Automatic subsystem identification in statistical energy analysis

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Abstract

An automatic methodology for identifying SEA (statistical energy analysis) subsystems within a vibroacoustic system is presented. It consists in dividing the system into cells and grouping them into subsystems via a hierarchical cluster analysis based on the problem eigenmodes. The subsystem distribution corresponds to the optimal grouping of the cells, which is defined in terms of the correlation distance between them. The main advantages of this methodology are its automatic performance and its applicability both to vibratory and vibroacoustic systems. Moreover, the method allows the definition of more than one subsystem in the same geometrical region when required. This is the case of eigenmodes with a very different mechanical response (e.g. out-of-plane or in-plane vibration in shells).

Keywords: Statistical energy analysis, vibroacoustics, modal analysis, cluster analysis, substructuring

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List of symbols

e_{ij}	Energy density for cell i and mode j
$\tilde{e}_{ij}$	Normalised energy density for cell i and mode j
E	Young's modulus
$\bar{E}_j$	Mean energy density of the domain for mode j
h	Plate thickness
i	Cell counter
j	Eigenmode counter
n	Modal density
N	Number of cells
S_j^2	variance of e_{ij} for mode j
$\mathbf{x}_i$	Energy vector associated to cell i
Γ_D	Dirichlet boundary
η_{ii}	Internal loss factor
η_{ij}	Coupling loss factor
λ_{plate}	Wavelength at the plate
ν	Poisson's ratio
ρ	Density
$\mathbf{1}$	Vector with all the components equal to 1
ω	Angular frequency

1 Introduction

The solution of acoustic and vibroacoustic interior problems is still nowadays an important challenge for researchers and industry. As discussed in [1], the intrinsic difficulty of the problem is not only the high computational cost of deterministic models, but also modelling aspects such as the uncertainty of material parameters (specially damping) or the simulation of real boundary conditions. A classical modelling technique in the high-frequency range is the statistical energy analysis (SEA) framework, which is based on the power flow between different parts of the problem domain, called subsystems. A proper subdivision of the domain into weakly coupled subsystems consisting of modes with similar energetic behaviour is crucial for the good performance of SEA. This subdivision combined with modelling aspects such as the correct evaluation of coupling loss factors or the internal losses determine later the quality of the SEA predictions [2]. This paper presents a methodology for identifying the optimal subdivision of a domain into subdomains, such that they can be used as SEA subsystems. The main contributions associated to this methodology are:

- The automatic choice of the optimal subdivision (number and geometry of subsystems).
- The applicability of the methodology for problems consisting of both fluid and solid domains.
- The possibility of preprocessing the eigenmodes of the problem depending on their nature. This allows a subsystem definition based on the mechanical be-

haviour and different subsystems can coexist in the same part of the domain (e.g. out-of-plane or in-plane vibration in shells).

SEA subsystems are defined by Lyon [3] as "groups of 'similar' energy storage modes. These modes are usually modes of the same type (flexural, torsional, etc.) that exist in some section of the system". In order to classify a certain set of modes as a subsystem, they must fulfil two main criteria [2, 3]:

1. Similarity: all the modes must have a similar energetic response in front of any possible excitation.

2. Significance: they must play an important role in the transmission, dissipation or storage of energy of the problem.

In many typical applications, subsystem identification is straightforward. Common building elements like beams or thin plates clearly fulfil the requirements just discussed. However, SEA may also be a powerful tool for dealing with vibroacoustic problems with complex shapes and non-conventional configurations. Examples of this kind of problems might be found, for instance, in the automotive [4–6], railway [7, 8] or aerospace [9] industries, or even in some new architectonic designs and buildings [10]. In these cases the domain subdivision is not so clear and often the decomposition is done following material or geometrical criteria, see Chen et al. [11] or Forssén et al. [7].

Different authors have worked on domain substructuring for energy models. As an example, Kassem et al. [12] propose a strategy for their local vibroacoustic energy model based on searching the validity frequencies for a certain substructuring, and Kovalevsky and Langley [13] propose two different strategies for recognising the elements of their finite element/statistical energy analysis model, based on the Green's functions of the problem.

In the particular case of the identification of SEA subsystems, Fahy [14] studies qualitatively the effect of subdividing the cavity inside a car into different subsystems. He concludes that this can be done, in particular at the region below the seats, but recommends the use of experimental information for checking the robustness of the approach.

Gagliardini et al. [5] propose a strategy for identifying SEA subsystems. It is based on the energetic transfer functions obtained between points of the domain for different excitations. This analysis involves solving the vibratory problem for every excitation in a particular frequency band.

Totaro and Guyader [15] propose an original strategy based on cluster analysis. It requires the numerical simulation of the vibratory problem for a representative set of excitations. They discretise the domain with finite elements and perform a cluster analysis of these elements. The analysis is based on a set of energy transfer functions obtained for different excitations, and a principal component analysis of these functions is performed before the cluster analysis, to reduce the data size. The final decision of the optimal amount of clusters is done in terms of an external parameter called *mutual inertia ratio*.

These methods have only been applied to purely vibratory or purely acoustic systems, but not to vibroacoustic problems. Another limitation of these techniques

is that they perform a purely spatial subdivision into subsystems. This means that a certain point of the domain cannot belong to two different subsystems at the same time. However, a certain region of the domain may present two types of modes with very different energetic responses to excitations, as discussed by Lyon [3], Maidanik [16] or McCollum and Cuschieri [17]. For example, a structure consisting of thin shells may present both flexural and in-plane modes for the frequency range of interest.

The strategy presented in this paper is based on a modal analysis of the problem. The domain is divided into small cells, and these cells are classified with a cluster analysis as done in [15]. The difference here is that the analysis is based on the energies for a set of eigenmodes of the problem instead of the frequency-dependent response due to some particular excitation (i.e. point forces). This approach is intimately related to the definition of subsystem proposed by Lyon, and allows as a novelty the possibility of preprocessing the modes for defining more than one subsystem at a certain spatial region. Some added advantages of this strategy are that no excitations are required for the analysis (there are no issues of excitation selection), its applicability to vibroacoustic problems, its low computational cost (it only requires the computation of a few eigenmodes) and the use of the information provided by the cluster analysis to choose the amount of subsystems. Therefore, the choice is independent of any external parameter such as the mutual inertia ratio used in [15]. Moreover, the use of the eigenmodes is a natural way to obtain the more meaningful information of the mechanical or vibroacoustic problem. Therefore, there is no need to filter the samples in a preprocess. This contributes to the low computational cost of the problem, which is particularly important because the identification of subsystems is a preprocessing step in SEA calculation, and one of the main features of SEA is its low computational cost. Finally, it is worth mentioning that subsystem identification is not only a topic of interest for SEA applications like [18] but also in the analysis of mechanical systems/problems in general, see for example [19].

The paper is structured as follows: first, the methodology for dividing the problem domain into subsystems is presented in Section 2, detailing the main elements of the required cluster analysis. Then, an extension of the methodology for dealing with different significant types of modes on the same part of the domain is described in Section 3. Section 4 shows a set of examples where the performance of the methodology is illustrated, and the main conclusions of this research are summarised in Section 5.

2 Methodology for dividing the domain into subsystems

In this section, the case where only one type of modes fulfils the significance criterion at each part of the domain is analysed. Hence, the goal is to obtain a geometrical decomposition of the domain into subsystems. If different types of significant modes coexist in the same part of the domain, further considerations need to be done, as discussed in Section 3.

The methodology proposed here for performing the geometrical decomposition is based on dividing the domain into cells and performing a hierarchical cluster analysis

of these cells based on the system eigenmodes. The main steps of this procedure are described below.

2.1 Cluster analysis based on the system eigenmodes

The idea of cluster analysis was first introduced by Tryon [20]. It consists in grouping a set of elements in such a way that elements in the same group (called cluster) are more similar (in some sense or another) to each other than to those in other groups, Everitt et al. [21]. The similarity between elements is measured as the distance between them in the sample space. The dimension of this space coincide with the number of samples used for the analysis. The main ingredients for the hierarchical cluster analysis are: the cluster elements, the sample space, the distance function and the clustering algorithm.

2.1.1 The cluster elements

The elements to be classified with the analysis are *cells of size equal or greater than half the maximum wavelength considered*. This maximum wavelength is associated to the lowest frequency of the modal analysis, and the domain is divided into cells whose size is equal or greater to this length.

The cell concept must be understood here as the division of the problem domain into zones. These zones can be defined randomly or using information about the geometry (i.e. the junction of a T-shaped structure). However, this is not required and some uniform splitting of the problem geometry should be enough. Once these zones are defined an averaged output can be assigned to each of them.

The cells are an intermediate layer between the modelling technique and the cluster analysis. The output average is performed in a different way depending on the modelling technique or the discretisation needed to perform the calculations. So, if the finite element method is used the cells can be defined as: *i)* groups of elements; or *ii)* any other patches of the domain (non-conforming with the finite element mesh). In other modelling techniques the output assigned to each cell can be a simple average of the solution. For example, the vibroacoustic problems of Section 4.3 have been solved by means of the finite strip method [22]. The vibration and pressure fields in the three-dimensional space are obtained as a combination of spatial discrete interpolation in a problem section and a modal combination in the third dimension. For this numerical technique the need of a clear definition of the cells becomes evident. A regular square grid has been used.

Another important aspect of the cell concept is that it can be understood as a regularisation of the post-process in order to avoid local effects. These can be caused by the boundary conditions (i.e. zones of an structure close to the supports) or by the nodes of the solution. In [15], the cluster elements are the finite elements. However, a fine mesh can present huge differences between the averaged energies of the elements (i.e. elements placed at nodes or anti-nodes of a pressure/vibration wave). In a cluster analysis based on the system eigenmodes, this may lead to a wrong subdivision, where elements at nodes or anti-nodes of the waves, or strongly influenced by the boundary conditions, might be identified as different subsystems. The averaged behaviour of

the waves is better captured if cells with a size equal or larger than half a wavelength are considered.

2.1.2 The sample space

This space is formed by the normalised energetic contributions to different eigenmodes of the problem. Therefore, the dimension of the space is equal to the number of eigenmodes considered. These modes are computed with the finite element code Cast3M [23] for vibratory problems and with an in-house code based on the finite strip method for vibroacoustic problems [22]. The eigenmodes of the problem are good samples for the cluster analysis, because each of them provides an independent and significant case to analyse. The term "normalised energetic contribution" is an abuse of notation, given that there is no energy associated to a particular eigenmode without excitation. However, given a criterion for the definition of the displacement, velocity or strain field associated to an eigenmode, a magnitude analogous to the energy associated to the eigenmode can be computed.

The approach used here contrasts with the approach of [15], in which the samples are obtained by computing the energy transfer functions for different positions of the excitation and at every Hz within a third octave band of interest. Two variables, the position and the frequency of the excitation, are combined to generate a representative set of samples. Then, they project the results of the simulations in their principal components, in order to extract the most representative information and reduce the number of dimensions of their space.

The approach based on a set of eigenmodes of the problem reduces the computational cost in the sense that the most representative frequencies within a certain range are obtained directly, and independently of the possible excitations to the system.

To obtain the normalised energetic contribution to each mode of a certain cluster element i, first its averaged energy density (the total cell energy divided by the area of the cell) is calculated, and a map of energy densities e_{ij} is created for every mode j. The analysis, however, is not performed directly on the energy densities. The most important variable is not the energy itself but the normalised difference between the energy at each cell and the averaged energy of the system, as defined by [15] or [24] in order to study the energy equipartition in SEA problems. Therefore, a new map is produced, computing for each cell i and mode j the value of $\tilde{e}_{ij}$ as

$$\tilde{e}_{ij} = \frac{e_{ij} - \bar{E}_j}{S_j \sqrt{N}}, \tag{1}$$

where

$$\bar{E}_j = \frac{1}{N} \sum_{i=1}^{N} e_{ij} \tag{2}$$

is the mean energy density in the domain for mode j,

$$S_j^2 = \frac{1}{N} \sum_{i=1}^{N} \left(e_{ij} - \bar{E}_j \right)^2 \tag{3}$$

is the variance of e_{ij} for mode j and N is the number of cells. A vector belonging to the sample space is associated to a certain cluster element, and has as j-th component the normalised energy density of that element for mode j.

The magnitude used for defining the sample space is also different in the approach of [15]. Since they obtain the values of the energies associated to particular excitations, they choose to perform the cluster analysis based on the energy transfer functions, in order to eliminate the effect of the excitation on the energies. In the strategy presented here, the lack of excitation has led to the decision of defining the sample space in terms of the energies.

2.1.3 The distance function

Once the energy maps are known, the energy vector $\mathbf{x}_i$ associated to a cell (cluster element) i is defined as the vector whose j-th component is the normalised energy density of that cell for mode j, $\tilde{e}_{ij}$.

Then, *the distance between two cluster elements m and n is computed as the correlation distance between their energy vectors*

$$d(\mathbf{x}_m, \mathbf{x}_n) = 1 - \frac{(\mathbf{x}_m - \bar{\mathbf{x}}_m)(\mathbf{x}_n - \bar{\mathbf{x}}_n)^\top}{\sqrt{(\mathbf{x}_m - \bar{\mathbf{x}}_m)(\mathbf{x}_m - \bar{\mathbf{x}}_m)^\top}\sqrt{(\mathbf{x}_n - \bar{\mathbf{x}}_n)(\mathbf{x}_n - \bar{\mathbf{x}}_n)^\top}}, \tag{4}$$

where

$$\bar{\mathbf{x}}_m = \left(\frac{1}{N_m}\sum_j x_{mj}\right)\mathbf{1}, \tag{5}$$

where $\mathbf{1}$ is a vector with all the components equal to one.

The correlation distance is chosen in this work instead of other measures such as the Euclidean distance because the difference in magnitude between the vectors is not as interesting as the dissimilarity between their directions. The correlation distance between two vectors specifically focuses on the difference between their directions, eliminating the influence of their magnitudes with a normalisation. The correlation distance d ranges between 0 and 2. These two values are reached when the direction of vectors $(\mathbf{x}_m - \bar{\mathbf{x}}_m)$ and $(\mathbf{x}_n - \bar{\mathbf{x}}_n)$ is very similar. On the contrary, $d = 1$ means that their direction is very different.

2.1.4 The clustering algorithm

The clustering is performed hierarchically. That means that the cluster elements are grouped progressively, in terms of the correlation distance between them. Once two elements are joined, they create a new element, whose distance to the others is measured as the average distance of the members, see Fig. 1. The elements are paired into binary clusters, and the newly formed elements are grouped into larger clusters until a hierarchical tree (dendrogram) is formed, see Fig. 2. On these figures the hierarchical clustering process is recreated for a simple case. For illustrative purposes the example is done using the Euclidean distance. However, in the proposed algorithm, the correlation distance is used.

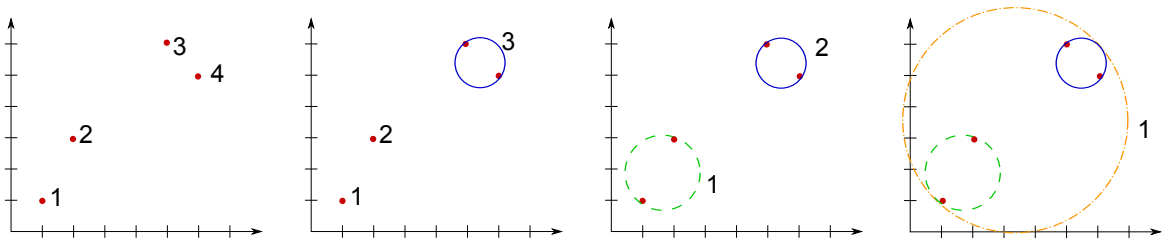


Figure 1: Grouping process.

2.1.5 The number of clusters

The dendrogram of the analysis shows the amount of clusters associated to each correlation distance. For instance, in Fig. 2, a distance of one unit corresponds to four clusters and a distance of three units corresponds to two clusters. In the same way, the range of distances associated to each amount of clusters can be computed. In this methodology, the optimal amount of clusters is defined such that it corresponds to the widest distance range. In the case of Fig. 2, the optimal subdivision corresponds to two clusters, because the largest range of distances is equal to 2 units, and is associated to the part of the dendrogram where only two clusters remain.

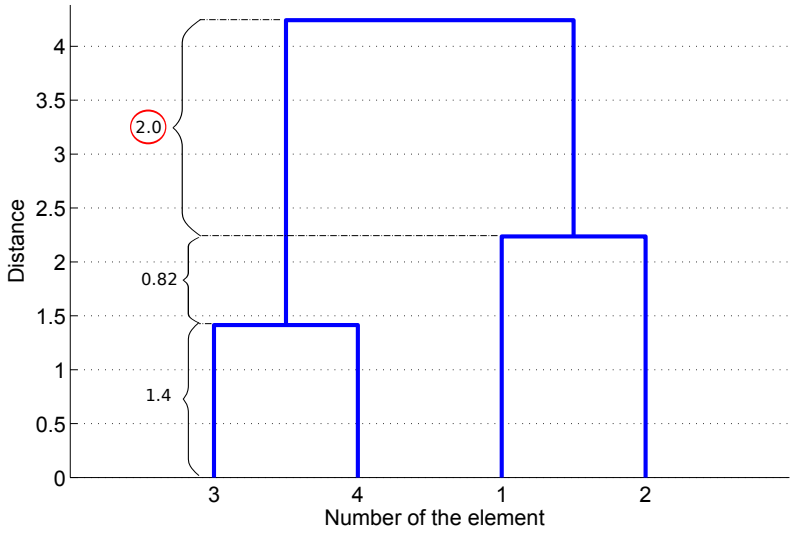


Figure 2: Distance range for each distribution.

At the beginning of the process the amount of cluster elements is large. Therefore, the dendrogram has a more complex structure, with more branches in the lower part as shown in Fig. 6. However, the most interesting part is the upper one, where the larger distance ranges appear and a reduced number of subsystems can be identified.

2.2 Computation of the energy

The energy distribution used for the cluster analysis may be either the kinetic energy, the strain energy or the total energy resulting from adding the other two, see Lyon [3]. In the vibratory examples of Section 4.1, the averaged total energy at each cell is computed and normalised to perform the cluster analysis.

In the pressure-displacement formulation for the vibroacoustic problems the outputs that can be obtained without additional post-process are the pressure energy in acoustic domains (no need to compute velocities) and the kinetic energy for the vibratory part (displacement field available). This choice of energies has been validated by means of one-dimensional vibroacoustic tests. Subsystems are properly identified as it is shown here for three-dimensional examples.

In [15] it is reported that the decomposition into subsystems with the kinetic energy is different from that with the total energy, specially for low frequencies and curve-shaped structures. A possible reason for this phenomenon is the different response provided by different types of modes coexisting in the same spatial region. Typically flexural modes store predominantly kinetic energy, and in-plane modes, strain energy. Due to this, comparing only their kinetic energy might be misleading. However, if different types of modes are analysed separately, this problem disappears. This topic is addressed with the methodology described in Section 3 and, therefore, should not affect the choice of the energy type here.

2.3 Robustness of the decompositions

An important aspect in the subsystem identification is the robustness of the method when changing the cluster elements. If the division into subsystems is clear for a certain problem, a cluster analysis performed with slightly different cells should lead to the same subdivision. If it does not, this subdivision should not be chosen.

The choice of the number of subsystems forming a certain system can be improved following this idea: if the cluster analysis is repeated with larger cells and the optimal subdivision obtained is the same, then its optimality is confirmed. Otherwise, the system may have an intermediate behaviour between two options. To deal with this case, more cluster analyses should be performed changing the cell size, and the most probable decomposition should be identified. For this subdivision, the coupling loss factors between each pair of subsystems should be estimated numerically, see for example [25–28], and compared with their internal loss factors. If the coupling loss factor between two possible subsystems is larger than their respective internal loss factors, the SEA hypothesis of weak coupling is not fulfilled and, therefore, they should be grouped as a single subsystem.

2.4 Step-by-step summary of the methodology

The step-by-step strategy proposed here to identify the subsystems within a certain system is the following:

1. Obtain a representative set of the eigenmodes of the problem.

2. Divide the domain into cells and compute the total energy density in the cells for each mode. These cells should have size equal or greater than half the largest wavelength.

3. Compute the normalised energies $\tilde{e}_{ij}$ at the cells.

4. Perform the hierarchical cluster analysis and obtain the associated dendrogram.

5. Select the amount of clusters with the wider distance range.

6. Repeat the analysis with larger cells and check if the optimal subdivision is still the same.

7. If the optimal subdivision coincides, the problem is solved. Otherwise, compute the coupling loss factors and base the decision on the fulfilment of the weak coupling hypothesis.

2.5 Other aspects of the analysis: consistency, damping and validity for SEA use

The systematic methodology summarised in Section 2.4 provides reasonable results for the examples in Section 4. However, when applying it to more complex configurations, other aspects may be relevant:

- The proposed methodology assumes that a unique subdivision of the system holds for the whole frequency range of analysis. Frequency-dependent subdivisions can be considered by repeating the analysis for different groups of modes, corresponding to different frequency ranges. In this way the optimal subdivision for each range can be found.

- In case the strategy leads to an optimal subdivision involving disjoint subsystems, which are non-physical, they must be divided providing a new subdivision with physical meaning. This is a very exceptional case, only likely in problems with periodic geometries or consisting of disjoint components with identical features.

The algorithm for the cluster analysis is only based on the eigenmodes of the system and does not make use of the eigenfrequencies. If hysteretic damping is added to the system, the eigenfrequencies become complex numbers, but the eigenmodes remain the same: all the damping information is seen through the imaginary part of the eigenfrequency, which has the meaning of the attenuation of the mode, and for constant damping this imaginary part does not differ much from one eigenfrequency to another. The invariability of the eigenmodes has an important implication: the cluster algorithm proposed here does not depend on the problem damping. It is interesting from the point of view of algorithm simplicity. However, the role of damping in the physical system must be checked a posteriori.

The decomposition obtained with the proposed algorithm can have multiple applications. It can be used, for example, as a domain splitting in order to perform parallel

computations or make use of a sub-structuring technique [29–31]. However, for using it in the framework of a SEA model, some extra verifications are required. These verifications are necessary in order to take into account the effect of the subsystem damping and check if the subdivision is consistent with SEA hypotheses [2, 24, 32–34].

The algorithm proposed in this paper is based on the definition of SEA subsystems. The cluster analysis groups cells (or zones in the problem domain) with a similar modal behaviour. Moreover, their behaviour and response must be clearly different from that of the other parts of the domain (i.e. structural mode with the largest vibrations mainly concentrated in one of the parts of the structure). The main implications of this are the following:

- If the cells are grouped by similarity in their modal behaviour with respect to the other parts of the domain, their relationship with other parts of the domain will most probably be also similar (hypothesis 3.1.3 in [2]).

- If the subsystems are defined by similarity in the modal behaviour and the damping is constant in that zone, it is very likely that the modes of the subsystem have almost the same damping (hypothesis 3.1.4 in [2]).

- The subsystems are defined by indirectly enforcing their independence with respect to other zones. This is equivalent to searching the optimally uncoupled subsystems. Therefore low coupling loss factors are expected. The strength of the coupling is inversely related with the length of the distance ranges in the dendrograms (hypothesis 3.3.2 in [2]).

Even if the obtained subsystems are good candidates for being SEA subsystems, some SEA hypotheses depend on aspects that are not considered during the subdivision process and must be verified a posteriori. The most critical points are:

- The modal overlap must be large: $\omega\, n(\omega)\eta_{ii}(\omega) \gg 1$ with $n(\omega)$ the modal density. This must be checked a posteriori but does not require much extra work within the framework of the algorithm presented here (hypothesis 3.1.1 in [2] and Eq. (14) in [32]).

- The interaction between subsystems must be weak: $\eta_{ij}/\eta_{ii} \ll 1$ (hypothesis 3.3.2 in [2] and the light coupling condition in Eq. (18) of [32]). This depends on the damping and the coupling loss factors, which must be estimated a posteriori. However, the obtained subdivision is expected to minimise the values of the CLF for the given geometry.

Most of the drawbacks mentioned above are not exclusive of the subdivision algorithm proposed here. They are common verifications of a standard SEA modelling process. Other SEA hypotheses like the nature of the excitation (rain-on-the-roof excitation that guarantees equipartition of energy in all subsystem modes and/or a diffuse field), the conservative behaviour of the coupling, the linearity of the damping mechanism or the dependence of the problem response only on the modes in the band, do not depend on the subdivision strategy but on the physical properties of the medium or the excitation type. Therefore, they would not be strongly affected by the definition of the subsystems.

3 Extension of the methodology: different types of significant modes

The case of having dissimilar and significant groups of modes coexisting in the same physical region is considered here. One of the most typical cases in building acoustics is the presence of in-plane and transverse waves together in the same area (for instance in plates or shells). To account for the effect of these two types of modes separately, the analysis is done in terms of the kinetic energy of the problem eigenmodes. In this sense, two magnitudes are computed for each eigenmode at every point: the kinetic energy associated to the normal velocity and the kinetic energy associated to the in-plane velocity. To do so, the vector normal to the plate or shell is computed at each point and, for each mode, the vibration velocity is decomposed into its normal and in-plane components. With them, the normal and in-plane kinetic energies can be computed.

Once the two kinetic energy maps are known for each eigenmode, every mode is classified as a *normal mode* or an *in-plane mode*. To do so, the averaged value of each type of energy along the whole domain is computed for every mode. If the averaged normal energy is larger than the in-plane one, the mode is classified as normal. Otherwise, it is classified as an in-plane mode.

After classifying the modes in two sets, the size of each set is computed. If one set is much larger than the other, the modes of the small one do not fulfil the principle of significance. Therefore they should not be taken into account and the analysis described in Section 2 must be applied only to the largest group of modes.

If, on the contrary, the two sets have a non-negligible size, both of them should be taken into account, and the analysis described in Section 2 should be done separately for each group of modes. Therefore, two geometric decompositions of the domain are obtained, one for each set of modes. The global amount of subsystems is obtained as the sum of the in-plane and transverse subsystems. For these analyses, the same considerations done in Section 2.3 regarding the robustness and the strength of the coupling still apply.

Since the decision of taking different sets of modes into account depends on the percentage of modes of each type, the frequency range of analysis plays an important role in the decision. It is important to remark that the lack of modes of a certain type within a particular frequency range does not guarantee that they will not appear at higher frequencies. If the amount of modes of a certain type is too small within the frequency range considered, their lack of relevance should be verified by checking more modes for other frequency ranges (higher frequencies), before deciding to ignore them definitely.

The procedure described here is restricted to systems consisting of plates or shells. Systems of this kind are very common in vibroacoustic problems. Therefore, it is interesting to develop a methodology for identifying their SEA subsystems. However, analogous analyses can be performed for other types of elements, if the directions governing the main types of waves are known in advance. For instance, for the case of a system of beams, energies associated to its flexural, axial, and even torsional waves can be computed separately in order to detect which of these phenomena are

significant within the frequency range considered. Once the significant sets of modes are identified, the analysis of Section 2 is applied to each of them.

4 Simulations

The presented methodology is validated here by means of dealing with several simple examples. In these cases, it is possible to base the subsystem identification on the intuition. However, they have been chosen precisely in order to illustrate that the proposed approach is able to detect the expected subsystem distribution without any a priori information.

4.1 Vibratory systems consisting of thin plates

4.1.1 T-shaped structure

The first example deals with the T-shaped structure of Fig. 3. The material properties of the structure are described in Table 1, and the displacement on the plates is blocked at the boundaries Γ_D.

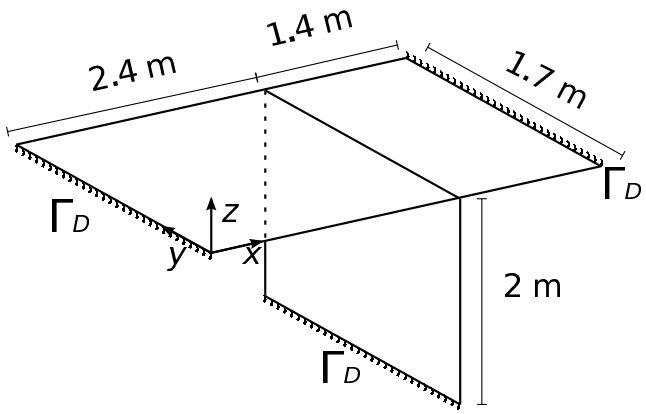


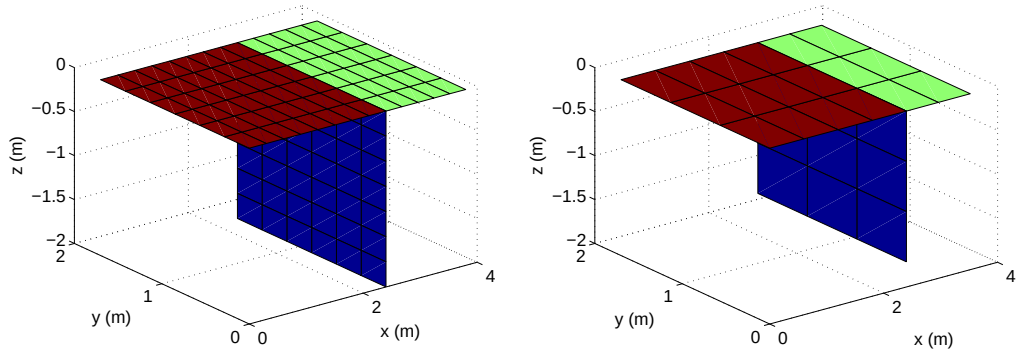
Figure 3: Sketch of the T-shaped structure.

Variable	Symbol	Value
Young's modulus	E	30 GPa
Density	ρ	2400 kg m^{-3}
Poisson's ratio	ν	0.2
Thickness	h	50 mm

Table 1: Properties of the adjacent plates.

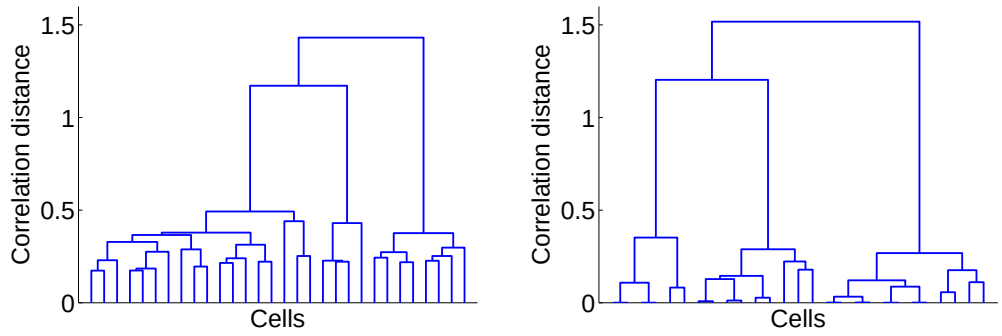
In this example, 37 eigenmodes associated to frequencies located between 1100 and 1500 Hz are considered. Preliminary numerical tests with different amounts of modes indicate that this amount provides a good compromise between computational cost and representativity of the set of modes. In Fig. 4, the optimal decomposition

both for cells of size $\lambda_{\text{plate}}/2$ and λ_{plate} is shown, where λ_{plate} is the wavelength of the first mode used in the analysis. Fig. 5 shows the dendrograms associated to both decompositions. They show clearly that the optimal decomposition consists of three subsystems in both cases.



(a) Cell size: $\lambda_{\text{plate}}/2$ (b) Cell size: λ_{plate}

Figure 4: Optimal subsystem distribution for the T-shaped structure.



(a) Cell size: $\lambda_{\text{plate}}/2$ (b) Cell size: λ_{plate}

Figure 5: Dendrogram of the analyses for the T-shaped structure.

If the same analysis is performed using the Euclidean distance instead of the correlation one, the resulting dendrogram of the example with cell size $\lambda_{\text{plate}}/2$ is shown in Fig. 6. The chosen subdivision is the same, but the choice is clearer with the correlation distance, since the option of three subsystems represents 47.5% of the possible distances, while for the Euclidean distance, this range is only of 33.3%.

4.2 Different types of modes in the same region: vibratory systems consisting of thick plates

The performance of the methodology presented in Section 3 is tested here for the example of the L-shaped structure shown in Fig. 7. The material properties of the

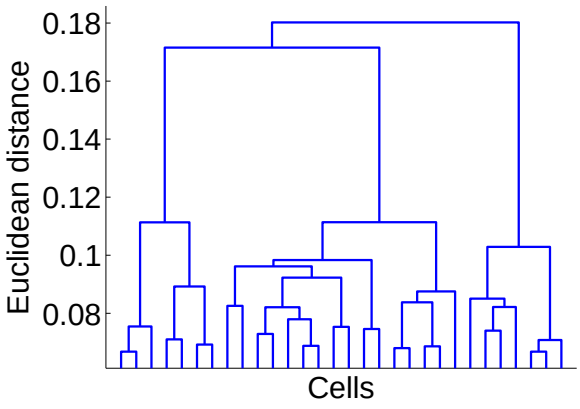


Figure 6: Dendrogram of the analysis for the T-shaped structure using Euclidean distance.

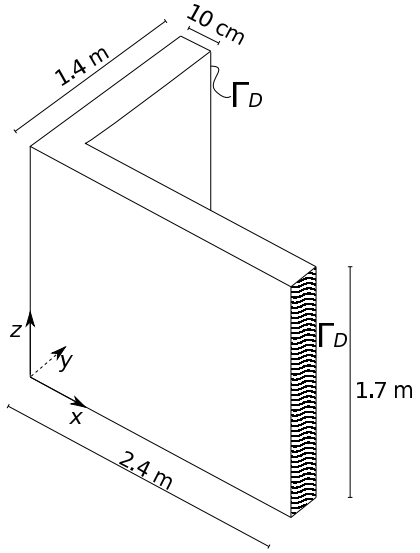


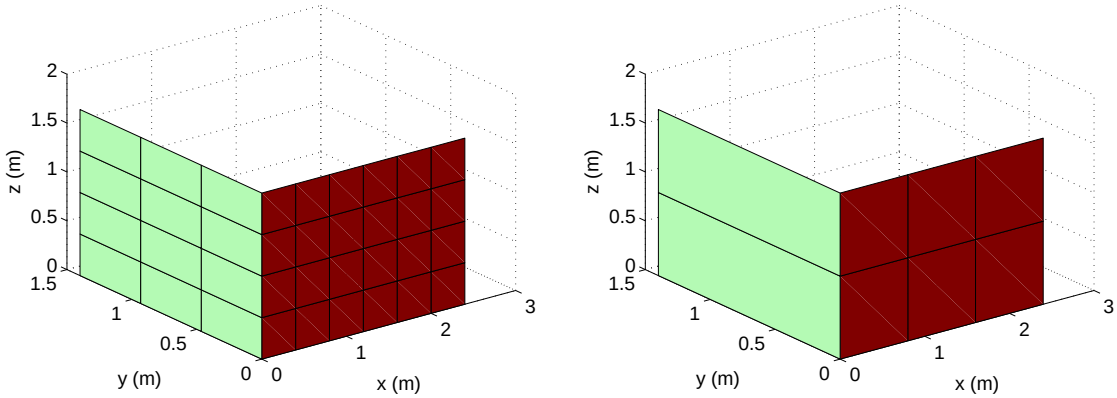
Figure 7: Sketch of the thick L-shaped structure.

structure are the same as in Table 1, but the thickness of the plates has been changed.

For this case, 25 modes between 1100 and 2000 Hz are selected. The normal and in-plane kinetic energy maps are obtained for each of them. After comparing the two types of energies for every mode, 19 of them (76%) are classified as normal modes and 6 as in-plane modes (24%). Due to the non-negligible amount of in-plane modes, it is considered that there is at least one in-plane subsystem.

Once the two types of modes are differentiated, the analysis described in Section 2 is performed with the 19 normal modes. The optimal number of subsystems for the normal modes is 2, as can be seen in Fig. 8.

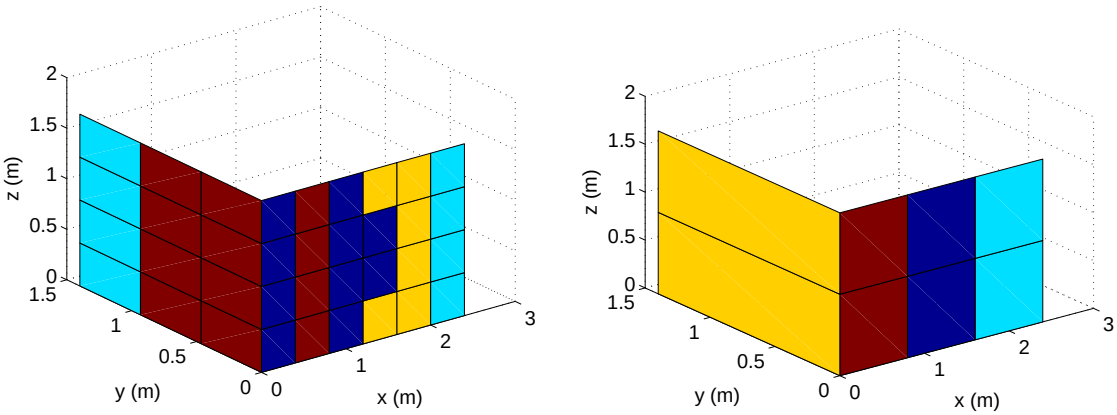
The analysis of Section 2 is also done for the 6 in-plane modes. In this case, the identified regions are not the same for the two different cell sizes, see Fig. 9.



(a) Cell size: $\lambda_{\text{plate}}/2$ (b) Cell size: λ_{plate}

Figure 8: Optimal subsystem distribution for the transverse modes of the thick L-shaped structure.

This strongly suggests that there is only one subsystem. The decision of the optimal amount of subsystems must be also based on the strength of the coupling.



(a) Cell size: $\lambda_{\text{plate}}/2$ (b) Cell size: λ_{plate}

Figure 9: Optimal subsystem distribution for the in-plane modes of the thick L-shaped structure.

Therefore, the global system consists of, at least, three subsystems: two for the normal (flexural) modes of the two plates, and another one (or more) for the in-plane modes of the system.

4.3 Vibroacoustic systems

The same type of analysis has been used for identifying SEA subsystems in vibroacoustic systems. In this case, the considerations related to the types of energy used for the analysis made in Section 2.2, must be taken into account.

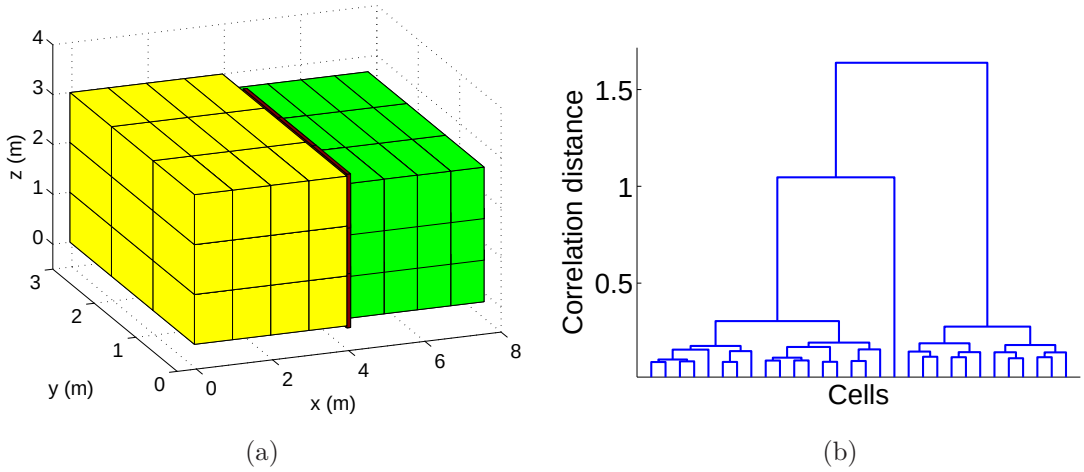
4.3.1 Single wall

The first example considered consists of two rooms divided by a thin wall. The dimensions of the rooms are 4 m × 3 m × 3 m and 3.5 m × 2.8 m × 2.5 m. The wall measures 2 m × 2.5 m and its main properties are summarised in Table 2. Due to the coupling of the structural and the acoustic domains, the amount of modes increases dramatically. For this analysis 939 coupled modes between 500 Hz and 700 Hz have been used.

Variable	Symbol	Value
Young's modulus	E	29.4 GPa
Density	ρ	2500 kg m^{-3}
Poisson's ratio	ν	0.25
Thickness	h	10 cm

Table 2: Properties of the wall.

For this problem, the optimal subdivision obtained with the method coincides with the three basic elements of the problem: the two rooms and the wall, see Fig. 10.



(a) (b)

Figure 10: Optimal subsystem distribution for two rooms divided by a single wall.

4.3.2 Double wall

The second vibroacoustic example corresponds to two rooms divided by a double wall. This double wall consists of two thin walls with an air cavity in between, as shown in Fig. 11.

The properties of the wall leaves are summarised in Table 3. The thickness of the two leaves are 13 and 26 mm respectively. They both measure 2.4 m × 2.4 m. The dimensions of the two rooms are 4 m × 3 m × 3 m and 3.5 m × 2.8 m × 3 m respectively. Different values of air cavity thickness have been considered, all of them leading to equivalent conclusions. 50 cm and 7 cm are used here to obtain the results.

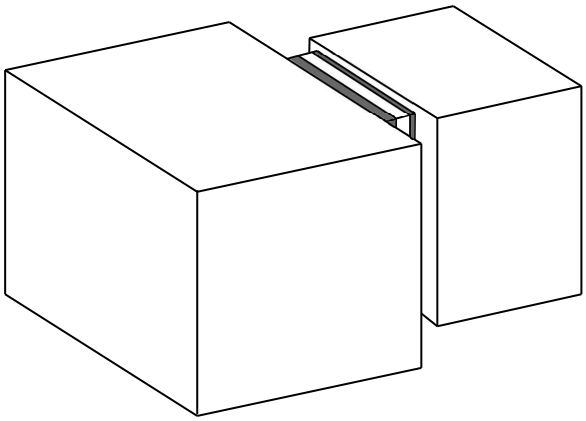


Figure 11: Sketch of the two rooms divided by a double wall.

All the modes associated to eigenfrequencies between 500 and 700 Hz have been used for the analysis. As an example, for the 50 cm thick cavity this implies an amount of 734 modes. In order to check the representativity of this frequency band, the same analysis has been performed with the 7303 modes located between 15 Hz and 2000 Hz, obtaining the same subdivision.

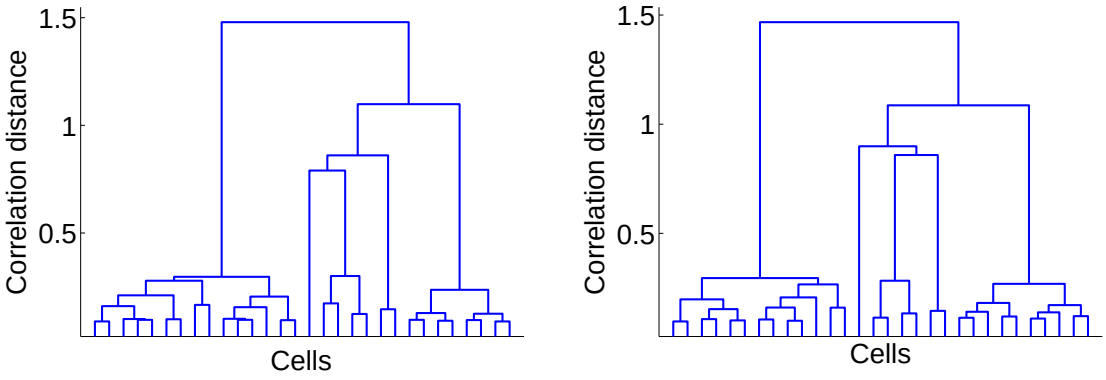
Variable	**Symbol**	**Value**
Young's modulus	E	2.5 GPa
Density	ρ	$692.3\,\mathrm{kg\ m^{-3}}$
Poisson's ratio	ν	0.25

Table 3: Properties of the wall leaves.

For these examples, the analysis of the dendrograms of Fig. 12 leads to an optimal decomposition into five subsystems for the two different values of the cavity thickness. These subsystems correspond, as expected, to the five basic elements of the system (two rooms, two plates and the air cavity).

The similarity of both dendrograms is a sign of the robustness of the method. For both examples the possible subdivisions into two and three subsystems coincide. The small diferences between both dendrograms are caused by the different suggested subdivisions into four subsystems: for the 70 mm thick cavity, this subdivision considers both leaves as one subsystem and, for the 50 cm thick cavity, the cavity comes together with one of the leaves, and the other leaf is a subsystem in itself. However, the subdivision into four subsystems is highly unstable, as its small distance range indicates, having almost no effect in the final choice.

As a comparison, the mutual inertia ratio introduced by Totaro and Guyader [15] has been computed for the different subdivisions provided by the cluster analysis. This parameter provides a measure of the coupling between clusters and is used in [15] for defining the optimal amount of clusters. It has been adapted here to the particular features of this approach by defining the distances required in the computation as the correlation distances between the energy vectors of the elements. Taking this into



(a) 50 cm thick cavity

(b) 7 cm thick cavity

Figure 12: Dendrogram for two rooms divided by a double wall.

account, Table 4 shows the maximum value of the mutual inertia ratio (MIR) for the different decompositions provided by the cluster analysis, ranging between 2 and 6 clusters.

Amount of subsystems	**50 cm**	**7 cm**
2	1.47	0.37
3	0.46	0.41
4	1.74	0.33
5	0.19	0.11
6	0.19	2.63

Table 4: Value of the MIR for the different amounts of subsystems and two different cavity thicknesses.

Results show that the MIR is an interesting parameter, able to measure the increment in the coupling occurring when moving from five subsystems to four, or from three (room - double wall - room) to two. However, it does not provide different results for the case of a cavity with a thickness of 50 cm and the subdivisions into five or six subsystems. It is coherent with the MIR definition that accounts for the most critical relationship between subsystems. This can be independent on the division of the other subsystems. In this case, the methodology based on the dendrograms provides a clearer and more reasonable decomposition.

It is also worth mentioning here that, if the correlation distance is replaced by the Euclidean distance for this specific problem, the algorithm provides wrong results, leading to a configuration in which a room is divided into two subsystems for the case of five clusters.

5 Conclusions

In this work a methodology for identifying SEA subsystems has been presented. The main conclusions associated to this research are the following:

- A cluster analysis based on the system eigenmodes can be used to subdivide a system into subsystems. The obtained subsystems are good candidates to be used in the SEA framework both for purely mechanical and vibroacoustic problems.

- The correlation distance is a good measure of the differences between the energetic behaviour of the different cells forming the domain.

- The use of a set of eigenmodes as samples for the cluster analysis avoids the need of projecting the results in their principal components. This leads to a lower amount of calculations compared to an excitation-dependent approach and, therefore, a lower computational cost.

- The dendrogram associated to the cluster analysis can be used to select the optimal amount of clusters. This information is clearer than the one provided by the mutual inertia ratio.

- The information provided by the modal analysis gives the possibility of classifying the modes of the system before doing the cluster analysis. In this way, modes associated to different types of waves can be treated separately. This preprocessing of the modes allows the detection of different subsystems sharing the same physical region.

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