Hamiltonian actions constitute a central object of study in symplectic geometry. Special attention has been devoted to the toric case. Toric symplectic manifolds provide natural examples of integrable systems and every integrable system on a symplectic manifold is a toric manifold in a neighbourhood of a compact fiber (Arnold–Liouville). The classification of toric symplectic manifolds is given by Delzant’s theorem in terms of the image of the moment map (Delzant polytope).

The goal of this talk is to present a comprehensive tour through different results concerning Hamiltonian group actions on Poisson manifolds (a generalization of symplectic manifolds) with an emphasis on the classification problems and the study of their rigidity.

We will first consider a simple class of Poisson manifolds which is close to the symplectic realm ($b$-symplectic manifolds) and sketch a proof of a Delzant theorem for toric $b$-symplectic manifolds taking as starting point the case of surfaces. These Poisson manifolds can be seen as symplectic manifolds with singularities and admit an adaptation of Moser’s path method from symplectic geometry.

Time permitting, we will end up this talk with a rigidity theorem for Hamiltonian group actions on Poisson manifolds. The tools needed to consider general Poisson manifolds require a generalization of Nash–Moser’s techniques native to geometric analysis.