Formulas to Estimate the VRP Average Distance Traveled in Elliptic Zones

F. Robuste, M. Estrada & A. Lopez Pita
CENIT – Center for Innovation in Transport
Technical University of Catalonia
Jordi Girona 29, 2A - 08034 Barcelona (Spain)
Tel: +34 93 413 76 67
Fax: +34 93 413 76 75
f.robuste@upc.es

Abstract

This paper defines some compact expressions to evaluate the average distance traveled in VRP problems in circular and elliptic zones. These formulas have been carried out empirically from the results obtained by the application to a set of problems of heuristic algorithms (Clarke & Wright, Fisher & Jaikumar and Gillet & Miller) and Daganzo's method based on continuous approximations of the demand spatial distribution.

The problems are solved and designed automatically by a computer. Vehicle capacity has been the only constraint considered in the tour design. If N is the number of points and C is the vehicle capacity, it has been shown that the Clarke and Wright algorithm provides the best solutions especially when $N<50$ or $N/C<14$. On the other hand, solutions with Daganzo's method are not adequate when $N/C \leq 3$.

1. INTRODUCTION

One of the most productive research areas in logistics analysis has been the chance to predict the tour lengths in a specific delivering system. Daganzo (1984) carries out expressions to evaluate the length of tours in TSP and VRP by continuous approximations about the distribution of the points.

In VRP problems, given a delivery region of area *A* that contains *N* scattered points, a fleet of delivery vehicles (each of them capable of serving C points) has to visit all demand points minimizing the total distance covered, starting and ending in a depot.

When we deal with VRP problems it is assumed that N>C. In this kind of logistic problem, Daganzo (1984) derives expressions proportional to $(AN)^{½}$ and the line haul between the middle service zone and the warehouse.

Robusté et al. (1990) argue for the suitability of these formulas in elongated zones (these sort of regions are generally used in VRP problems) and derive another compact formula for the sweep method (Gillet & Miller) including the term N/C^2.

This paper develops compact expressions for different shape zones of VRP from the results obtained by the application of some heuristics algorithms: Clarke & Wright, Fisher & Jaikumar and Gillet & Miller. In this way, the formulas given in references Daganzo (1984) and Robusté et al. (1990) are empirically proven and we present conclusions regarding the suitability of each heuristic.

2. VRP IN GEOMETRICAL PROBABILISTIC ANALYSIS

Generally, the delivery cost of any vehicle has a term proportional to the distance traveled (vehicle in movement) and another term proportional to the time (independent of the distance). If we assume a proper use of vehicle during the delivery service, the temporal cost could be determined as a fixed cost. In Daganzo (1991) it is supposed that the cost of a vehicle of *C* capacity in a *N* stops tour, loading *V* goods (V≤C) are linearly dependent on *N* and *V* with specific unit costs that depend on whole distance in turn:

$$(Cost \text{ of } N \text{ stops}) \approx c_s\ N + c_d\ N\ d + c_s^{'}\ V + c_d^{'}\ V\ d \tag{1}$$

However, in a specific VRP problem where *N* and *V* are fixed, we can only optimize the variable distance *d* (apart from the unit cost) in order to minimize the whole transportation cost. In this case, the solution complexity is fed by the non-continuous domain of the problem and boundary constraints. These problems could be avoided by continuous approximations of spatial distribution demand.

In fact, if δ is the average density of points in the area (δ=N/A), δ may be expressed as a continuous function with a smooth variation (it can be assumed by a constant value in each sub-region). Given a constant density of points δ in a spatial region, the probability that the area *A* contains *N* points follows a Poisson process as long as the location of visit points are independent:

$$P_N(A) = \frac{(\delta\ A)^N\ e^{-\delta A}}{N!} \qquad \text{A} \geq 0 \quad , \quad \text{N} = 0,1,2,... \tag{2}$$

In all stochastic Poisson processes, the expected value and variance are both identical to:

E(N)=Var(N)=δA (see more details in Novaes, 1989).

VRP – Vehicle Routing Problem.

These sort of problems have several constraints in their variables that result in a several sub- classifications problems according to the number of origins and destinations, the period of time in which pick-up or deliveries have to be done, uniformity in delivery vehicles, etc.

As in the elementary VRP, it is supposed that the distribution of points are uniformly scattered, there is a central depot in the region and the only constraint that takes part in the system is the capacity of the vehicles. In addition, it is assumed that all delivery points have the same demand.

In this way, the capacity is considered as the maximal number of points that each vehicle is able to serve in a tour assuming that we deal with a homogeneous fleet.

In that context, Daganzo (1984) makes up expression (3) to predict the total distance traveled in that problem, where *r* is the average distance between warehouse and the delivery points.

These formulas result from a method based on covering area *A* with swaths of optimal width $2w$ aligned with (oriented towards) the warehouse. Hence, each swath is split into two identical sub-swaths in longitudinal direction in order to generate near-optimal tours. Each tour starts from the depot and visits the points in the same order that they appear when we are covering one sub- swath in some way and return to the depot by the other.

$$D_{VRP-1} \approx \frac{2rN}{C} + 0.73\sqrt{AN} \quad \text{for } L_1 \text{ metric} \tag{3.a}$$

$$D_{VRP-2} \approx \frac{2rN}{C} + 0.57\sqrt{AN} \quad \text{for } L_2 \text{ metric} \tag{3.b}$$

In Robuste et al. (1990), the suitability of expression (3) is argued depending on the relation between some of the problem variables. Particularly, it has been proved that expression (3) provides good approximations when the distribution problem satisfies inequality (4):

$$7 < C < 1.5\left(\frac{N}{C}\right) \tag{4}$$

However, when inequality (4) is not satisfied, the following expression should be applied in compact zones, where k =0.45 for square zones and k= 0.55 for rectangular zones (when l/L=0.6 it is accomplished, *l* being the smaller side, and *L* the larger side of the rectangle).

$$D \approx \left[0.9 + k\frac{N}{C^2}\right]\sqrt{AN} \tag{5}$$

3. METHODOLOGY: ALGORITHMS AND PROBLEM GENERATION

In order to determine compact expressions estimating the total distance covered, a set of problems has been generated in regions with different shapes to perform the following heuristics: Clarke & Wright, Gillet & Miller and Fisher & Jaikumar. In addition, we consider the application of the swaths method in these problems to evaluate the formulas given in Robuste et al. (1990).

Particularly, we have considered zones of circular shape (a well-studied application morphology of these algorithms), as well as elliptic shape. This allows us to obtain results in near-circular zones or elongated zones depending on the value of their semiaxis.

In this way, we define a new variable β known as slimness which evaluates how elongated the zone shapes are. If $\beta \approx 1$ the zone shape will be quite similar to a circle. However, if $\beta << 1$ it will represent that the delivery region is structured in a main direction (*a* semiaxis).

$$\beta = \frac{b}{a} \qquad\qquad 0 \le \beta \le 1 \tag{6}$$

where:
a: major elliptic semiaxis
b: minor elliptic semiaxis

The set of problems is generated combining different values of the area and the number of scattered points to cover a large domain of the variable δ (*N* values have chosen between 21 and 184 delivery points).

In whole problems, the point position has been expressed in radial coordinates and they have been randomly generated by a computer taking into account that density of points should be kept constant in the whole region.

Hence, we split the circular and elliptic regions into various rings in order to determine an exact number of points according to each ring's area (it should be realized that for the same ring width, areas of external rings are greater than those of internal rings).

On the other hand, the vehicle capacity *C* has a minimal value of 2 (smaller values require that a single delivery point needs to be served by one or more full load vehicles). Its maximum value has been set to carry out the tour with 2 vehicles.

Finally, when we deal with elliptic problems, the considered values for β have been 0.2, 0.4, 0.6 and 0.8.

The set of problems generated have been solved by each reference algorithm using a computer. Their programming has followed the basic recipes that someone might look up in a logistic distribution book according to the constraints given in chapter 2. (The aim of this paper is not to perform an extended analysis of the properties of these algorithms). It has only considered L_2 metric in the distance calculations of each algorithm. Numerical results in elliptic zones are summarized in Table 1.

4. RESULTS. FORMULAS PREDICTING TOTAL DISTANCE TRAVELED.

All available data of the problems generation and resolution have been used in a regression model software to provide empiric formulas for each algorithm.

A first set of problem solutions (greater than 30 examples) has been used as a source to develop a multi-variant regression analysis. The most significant issues that have been analyzed to accept or reject each regression model are: the multiple correlation coefficient, relationship between those solution values resulting from the model with error values, linear independence of the variables, and the probability $P(H_o)$ to accept the model when all variable coefficients are equal to 0.

In this way, it has been established that the multiple correlation coefficient (r^2) associated with each formula has to be greater than 0,98 in order to ensure that numerical results are given by the prediction variables. Besides, we have also studied the quotient between the sum of the square value of the regression model with the sum of square value of their residues. We have considered that if this quotient exceeds a threshold of 5 % of total value, the formula should be rejected.

In addition, the linear independence of variables has been analyzed carrying out a Pearson correlation. As a result, we have obtained a dependent coefficient that should approximate to 0 in order to achieve a proper performance of the model. Eventually, it has been made up a statistic estimator that follows *t-Student* distribution to ensure that the probability $P(H_o)$ is closer to 0.

The main statistical analysis of the regression models has been summed up in tables 2,3,4 and 5

On the other hand, we have made use of another set of problem solutions as a tool to compare the performance of each distance expression. In this way, it has been proven in each problem that the difference between values resulting from those empirical expressions and the values obtained from the algorithm program is less than a fixed value (it is expressed as a percentage).

4.1 Circular VRP

In this kind of problem, the best results are obtained by considering practically the same variables of expression (3) to determine compact formulas in order to evaluate the distance covered. Because of the difficulty to determine the linehaul distance *r* in some problems (when some delivery sectors do not contain the warehouse), we propose to consider the radius of the circular region instead.

It should be noted that the first term of expression (7) represents the distance covered in radial direction by whole vehicles to arrive at the delivery sector (*N*/*C* is the number of full vehicles).

However, the second term is associated with the distance traveled in the local delivery sector. There has been a lot of research showing that the total distance is proportional to $(AN)^{1/2}$ in TSP problems (only one vehicle visits all delivery points). Some of them are summarized in Lawler et al. (1985) or in Larson et al. (1981).

Robusté et al.	$D = 1.344\left(Radius\,\dfrac{N}{C}\right) + 0.580\sqrt{AN}$	(7.a)
Clarke & Wright	$D = 1.189\left(Radius\,\dfrac{N}{C}\right) + 0.680\sqrt{AN}$	(7.b)
Gillet & Miller	$D = 1.404\left(Radius\,\dfrac{N}{C}\right) + 0.613\sqrt{AN}$	(7.c)
Fisher & Jaikumar	$D = 1.137\left(Radius\,\dfrac{N}{C}\right) + 0.672\sqrt{AN}$	(7.d)

Despite the fact that coefficients of the second term are very similar in expressions (7.a) and (3.b), coefficients of the first term are quite different due to the inclusion of radius in the model instead of the linehaul distance.

Apart from that, it should be noted that it is not possible to define which is the best algorithm analyzing the coefficients of the expressions (7.a) to (7.d.)

On the other hand, equation (8) has been developed under those situations that do not satisfy expression (4). It has been demonstrated that the application of Daganzo's algorithm in these problems does not provide good solutions. The "swath pattern" does not fit properly the shape of a delivery region associated to a single vehicle because it is such a wide circular sector in these situations.

It must be noted that expression (5) was developed theoretically although it was never checked using empirical results. Hence, this algorithm overestimates the optimal distance of tours, especially when $N/C \leq 3$, which is consistent with the hypotheses underlying the development of expression (5) (see Robuste et al., 1990).

In order to analyze this performance, graphic solutions of a specific circular VRP problem have been depicted in figures 1, 2, 3 and 4, applying the reference algorithms. The considered variable values have been: N=57, C=30 and Radius=5.

Clarke & Wright	$D = 0.835\sqrt{AN} + 0.600\frac{N}{C^2}\sqrt{AN}$	(8.a)
Gillet & Miller	$D = 0.833\sqrt{AN} + 0.565\frac{N}{C^2}\sqrt{AN}$	(8.b)
Fisher & Jaikumar	$D = 0.829\sqrt{AN} + 0.543\frac{N}{C^2}\sqrt{AN}$	(8.c)

4.2. Elliptic VRP

As a result of the suitability of expression (3) and (7), we have now considered a new variable that represents the whole approaching distance to delivery sectors depending on the slimness of the elliptic region.

In this way, expression (9) has been proposed for elliptic regions in VRP in which the radius that took part in the first term in equation (8) has been replaced by $a(1+\beta^2)^{1/2}$. Hence, the term $a(1+\beta^2)^{1/2}$ is equal to the distance of the diagonal of rectangle with the same sides as the semiaxis of the ellipse.

Clarke & Wright	$D = 0.603a\sqrt{1+\beta^2}\frac{N}{C} + 0.725\sqrt{AN}$	(9.a)
Gillet & Miller	$D = 0.781\,a\sqrt{1+\beta^2}\frac{N}{C} + 0.656\sqrt{AN}$	(9.b)
Fisher & Jaikumar	$D = 0.715\,a\sqrt{1+\beta^2}\frac{N}{C} + 0.664\sqrt{AN}$	(9.c)

It should be noticed that if we consider circular regions, i.e. elliptic major semiaxis a is equal to radius and $\beta=1$; expression (9) tends to strengthen the second term $(AN)^{1/2}$ compared to the coefficients of expression (7). It could be reasonable because these formulas result from elliptic regions with slimness range values between 0.2 and 0.8. Hence, when $\beta \ll 1$ the approaching distance through the sectors located near the minor semiaxis is not as important as the distance covered perpendicularly to the minor semiaxis direction (it is associated with $(AN)^{1/2}$ variable).

5. CONCLUSIONS

Some empiric formulas for VRP problems in circular and elliptic regions have been provided. Their associated linear correlation coefficients (r^2) are greater than 0,996 in all cases and linear independence of the variables of each model is guaranteed.

Despite the fact that no one can determine any algorithm that provides the minimum distance traveled analyzing expressions (7), (8) and (9), it may be shown some specifications in a particular domain of system variables.

In fact, Clarke and Wright's algorithm provides the best solutions specially when we deal with problems where N<50 or N/C>14. On the other hand, it is recommended the use the Fisher & Jaikumar algorithm when the number of points are much greater than 50 and N/C<14.

Finally, expressions given in Daganzo (1984) which were deduced theoretically are quite similar to empirical expressions obtained in this paper although the “swath method” gives the worst results: the formula (5) deduced in Robusté et al. (1990) and Daganzo’s method should not be used when N/C≤3.

6. REFERENCES

Daganzo, C.F. (1984) The length of tours in zones of different shapes. *Transportation Research,* 18B:5,135-145.

Daganzo, C.F. (1991) *Logistics Systems Analysis*, Springer-Verlag, Berlin.

Estrada, M. (2001) *Distance estimation in VRP in logistics*. Graduating thesis. Civil Engineering School, Technical University of Catalonia at Barcelona, Spain. (in Spanish)

Larson, R.C. y A.R. Odoni (1981) *Urban Operations Research*. Prentice-Hall.

Lawler, E.L., J.K. Lenstra, A.H.G. Rinnooy Kan, y D.B. Shmoys (1985) *The Traveling Salesman Problem,* John Wiley & Sons, New York.

Novaes, A.G. (1989) *Sistemas Logísticos: Transporte, Armazenagem e Distribuiçao Física de Productos.* Editora Edgard Blücher Ltda., Sao Paulo.

Robuste, F., C.F. Daganzo y R.R. Souleyrette (1990) Implementing vehicle routing models. *Transportation Research,* 24B:4, 263-286.

List of Tables & Figures

TABLE 1 Numerical results in elliptic VRP

TABLE 2 Descriptive statistics of Clarke & Wright algorithm model in elliptic VRP

TABLE 3 Correlations of Clarke & Wright algorithm model in elliptic VRP

TABLE 4 Model summary of Clarke & Wright algorithm in elliptic VRP

TABLE 5 ANOVA of Clarke & Wright algorithm in elliptic VRP

FIGURE 1 Resolution of a circular VRP problem using the Daganzo algorithm (N=57, C=30 and Radius=5)

FIGURE 2 Resolution of a circular VRP problem using the Clarke & Wright algorithm (N=57, C=30 and Radius=5)

FIGURE 3 Resolution of a circular VRP problem using the Gillet & Miller algorithm (N=57, C=30 and Radius=5)

FIGURE 4 Resolution of a circular VRP problem using the Fisher & Jaikumar algorithm (N=57, C=30 and Radius=5)

TABLE 1 Numerical results in elliptic VRP

	N	C	Minor Semiaxis	Major semiaxis	Area	Beta	Vehi-cles	Distance. A. Clarke	% Overp.	Distance. A. Gillet	% Overp.	Distance. A. Fisher	% Overp.
1	15	5	1,41	7,07	31,42	0,2	3	30,09	0,00	35,16	16,84	32,05	6,51
2	26	6	1,41	7,07	31,42	0,2	5	42,76	0,00	48,88	14,31	44,55	4,17
3	39	7	1,41	7,07	31,42	0,2	6	50,05	0,00	64,73	29,31	54,81	9,49
4	57	15	1,41	7,07	31,42	0,2	4	48,34	0,00	54,21	12,15	51,24	6,00
5	71	16	1,41	7,07	31,42	0,2	5	51,47	0,00	63,88	24,10	54,11	5,12
6	28	3	2,00	5,00	31,42	0,4	10	52,71	0,00	63,20	19,91	57,55	9,18
7	46	4	2,00	5,00	31,42	0,4	12	67,41	0,00	75,84	12,51	68,49	1,61
8	74	14	2,00	5,00	31,42	0,4	6	53,91	0,00	56,50	4,81	55,28	2,54
9	86	21	2,00	5,00	31,42	0,4	5	51,35	0,00	54,65	6,43	52,56	2,37
10	93	35	2,00	5,00	31,42	0,4	3	48,65	3,46	49,24	4,72	47,02	0,00
11	31	8	2,45	4,08	31,42	0,6	4	33,63	0,00	35,89	6,71	35,69	6,14
12	49	9	2,45	4,08	31,42	0,6	6	41,95	0,00	45,54	8,55	43,72	4,22
13	67	10	2,45	4,08	31,42	0,6	7	52,59	0,00	55,24	5,04	55,17	4,90
14	86	28	2,45	4,08	31,42	0,6	4	49,02	2,47	51,60	7,87	47,83	0,00
15	99	52	2,45	4,08	31,42	0,6	2	45,32	0,00	47,73	5,33	52,76	16,40
16	25	8	2,83	3,54	31,42	0,8	4	30,09	0,00	30,30	0,73	31,06	3,25
17	44	9	2,83	3,54	31,42	0,8	5	38,80	0,00	40,88	5,36	39,57	1,96
18	61	10	2,83	3,54	31,42	0,8	7	48,84	0,00	51,92	6,30	51,42	5,29
19	94	30	2,83	3,54	31,42	0,8	4	50,31	4,33	50,84	5,42	48,22	0,00
20	112	51	2,83	3,54	31,42	0,8	3	49,61	0,48	54,08	9,52	49,38	0,00
21	22	5	4,00	20,00	251,33	0,2	5	105,44	0,00	150,79	43,02	114,66	8,75
22	36	6	4,00	20,00	251,33	0,2	6	144,96	0,00	174,14	20,13	161,15	11,17
23	66	7	4,00	20,00	251,33	0,2	10	210,65	0,00	276,57	31,29	225,14	6,87
24	74	35	4,00	20,00	251,33	0,2	3	122,15	0,00	151,49	24,02	129,04	5,64
25	87	40	4,00	20,00	251,33	0,2	3	139,61	0,67	159,31	14,87	138,69	0,00
26	30	4	5,66	14,14	251,33	0,4	8	133,03	1,72	150,80	15,31	130,78	0,00
27	51	9	5,66	14,14	251,33	0,4	6	138,56	0,00	153,59	10,85	142,25	2,67
28	69	14	5,66	14,14	251,33	0,4	5	147,94	4,72	147,23	4,22	141,27	0,00
29	85	25	5,66	14,14	251,33	0,4	4	138,59	2,08	135,77	0,00	141,30	4,07
30	100	39	5,66	14,14	251,33	0,4	3	133,74	0,76	144,87	9,15	132,73	0,00
31	38	11	6,93	11,55	251,33	0,6	4	88,24	0,00	97,47	10,46	93,32	5,75
32	52	10	6,93	11,55	251,33	0,6	6	131,75	2,18	134,72	4,48	128,94	0,00
33	94	34	6,93	11,55	251,33	0,6	3	135,35	4,28	129,79	0,00	140,72	8,42
34	111	17	6,93	11,55	251,33	0,6	7	171,39	2,32	167,52	0,00	168,22	0,42
35	124	13	6,93	11,55	251,33	0,6	10	218,58	0,12	218,32	0,00	**	0,00
36	40	12	8,00	10,00	251,33	0,8	4	98,95	3,37	111,21	16,18	95,72	0,00
37	59	5	8,00	10,00	251,33	0,8	12	192,17	0,00	207,34	7,89	195,14	1,55
38	84	14	8,00	10,00	251,33	0,8	6	155,40	2,47	154,85	2,11	151,66	0,00
39	115	17	8,00	10,00	251,33	0,8	7	185,47	10,74	177,96	6,26	167,48	0,00
40	134	29	8,00	10,00	251,33	0,8	5	176,53	5,23	174,29	3,90	167,75	0,00
41	24	10	7,87	39,37	973,89	0,2	3	172,83	0,00	193,43	11,92	174,68	1,07
42	63	6	7,87	39,37	973,89	0,2	11	447,08	0,00	499,38	11,70	**	0,00
43	79	15	7,87	39,37	973,89	0,2	6	330,55	0,00	379,04	14,67	352,41	6,61
44	96	27	7,87	39,37	973,89	0,2	4	311,80	2,60	332,05	9,26	303,91	0,00
45	103	24	7,87	39,37	973,89	0,2	5	341,24	0,00	357,98	4,91	352,33	3,25
46	29	3	11,14	27,84	973,89	0,4	10	295,21	0,00	341,39	15,65	332,68	12,69
47	45	7	11,14	27,84	973,89	0,4	7	281,54	0,00	324,19	15,15	301,78	7,19
48	68	16	11,14	27,84	973,89	0,4	5	264,59	0,00	279,88	5,78	266,80	0,83
49	99	28	11,14	27,84	973,89	0,4	4	296,00	5,37	280,91	0,00	285,68	1,70
50	107	61	11,14	27,84	973,89	0,4	2	260,13	0,38	276,57	6,72	259,15	0,00
51	33	7	13,64	22,73	973,89	0,6	5	194,71	0,00	213,09	9,44	219,15	12,55
52	56	5	13,64	22,73	973,89	0,6	12	370,42	0,00	399,60	7,88	410,50	10,82
53	74	19	13,64	22,73	973,89	0,6	4	253,93	1,73	257,14	3,02	249,60	0,00
54	105	23	13,64	22,73	973,89	0,6	5	322,47	3,51	313,55	0,65	311,53	0,00
55	118	34	13,64	22,73	973,89	0,6	4	318,82	4,00	316,90	3,37	306,57	0,00
56	43	10	15,75	19,69	973,89	0,8	5	217,85	0,00	235,36	8,04	229,01	5,12
57	74	12	15,75	19,69	973,89	0,8	7	295,12	0,00	321,67	9,00	309,00	4,70
58	106	41	15,75	19,69	973,89	0,8	3	287,20	7,17	278,50	3,93	267,98	0,00
59	116	26	15,75	19,69	973,89	0,8	5	330,85	5,39	313,93	0,00	315,08	0,37
60	139	61	15,75	19,69	973,89	0,8	3	301,77	0,51	310,37	3,38	300,24	0,00

TABLE 2 Descriptive statistics of Clarke & Wright algorithm model in elliptic VRP

	Mean	Standard deviation	N
Distance	163.3252	110.1525	60
$a(N/C)(1+\beta^2)^{1/2}$	95.6398	82.7524	60
$(AN)^{1/2}$	148.3260	102.7825	60

TABLE 3 Correlations of Clarke & Wright algorithm model in elliptic VRP

		Distance	$a(N/C)(1+\beta^2)^{1/2}$	$(AN)^{1/2}$
Pearson Correlation	Distance	1.000	.822	.923
	$a(N/C)(1+\beta^2)^{1/2}$	.822	1.000	.545
	$(AN)^{1/2}$	.923	.545	1.000
Sign.	Distance	.	.000	.000
	$a(N/C)(1+\beta^2)^{1/2}$	.000	.	.000
	$(AN)^{1/2}$	.000	.000	.
N	Distance	60	60	60
	$a(N/C)(1+\beta^2)^{1/2}$	60	60	60
	$(AN)^{1/2}$	60	60	60

TABLE 4 Model summary of Clarke & Wright algorithm in elliptic VRP

R	R2	R2 corrected	Error of estimation	Change of statistics			Durbin-Watson
				Change in R^2	Change in F	Sign. of change in F	
.998	.996	.996	6.8781	.996	7537.686	.000	1.894

TABLE 5 ANOVA of Clarke & Wright algorithm in elliptic VRP

Model	Sum of square values	gl	Square mean	F	Sign
Regresion Residue Total	713183.664 2696.548 715880.213	2 57 59	356591.832 47.308	7537.686	.000

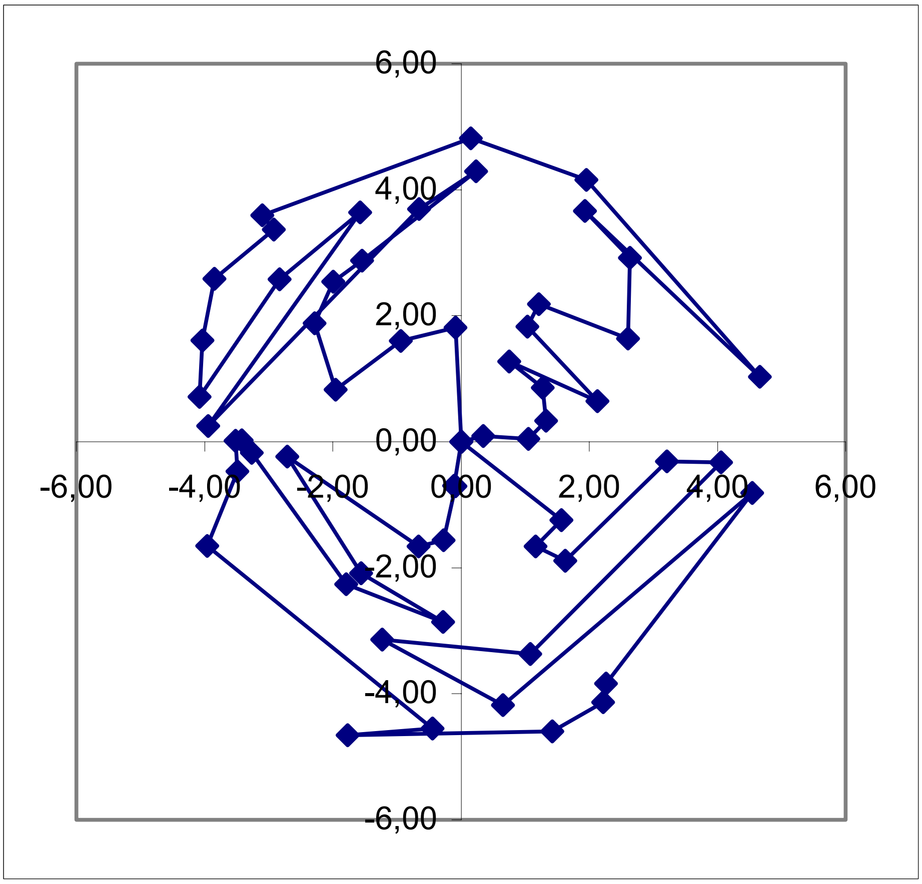

FIGURE 1 Resolution of a circular VRP problem using the Daganzo algorithm (N=57, C=30 and Radius=5). Source: M. Estrada (2001).

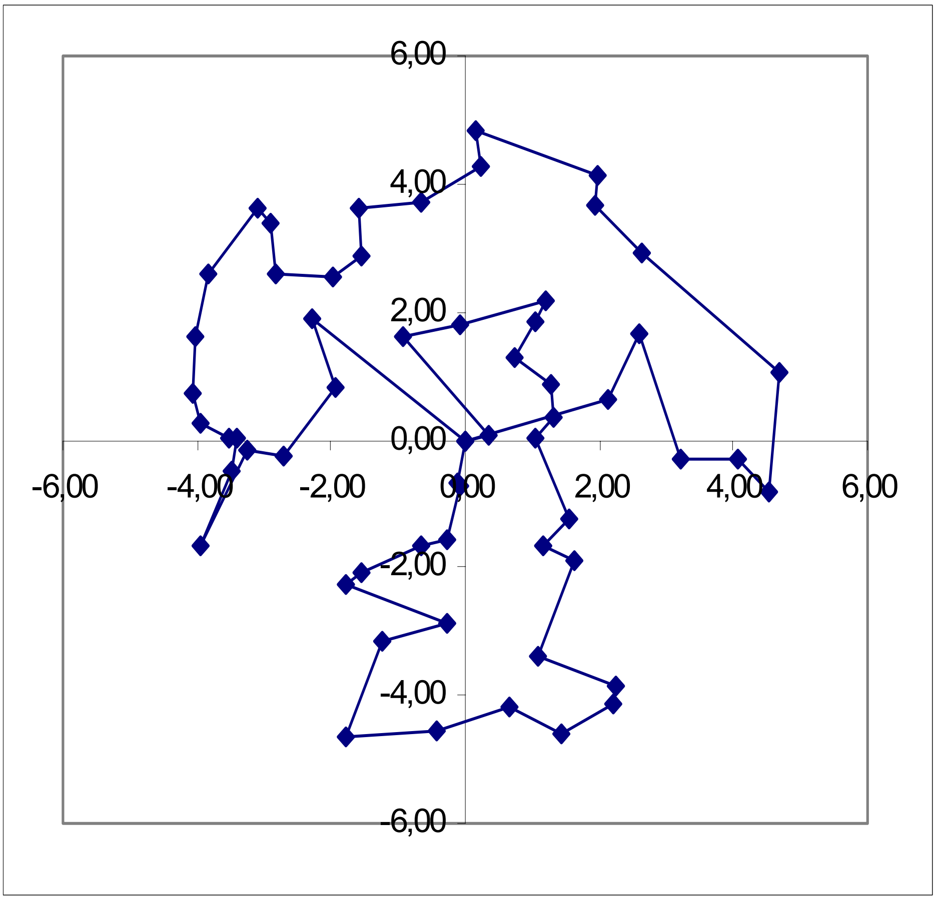

FIGURE 2 Resolution of a circular VRP problem using the Clarke & Wright algorithm (N=57, C=30 and Radius=5). Source: M. Estrada (2001)

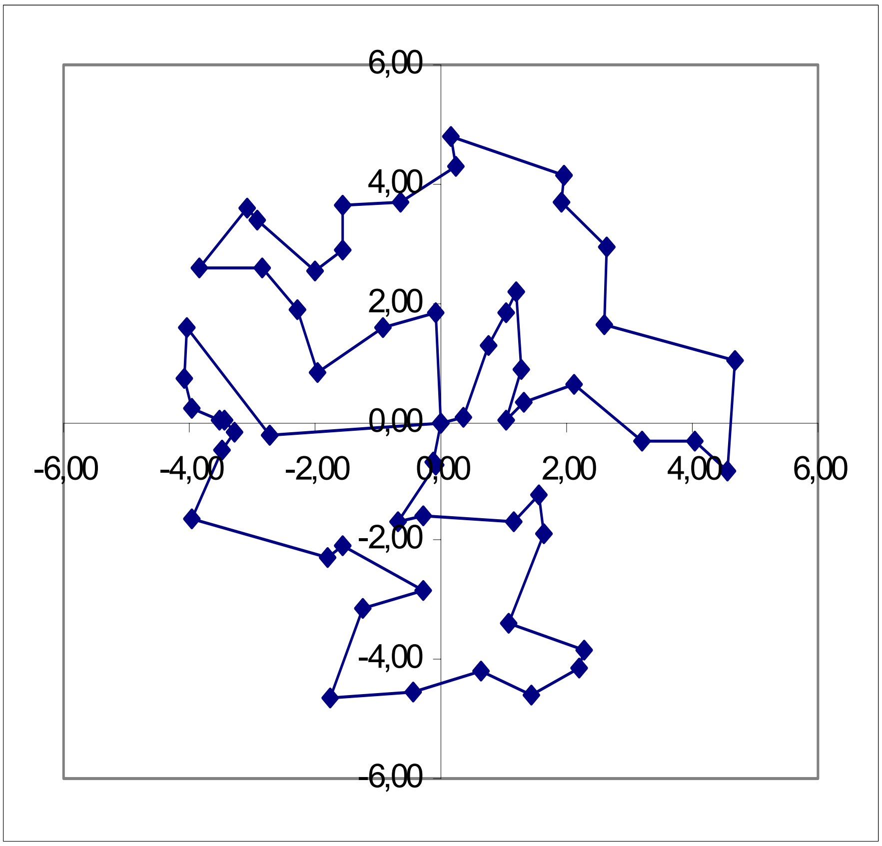

FIGURE 3 Resolution of a circular VRP problem using the Gillet & Miller algorithm (N=57, C=30 and Radius=5). Source: M. Estrada (2001)

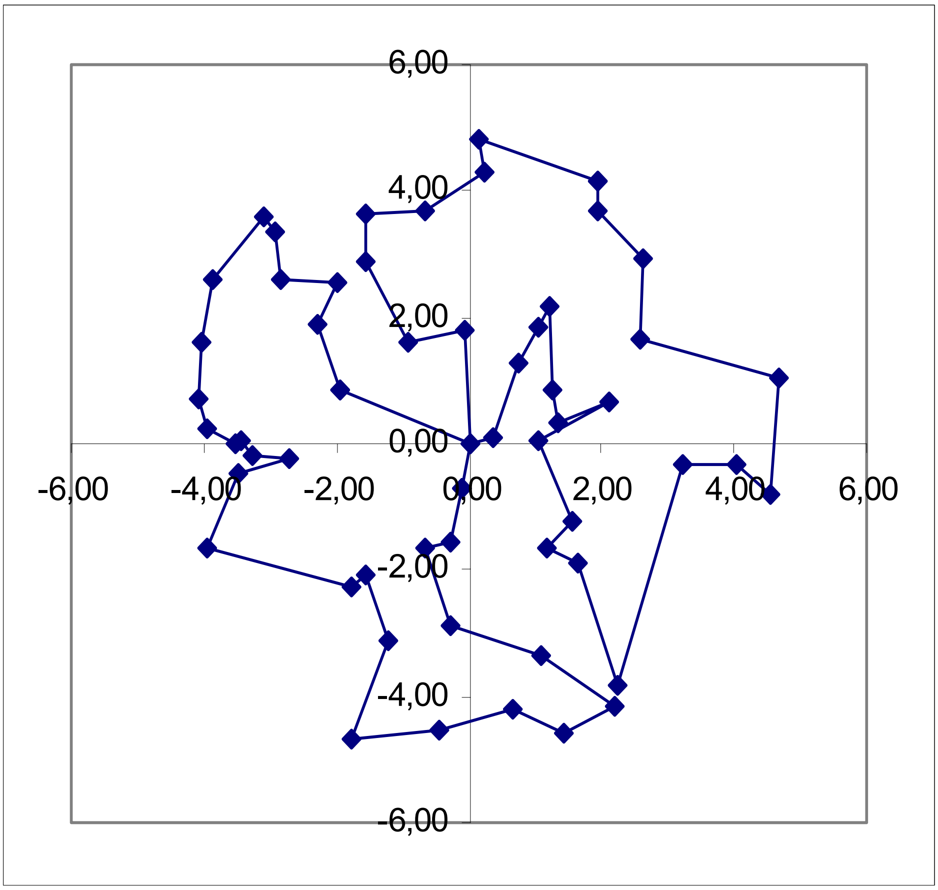

FIGURE 4 Resolution of a circular VRP problem using the Fisher & Jaikumar algorithm (N=57, C=30 and Radius=5). Source: M. Estrada (2001)