.
- The fact that $G_p(z)$ is minimum-phase allows $G_x(z) = k_r G_0^{-1}(z)$, with $k_r = 0.3$.

These settings yield the control law:

$$\begin{aligned} u_k &= 0.25e_k + 0.015e_{k-23} + 0.70e_{k-24} - 0.84e_{k-25} \\ &\quad -0.018e_{k-26} + 0.02u_{k-24} + 0.96u_{k-25} \\ &\quad +0.02u_{k-26} \end{aligned}$$

with $e_k = r_k - y_k$, where y_k is the system output (speed) and r_k is the reference.

4.3 Stability analysis

Although the controller is designed to regulate the speed at 8rev/s, in practice it will be necessary to move from this design point. In this subsection the closed-loop stability under aperiodical sampling time condition is analyzed.

Let us assume that we are interested in varying the speed reference in the interval $[6, 11]$ rev/s: this entails a sampling period variation in the interval $\mathcal{T} = [0.00363, 0.00666]$ s. Figure 6 depicts the maximum modulus eigenvalue of the closed-loop matrix A_k for a range of the sampling period T_k that includes $\mathcal{T}$. It can be noticed that the closed-loop system is stable if the sampling time is kept constant at any $T_k \in \mathcal{T}$. Unfortunately, this is not a valid stability proof for the time-varying sampling period case.

The stability analysis that stems from Proposition 4 includes the solution of the LMI (5), which is known to be feasible, and the checking of the negative-definite character of $L_{T_k}(P_N)$. Figure 7 shows the evolution of the maximum modulus eigenvalue of $L_{T_k}(P_N)$ when solving for $\alpha = 4645$, and also for 50000 uniformly distributed values of T_k. Therefore, it can be presumed that the closed-loop system may operate in a speed range of $[7.13, 8.32]$

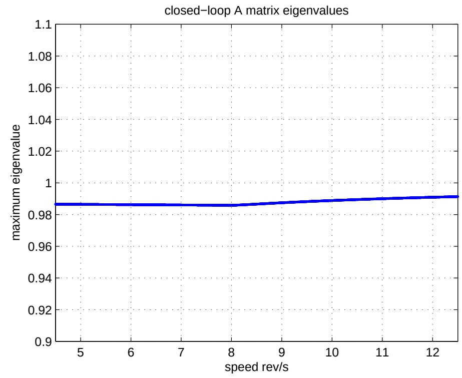

Figure 6: Maximum modulus eigenvalue of the closed-loop matrix A_k for a large range of T_k that includes $\mathcal{T}$.

rev/s with dynamically preserved stability and $\alpha = 50$. This speed interval is obviously very narrow and operation conditions are limited to a sampling period interval $\mathcal{I}_N$ such that $\mathcal{T} \not\subseteq \mathcal{I}_N$. Once at this point, it is important to remember that this test comes not from a necessary condition but from a sufficient condition, so moving out of this interval does not necessarily imply instability.

In order to guarantee a broader stability interval the second the method described in section 3 may be applied. Therefore, 40 uniformly distributed point are selected in $\mathcal{T} = [0.00363, 0.00666]$ rev/s. This points are used to construct the set of LMIs (6), and a feasible solution $P = P_G$ with $\alpha = 100$ is obtained. Figure 8 depicts the maximum modulus eigenvalue of $L_{T_k}(P_G)$, detailing in green color the 40 points leading to the LMI formulation. The maximum modulus eigenvalue of $L_{T_k}(P_G)$ corresponding to a finer grid consisting of 50000 uniformly distributed point are drawn in blue color. These points are used to check the sign of $L_{T_k}(P_G)$ in the intervals between the points defining the LMI set. It can be seen that $L_{T_k}(P_G) < 0$ for every

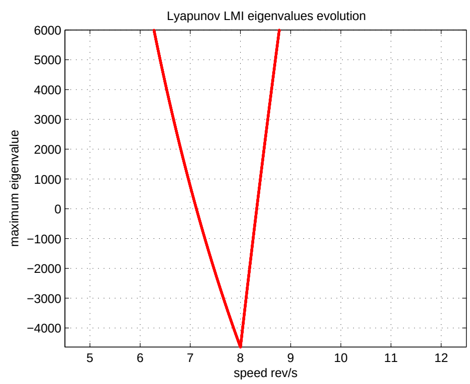

Figure 7: Maximum eigenvalue of $L_{T_k}(P_N)$ with $\alpha = 4645$ and $T_s^N = 0.005$ s.

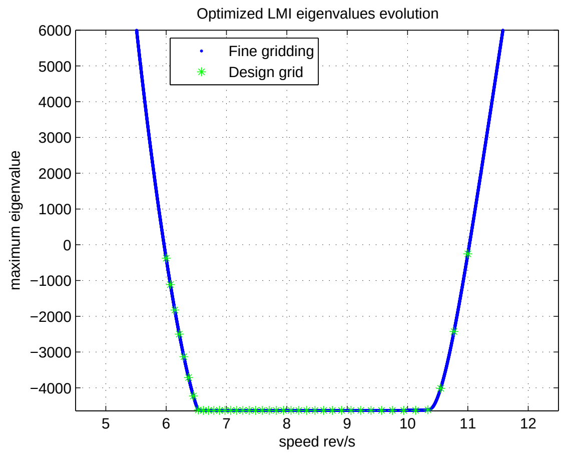

Figure 8: Maximum eigenvalue of $L_{T_k}(P_G)$ with 40 points in the range $[6, 11]$ rev/s to define the LMI set.

point in this finer grid of the interval $\mathcal{T}$; hence, stability is dynamically preserved therein. This method extends the previously obtained stability interval $[7.13, 8.32]$ rev/s, thus providing less conservative results. Further extensions of the new interval could also be feasible.

4.4 Experimental results

Figure 9 contains the experimental results of the repetitive controller designed in section 4.2. During the time interval $[0, 10]$ s, the reference is maintained constant at the nominal value of $\bar{\omega} = 8$ rev/s: it is important to realize that, in comparison to the uncompensated speed profile of Figure 5, now disturbances are almost rejected. Figure 10 contains both the uncompensated and compensated speed spectrum when working at the nominal speed; note that most relevant harmonics are eliminated or highly attenu-

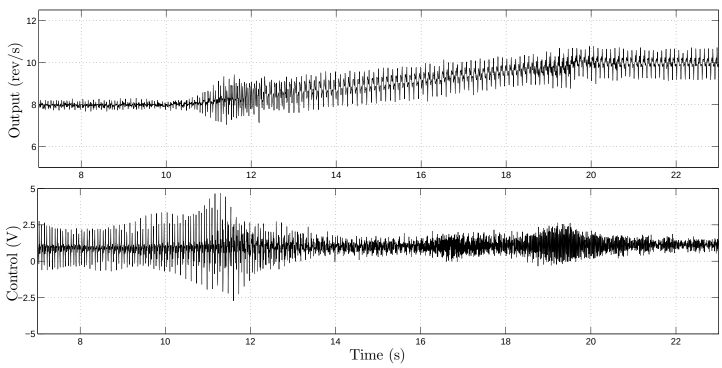

Figure 9: Closed-loop system behavior using a repetitive controller and with sampling period fixed at the nominal value ($\bar{T} = 5$ms).

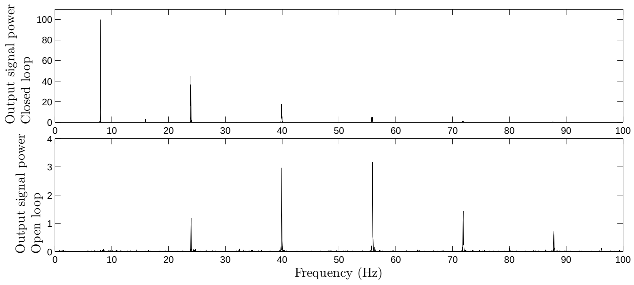

Figure 10: Frequency spectra in open and closed-loop working at the nominal speed ($\bar{\omega} = 8$ rev/s).

ated due to the repetitive control action. At time 10 s a ramp reference change, beginning at $\omega = 8$ rev/s and finishing at $\omega = 10$ rev/s, $t = 20$ s, is introduced in the system: Figure 9 reveals that the system can no longer reject the disturbances. In addition, the action generated by the control law is also portrayed in Figure 9: it can be seen that the controller generates the necessary action to compensate disturbances when working at the nominal speed, while when it works at the new speed the control action is reduced so disturbances cannot be properly compensated.

Figure 11 shows the same experimental results using an adaptive sampling rate, which is accommodated to the desired reference. One may observe that, after a short transient, the controller is capable of preserving the system performance and both references are tracked with small steady state. At the bottom of Figure 11 the control action generated by the control law shows now a proper amplitude in both cases, so the controller is capable of rejecting the disturbances.

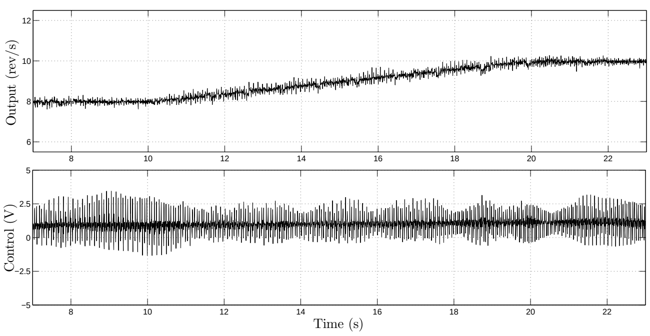

Figure 11: Closed-loop system behavior using a repetitive controller with adaptive sampling rate.

5 Conclusions

This report analyzes the stability of a closed loop system containing a digital repetitive controller working under time-varying sampling period. The analysis is carried out using an LMI gridding approach. The theoretically predicted results are experimentally validated through an educational laboratory plant in which rejection of a periodic disturbance is successfully achieved.

In sight of Remark 2, further research actually on course is lead to improve the quality of the stability analysis by turning necessary conditions onto sufficient conditions. The study, which is based on the recently published approaches [27, 28], treats the time-varying parts of the system description as norm bounded uncertainties. This allows the use of robust control techniques that may eventually provide closed intervals containing the gridding points where stability is definitely ensured.

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