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Incorporating Working Conditions to a Mixed-Model Sequencing Problem

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Abstract: Beginning with a variation of the sequencing problem in a mixed-products line (MMSP-W: Mixed-Model Sequencing Problem with Workload Minimization), we propose two new models that incorporate the working conditions into the workstations on the line. The first model takes into account the saturation limit of the workstations, and the second model also includes the activation of the operators throughout the working day. Two computational experiments were carried out using a case study of the Nissan motor plant in Barcelona with two main objectives: (1) to study the repercussions of the saturation limit on the decrease in productivity on the line and (2) to evaluate the recovery of productivity on the line via activation of operators while maintaining the same quality in working conditions achieved by limiting the saturation. The obtained results show that the saturation limitation leads to suppose an important increase of work overload, which means average economic losses of 28,731.8 euros/day. However, the consideration of activity reduces these losses by 62.7%.

Keywords: Sequencing operations; mixed-products lines; Work overload, Saturation, Activity factor; MMSP-W

1 Preliminaries

Currently, many production systems exist in which the manufacture or assembly of an entire product (or a subcomponent of the product) is carried out on the production line. At the same time, the increasing market requirements demand that companies offer a wide range of products with different options. This situation is commonly found in the automotive industry in which different products are manufactured and belong to the same family but have variable characteristics that require different component consumption and resource use. Obviously, not all vehicles share the same type of motor, and not all are equipped with the same components.

One clear example of this type of mixed-product line, also known as Mixed-Model Assembly Line (*MMAL*), is found in the engine lines or in the assembly lines in which the different components (seats, steering wheels, pedals, etc.) are incorporated into the vehicle body. The variety in this type of product requires that production lines and assembly lines are flexible and able to adapt to the different types of products assembled without incurring excessive costs.

Therefore, to increase flexibility and reduce costs in terms of both workforce and storage, the assembly lines for mixed products face two basic problems: (1) the balance of the line which is known in the literature, Salveson (1955), as an Assembly Line Balancing Problem (*ALBP*) and exists in many variations (Becker and Scholl (2006); Battaïa and Dolgui (2013)); and (2) the sequencing of mixed products in production lines and workshops.

In problem number two, we classify the problems depending on the variability in the processing times of the operations required to produce or assemble different types of products. Our ideology contains the following classifications:

(i) Permutation problems, i.e., Flow-Shop. This type of problem applies if the processing times of the units are heterogeneous in the stages of the production process in a workshop (Bautista, Cano, Companys et al. (2012), Pan and Ruíz (2013)).

(ii) Economic Lot Scheduling Problems (ELSP). Because it is often convenient to sequence the units in batches, the processing time of any operation depends on the number of units that make up a given batch of pieces. It is necessary to determine the size of the lots, which depends on the balance between the launching costs of the line and the holding costs for the stock of parts (Elmaghraby (1978), Raza and Akgunduz (2008)).

(iii) Mixed-Model Sequencing Problems (MMSP). In this type of problem, the processing times of the mixed products are homogeneous during the stages of the production process. The aim is to establish a manufacturing order for the products (and this order must be maintained much as possible). These problems appear in the supply chain of production systems governed by the Just In Time (JIT, Toyota) and Douki-Seisan (DS, Nissan) ideologies. The literature contains various studies dedicated to these problems (Solnon et al. (2008), Boysen et al. (2009a,b), Bautista and Cano (2011)).

From our point of view, the *MMSP* can be classified according to the optimization criterion that affects one or more elements of the production system:

a. Minimization of the level of stock of products and components. This category contains the Product Rate Variation Problem (*PRVP*) proposed by Miltenburg (1989), the purpose of which is to minimize the variation of production rates as well as the Output Rate Variation Problem (*ORVP*) proposed by Monden (1983), the purpose of which is to minimize the variation in the component consumption rates.
b. Minimization of the work overload. Obviously, in the sequencing of mixed products, certain units will require selected workstation processing times that are greater than the average values. The cycle time that corresponds to the time assigned to any line workstation for processing of any type of product is determined from those average values. When the processor in a workstation does not have sufficient time to complete work on the task of the product that has been assigned, a work overload appears. Without extra effort, this situation ends up generating backlog. In this case, the objective is to minimize the uncompleted work, which is also known as work overload. One example of this type of problems is the Mixed-Model Sequencing Problem with Workload Minimization (*MMSP-W*) both the original version, Yano and Rachamadugu (1991), and its variants Bautista et al. (2012a,b).
c. Minimization of the number of subsequences with special options. These problems are focused on avoiding blockages caused by products that require additional work by offering special options. This problem is known as the Car Sequencing Problem (*CSP*) and was first described by Parrello et al. (1986).
d. Minimization of the cost of production processes with meeting customer demand (Lin and Chu (2013,2014)).

In this article, we focus on the (iii)-(b) category of problems and thus address the sequencing problems of mixed products in the production lines with the objective of minimizing the work overload (i.e., *MMSP-W*: Mixed-Model Sequencing Problem with Workload Minimization).

Although this type of problem has been widely treated in the literature, certain specific aspects of the human resources involved in the production system have not yet been addressed.

In this work, the processors contain automated systems and operators. The operators are subjected to working conditions defined according to laws, rules, and contracts and also negotiations between the company and the workers´ representative. These conditions affect certain job characteristics, i.e., the length of working days, the saturation and occupancy rates of the processors, and the normal activity level of the operators.

In this article, our purpose is to incorporate certain working conditions into the problems known by the acronym *MMSP-W*.

This work is structured as follows. In the Section 2, we describe and illustrate the *MMSP-W* problem and certain of its variants. In Section 3, we analyze the working days and the job saturation line, and we distinguish between static and dynamic saturation. In Section 4, we formulate a new model known as $M4\cup3\_\eta$ from a variant of *MMSP-W,* which incorporates restrictions of dynamic saturation in the line workstations as a function of the product sequence. In Section 5, we develop the experimentation of this model based on a case study of the Nissan motor plant in Barcelona that measures the impact caused by the limitation of dynamic saturation in the stations over increasing the global work that has not been completed. In Section 6, we propose a series of corrective measures designed to reduce the global work overload. One of these corrective measures, i.e., the activation of the processors, is described in the Section 7. We consider this measure to formulate a new model $M4\cup3\_\dot{\alpha}I\_\eta$ as a result of the natural extension of the $M4\cup3\_\eta$ model. In Section 8, we evaluate the $M4\cup3\_\dot{\alpha}I\_\eta$ model. We use the same data as in Section 5 in the computational experiments with the objective of achieving a balance between productivity and ideal working conditions. Finally, we dedicate Section 9 to the conclusions and proposals for future work.

2 The MMSP-W

The *MMSP-W* consists of establishing the manufacturing order of T units of a set I of product types in an assembly line composed of a set K of workstations arranged in series. Each unit of product type i $(i=1,\dots,|I|)$ requires from each homogeneous processor (operator, robot…) of a workstation k $(k=1,\dots,|K|)$ a processing time $p_{i,k}$ measured at normal activity (α^N). Each processor has a normal working time. This time, known as the cycle time (c), is the standard time for which each processor is available to work on a product unit. Occasionally, to complete the work, an unit can be retained in the workstation k $(k=1,\dots,|K|)$ for a short time. This time is referred to as the time window or temporal window (l_k) and is longer than the cycle time $(l_k-c>0)$; therefore, it reduces the time available to work on the next unit of the sequence. When the temporal window is not sufficient to complete the entire work required, the work overload is generated.

In this way, the objective of the problem is to minimize the work overload or maximize the work completed by taking into consideration the variation in the processing times of the operations and assuming that the cycle time of the processors is determined based on the average of the times and demands of all product types.

This objective can be achieved in two ways, first, by minimizing the work that has not been completed (work overload) by the processors, which ensures that most of the work required is achieved, and second, by minimizing the downtime (idle time) of the processors.

Fig. 1 shows the effect produced in a workstation by three sequences of six units of two types of products (*A* with high load and *B* with low load). The first sequence (*AAABBB*) generates work overload, and the second sequence (*AABBBA*) generates idle time, whereas the third sequence (*ABABAB*) produces neither of this effects.

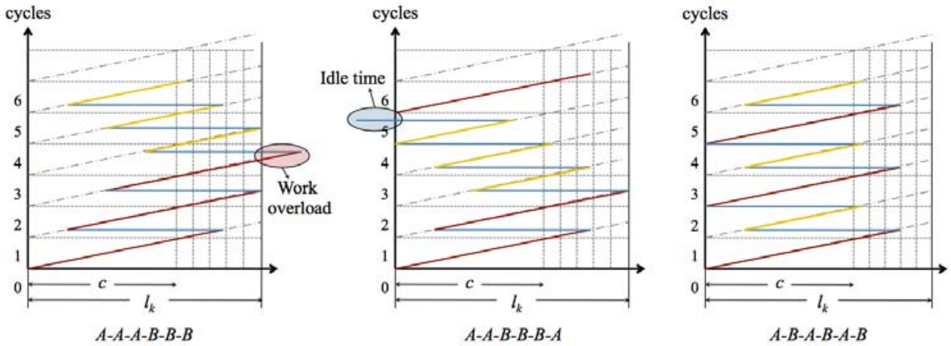

Fig. 1. Work overload, idle time, and completed work as a function of the sequence.

Many variants of the *MMSP-W* have been studied; including the $M1$ model proposed by Yano and Rachamadugu (1991) and the $M2$ model proposed by Scholl et al. (1998). From these models in which no links between stations are contemplated, Bautista and Suárez (2009) formulated two new models, i.e., $M3$ and $M4$, with links between stations and the assignment of a maximum time equal to l_k for all units of products. Next, Bautista et al. (2011) extended the $M4$ model to obtaining the $M4'$ model and, in addition to minimizing the work overload; to use the relative start instants; to consider the temporal window in all of the workstations for all products and links between workstations; this model takes into account that a workstation can be viewed as more than a homogenous processor. Finally, Bautista et al. (2012b) formulated two new equivalent models, the $M3\cup 4$ model and the $M4\cup 3$ model.

To summarize, Table 1 lists the differences between the models mentioned above.

	$M1$	$M2$	$M3$	$M4$	$M4'$	$M3\cup 4$	$M4\cup 3$
Objective function	$\max V$	$\min W$	$\max V$	$\min W$	$\min W$	$\max V$ / $\min W$	$\max V$ / $\min W$
Start instants	$s_{k,t}$	$\hat{s}_{k,t}$	$s_{k,t}$	$\hat{s}_{k,t}$	$\hat{s}_{k,t}$	$s_{k,t}$	$\hat{s}_{k,t}$
Process variables	$v_{k,t}$	$w_{k,t}$	$v_{k,t}$	$w_{k,t}$	$w_{k,t}$	$v_{k,t}$, $w_{k,t}$	$v_{k,t}$, $w_{k,t}$
Temporal Window $t=T$	$l_k\ \forall k$	$c\ \forall k$	$l_k\ \forall k$	$l_k\ \forall k$	$l_k\ \forall k$	$l_k\ \forall k$	$l_k\ \forall k$
Range for b_k	$b_k \geq 1$	$b_k = 1$	$b_k \geq 1$	$b_k = 1$	$b_k \geq 1$	$b_k \geq 1$	$b_k \geq 1$
Seasonable links	No	No	Yes	Yes	Yes	Yes	Yes

Table 1. Comparative study of the literature models.

To differentiate each model, we use the following characteristics: (1) the objective function with the minimization of the work overload (W) or the maximization of the completed work (V); (2) the variables associated with the start instants of the operations on absolute ($s_{k,t}$) and relative ($\hat{s}_{k,t}$) scales; (3) variables associated with the completed processing times, which are:

the applied processing time at normal activity ($v_{k,t}$) and the generated work overload ($w_{k,t}$); (4) the maximum time that the processors of workstations can work on the last product unit ($t = T$); (5) the range of the number of processors (b_k) by workstation; (6) the consideration of links between serial stations.

None of the above models contemplates such working characteristics as the level of occupation or saturation of the processors and the workers' activity.

3 Conditions of the Workday and Saturation

At this point we analyze the diary organization of the workday in the automotive industry to determine the productive working time available within the legal limits.

The working time must be understood according to Article 2 of the 2003/88/CE Directive of the European Parliament and the Council of 4 November 2003 concerning certain aspects of the organization of working time, defined as "Any period during which the worker is working, at the employer's disposal and carrying out his activity or duties, in accordance with national legislation and / or national practice ".

The duration of the working day is decided by the collective agreements or work contracts, according to Article 34 of the Statute of the Workers´ Rights. In general terms, after the analysis of various collective agreements in the automotive industry and at Nissan, we can establish that the work schedule with a horizon of one year is segmented into numbered weeks (52 or 53), and the number days considered as working days is approximately 227. The daily working time might be eight hours from Monday to Friday organized into three shifts that we describe as follows: from 6:00 to 14:00, from 14:00 to 22:00, and from 22:00 to 6:00 (not including the special shifts that can be set for weekends).

We next determine the useful work time within a working day by analyzing the non-productive work time:

- A total of 18 minutes for snack break.
  Both Article 4 of the 2003/88/CE Directive of the European Parliament and the Council of 4 November 2003 and Article 34 of the Statute of the Workers Rights recognize that if the duration of the working day exceeds 6 hours, a rest period should be established. This rest period is known as the snack break, which does not count as effective work time if it has not been established in this manner in the collective agreements or work contracts, according to Article 34.4 of the Statute of the Workers Rights.
- A total of 10 minutes for a scheduled pause after the snack break.
  This pause, which is added to the 18 minutes of the snack break, increases the rest period to a total of 28 minutes. This pause is applied to cover the fatigue that derives from the type of work that the worker performs and/or a supplement for personal needs.
- A pause of 0 to 14 minutes that workers choose according to their own criteria to cover fatigue and personal needs.
- Three scheduled pauses of 10 minutes, each of which is distributed throughout the working day for a total of 30 minutes.
- Three minutes intended for preparation of materials and tools.

- From 0 to 30 minutes of downtime.
  Within the downtime, we distinguish between stoppages that are payable, i.e., lack of work, forced waits, allowed movements, and breakdowns, and non-payable stoppages. Within the non-payable stoppages, we find causes that are directly attributable to the worker and to force majeure.

In total working with the minimums limits and maximums analyzed for the non-productive time, we calculate between 1 hour and 1 minute as the minimum non-productive time and 1 hour and 45 minutes as the maximum non-productive time. These limits translate into a minimum limit for useful time of work of 6 hours and 15 minutes and a maximum limit of 6 hours and 59 minutes within a working day of 8 hours.

The cycle time (c) is determined by establishing the effective hours of the workday (H) and the number of units (T) for manufacture through that workday. The cycle time is the time allowed for any workstation to process any operation, and thus, $c = H/T$.

The saturation to which processors (operators) are subjected throughout the workday in a production line is one of the conditions that the major automotive companies negotiate with the workers' representatives. European companies usually set limits for the maximum saturation $\left(\eta_{\max}^{\infty}\right)$ and the average saturation $\left(\eta_{med}^{\infty}\right)$.

We define the maximum saturation as the proportion of time used by the operation (workload) with greater additional processing time at a certain time of the workday with respect to the cycle time available to perform that operation. We define the medium saturation as the proportion of the required time for the entire work operation with respect to the time available to complete it. If we use work paces between the normal activity and 10% excess, the usual limits for these saturations would be $\eta_{\max}^{\infty} = 1.2$ y $\eta_{med}^{\infty} = 0.95$. It is necessary to distinguish between static saturation and dynamic saturation to determine if the saturation conditions are respected.

The static saturation is determined by a cycle time (c), a matrix of the processing times (at normal activity) of the operations $\left(\mathrm{P} := \left(p_{i,k}\right) : i \in I, k \in K\right)$, and a demand plan $\left(\vec{d} = \left(d_1, \ldots, d_{|I|}\right)\right)$ composed of T units of mixed products.

We also distinguish between medium static saturation and maximum static saturation.

The medium static saturation, $\eta_{med}^{\circ}\left(k, c, \vec{d}, \mathrm{P}\right)$, is associated with the workstation $k \in K$ the cycle time (c), the demand plan $\left(\vec{d}\right)$, and the set of process times (P). We define the medium static saturation as the proportion of time required by each processor to complete the required work with respect to the time available:

$$\eta_{med}^{\circ}\left(k, c, \vec{d}, \mathrm{P}\right) = \frac{1}{c \cdot T} \cdot \sum_{i=1}^{|I|} p_{i,k} \cdot d_i \qquad \left(k = 1, \ldots, |K|\right) \qquad (1)$$

The maximum static saturation, $\eta^{\circ}_{\max}(k,c,\mathrm{P})$, is associated with the workstation $k \in K$, the cycle time (c), and the set of processing times (P). We define the maximum static saturation as the proportion of time that requires each processor to complete the most laborious operation with respect to the cycle time:

$$\eta^{\circ}_{\max}(k,c,\mathrm{P}) = \frac{1}{c} \cdot \max_{i \in I}\{p_{i,k}\} \qquad (k = 1,\ldots,|K|) \qquad (2)$$

If we take into consideration the conditions established between the company and the workers´ representative, the following equation should be satisfied:

$$\eta^{\circ}_{med}\left(k,c,\vec{d},\mathrm{P}\right) \le \eta^{\infty}_{med} \qquad (k = 1,\ldots,|K|) \qquad (3)$$

$$\eta^{\circ}_{\max}(k,c,\mathrm{P}) \le \eta^{\infty}_{\max} \qquad (k = 1,\ldots,|K|) \qquad (4)$$

On the other hand, the dynamic saturation of the workstations depends on the sequence $(\pi(T) = \{\pi_1,\ldots,\pi_T\})$ for manufacturing of the products, the cycle time (c), and a matrix of times (at normal activity) corresponding to the work completed $(\mathrm{V} := (v_{k,t}) : k \in k,\, t = 1,\ldots T)$ at each workstation and position in the sequence. The medium and maximum dynamic saturation gives the following equations:

$$\eta^{\circ}_{med}(k,c,\pi(T),\mathrm{V}) = \frac{1}{c \cdot T} \cdot \sum_{t=1}^{|T|} v_{k,t} \qquad (k = 1,\ldots,|K|) \qquad (5)$$

$$\eta_{\max}(k,c,\pi(T),\mathrm{V}) = \frac{1}{c} \cdot \max_{1 \le t \le T}\{v_{k,t}\} \qquad (k = 1,\ldots,|K|) \qquad (6)$$

Therefore, to limit the medium and maximum dynamic saturation in the line workstations, we add to the *MMSP-W* models the following restrictions:

$$\sum_{t=1}^{|T|} v_{k,t} \le \eta^{\infty}_{med} \cdot c \cdot T \qquad (k = 1,\ldots,|K|) \qquad (7)$$

$$v_{k,t} \le \eta^{\infty}_{\max} \cdot c \qquad (k = 1,\ldots,|K|) \qquad (8)$$

Although many precautions related to the design of the methods and measured times could be applied by the *MTM* (Methods Time Measurement) system, it is possible that we will find a case in which the limit values of saturation are exceeded. This situation would be caused by a demand plan with a highly demanding load or an inappropriate fabrication sequence.

Violation of the maximum saturation is considered unacceptable and requires review by the department of times and measures to search for alternatives in the assembly process that will reduce processing times. This situation does not occur with medium saturation.

It is a fact that medium saturation, either static or dynamic, increases the admissible η_{med}^{∞} limit value in certain workstations, and this situation reflects that the processors will not have sufficient time to complete the required work. Therefore, a work overload will be generated, with a value that will grow against the reduction of the value η_{med}^{∞}.

On the one hand, the static work overload associated with each processor of the workstation $k \in K$ can be determined in the following manner:

$$\omega_0\left(k,c,\vec{d},\mathrm{P},\eta_{med}^{\infty}\right)=c\cdot T\cdot\max\left\{0,\ \eta_{med}^{\circ}\left(k,c,\vec{d},\mathrm{P}\right)-\eta_{med}^{\infty}\right\} \qquad \left(k=1,\ldots,|K|\right) \tag{9}$$

Therefore, to determine the static work overload of the line, we must add the static work overloads of the line workstations multiplied by their processors $\vec{b}=(b_1,..,b_{|K|})$:

$$W_0\left(c,\vec{d},\vec{b},\mathrm{P},\eta_{med}^{\infty}\right)=\sum_{k=1}^{|K|}b_k\cdot\omega_0\left(k,c,\vec{d},\mathrm{P},\eta_{med}^{\infty}\right) \tag{10}$$

On the other hand, the dynamic work overloads associated with the workstations and the line are determined in the following manner:

$$\omega(k,c,\pi(T),\mathrm{V},\eta_{med}^{\infty})=c\cdot T\cdot\max\left\{0,\eta_{med}\left(k,c,\pi(T),\mathrm{V}\right)-\eta_{med}^{\infty}\right\} \qquad \left(k=1,\ldots,|K|\right) \tag{11}$$

$$W\left(c,\vec{b},\pi(T),\mathrm{V},\eta_{med}^{\infty}\right)=\sum_{k=1}^{|K|}b_k\cdot\omega(k,c,\pi(T),\mathrm{V},\eta_{med}^{\infty}) \tag{12}$$

Obviously, the work overload value that derives from the dynamic saturation (W) will always be equal or greater than the work overload that derives from the static saturation (W_0). This situation occurs because we must add the effects produced by the variation of processing times in combination with the sequence in the dynamic situation.

4 MMSP-W with Saturation Constraints

We now formulate a new mathematic model $M4\cup3\_\eta$ by working from the above definitions and the $M4\cup3$ model for the *MMSP-W* presented by Bautista et al. (2012b). The $M4\cup3\_\eta$ model contains the following parameters and variables:

Parameters:

K	Set of workstations $(k=1,\ldots,\|K\|)$.
b_k	Number of homogeneous processors in each workstation k $(k=1,\ldots,\|K\|)$.
I	Set of product types $(i=1,\ldots,\|I\|)$.
d_i	Programmed demand of the product type i $(i=1,\ldots,\|I\|)$.
$p_{i,k}$	Processing time (normal activity) required of each homogeneous processor for one unit of a product type i $(i=1,\ldots,\|I\|)$ in workstation k $(k=1,\ldots,\|K\|)$.

T	Total demand $\sum_{i=1}^{\|T\|} d_i = T$.
t	Position index in the sequence $(t = 1,\dots,T)$.
c	Cycle time; standard time assigned for each homogeneous processor in the workstations $(k = 1,\dots,\|K\|)$ to process any product unit.
l_k	Temporal window; maximum time that each homogeneous processor of workstation k $(k = 1,\dots,\|K\|)$ is allowed to work on any unit of product; once the cycle has been completed, the maximum time that a unit of product can be retained in station k is $l_k - c > 0$.
η_{med}^{∞}	Allowable medium saturation by the processor of each workstation $(k = 1,\dots,\|K\|)$.
$\eta_{\max}^{\infty}$	Allowable maximum saturation by the processor of each workstation $(k = 1,\dots,\|K\|)$.

Variables:

$x_{i,t}$	Binary variable that takes on the value of 1 if the product unit i $(i = 1,\dots,\|I\|)$ is assigned to the position t $(t = 1,\dots,T)$ of the sequence and takes on the value of 0 otherwise.
$\hat{s}_{k,t}$	Positive difference between the real start time and the minimum start time of the t^{th} operation in the workstation k $(k = 1,\dots,\|K\|)$.
$v_{k,t}$	Processing time applied by each homogeneous processor (normal activity) to the t^{th} product unit sequenced in the workstation k $(k = 1,\dots,\|K\|)$.
$w_{k,t}$	Work overload generated in each homogeneous processor measured in units of time at normal activity by the t^{th} product unit sequenced in the workstation k $(k = 1,\dots,\|K\|)$.

$M4 \cup 3\_\eta$ Model:

$$\min W = \sum_{k=1}^{|K|}\left(b_k \sum_{t=1}^{T} w_{k,t} \right) \Leftrightarrow \max V = \sum_{k=1}^{|K|}\left(b_k \sum_{t=1}^{T} v_{k,t} \right) \quad (13)$$

Subject to:

$$\sum_{t=1}^{T} x_{i,t} = d_i \qquad i = 1,\dots,|I| \quad (14)$$

$$\sum_{i=1}^{|I|} x_{i,t} = 1 \qquad t = 1,\dots,T \quad (15)$$

$$v_{k,t} + w_{k,t} = \sum_{i=1}^{|I|} p_{i,k} \cdot x_{i,t} \qquad k = 1,\dots,|K|;\ t = 1,\dots,T \quad (16)$$

$$\sum_{t=1}^{T} v_{k,t} \le \eta_{med}^{\infty} \cdot c \cdot T \qquad k = 1,\dots,|K| \quad (17)$$

$$v_{k,t} \le \eta_{\max}^{\infty} \cdot c \qquad k = 1,\dots,|K|;\ t = 1,\dots,T \quad (18)$$

$$\hat{s}_{k,t} \ge \hat{s}_{k,t-1} + v_{k,t-1} - c \qquad k = 1,\dots,|K|;\ t = 2,\dots,T \quad (19)$$

$$\hat{s}_{k,t} \ge \hat{s}_{k-1,t} + v_{k-1,t} - c \qquad k = 2,\dots,|K|;\ t = 1,\dots,T \quad (20)$$

$$\hat{s}_{k,t} + v_{k,t} \le l_k \qquad k = 1,\dots,|K|;\ t = 1,\dots,T \quad (21)$$

$$\hat{s}_{k,t}, v_{k,t}, w_{k,t} \ge 0 \qquad k = 1,\dots,|K|;\ t = 1,\dots,T \quad (22)$$

$$x_{i,t} \in \{0,1\} \qquad i = 1,\ldots,|I|;\ t = 1,\ldots,T \tag{23}$$

$$\hat{s}_{1,1} = 0 \tag{24}$$

In the $M4\cup3\_\eta$ model, the objective function (13) expresses the equivalence between the minimization of the total dynamic work overload (W) and the maximization of the completed work (V). The constraints (14) represent the satisfaction of the programmed demand. The constraints (15) state that a product unit can only be assigned to one position of the sequence. The set of constraints (16) fix the relationships among the required processing time, the completed work, and the work overload in each workstation and in each moment. The constraints (17) restrict the medium saturation, and the constraints in (18) restrict the maximum saturation in the workstations. The sets (19)-(21) determine the start time of the operations at the workstations. The constraints (22) establish the negativity of the variables. The constraints (23) state that the variables that assign the units to the sequence position must be binaries. Finally, the equality (24) fixes the start time of the operations.

5 Exploitation of the $M4\cup3\_\eta$ Model

Next, we evaluate the effect produced by the limitation of the saturation over the increase in the partial and global work overload. We achieve this result by resolving a study case based on an engine assembly line at Nissan in Barcelona.

We use the $M4\cup3\_\eta$ model to obtain the sequences of the products starting from a set E of 23 instances corresponding to different demand plans. All plans consist of a day´s work divided into two shifts of 8 hours, each of which represents an uptime of 13.125*h* after deducting statutory breaks and rest periods (see Block I of Table 7 in Bautista et al. (2012b)). All plans must satisfy a total demand of $T = 270$ engines.

There are nine types of engines, ($|I| = 9$) with a different processing time $p_{i,k}$ for each one. The nine types of engines are grouped into three families according to the vehicle types: (1) 4x4 vehicles (p_1, p_2 y p_3), (2) vans (p_4 y p_5), and (3) average tonnage trucks ($p_6,\ldots,p_9$).

The engine assembly line is composed of 21 modules or workstations, $|K| = 21$ arranged in series with a unique homogeneous processor in each one ($b_k = 1, \forall k \in K$). This homogeneous processor is equivalent to a team of two workers with identical skills and tools and the same requirements for auxiliary equipment.

We also must consider other elements that are necessary for the experiment. These elements are the effective cycle time, i.e., $c = 175s$, and a temporal window, which is considered identical for all workstations and is equal to $l_k = 195s$. The latter allows a clearance of slightly over 10% of the cycle time to work on a product unit in the sequence at any workstation. Moreover, the limit for the maximum dynamic saturation is considered equal to $\eta^{\infty}_{\max} = 1.2$, and for the medium saturation, this value is $\eta^{\infty}_{med} = 0.95$.

To develop the $M4\cup3\_\eta$ model we use an LP Solver of the Gurobi Optimizer 4.5.0 in an Apple Macintosh iMac computer with an Intel Core i7 2.93-GHz processor, 8 GB of RAM memory, and a MAC OS X 10.6,7 operating system, thus limiting the CPU time for each production plan to 7200 s.

Table 2 shows the results achieved in this experiment and the results for the $M4\cup3$ model that does not consider the saturation constraint.

ε	$k\in K:\eta^{\circ}_{med}(k)\geq\eta^{\infty}_{med}$	$W_{4\cup3}$	W_{0_η}	$W_{4\cup3\_\eta}$	RPD_1
#1	4, 9, 10, 16, 17, 18	187.0	12315.0	12315.0*	6485.6
#2	4, 9, 10, 16, 17, 18	341.0	12458.0	12458.0*	3553.4
#3	4, 9, 10, 11, 16, 17, 18, 21	427.0	12210.0	12210.0*	2759.5
#4	4, 9, 10, 16, 17, 18	310.0	12470.0	12470.0*	3922.6
#5	4, 9, 10, 16, 17, 18, 21	633.0	13012.5	13012.5*	1955.7
#6	4, 9, 10, 16, 17, 18	413.0	12910.0	12910.0*	3025.9
#7	4, 9, 10, 16, 17, 18, 21	742.0	12722.5	12722.5*	1614.6
#8	4, 9, 10, 16, 17, 18	139.0	12018.0	12018.0*	8546.0
#9	4, 9, 10, 16, 17, 18	732.0	13363.0	13363.0*	1725.5
#10	4, 9, 10, 11, 16, 17, 18, 21	1208.0*	13122.0	13122.0*	986.3
#11	4, 9, 10, 11, 16, 17, 18	78.0	11792.5	11792.5*	15018.6
#12	4, 9, 10, 16, 17, 18	284.0	12246.0	12246.0*	4212.0
#13	4, 9, 10, 16, 17, 18	286.0	12551.0	12551.0*	4288.5
#14	4, 9, 10, 16, 17, 18	420.0	12646.0	12646.0*	2911.0
#15	4, 9, 10, 16, 17, 18, 21	433.0	12393.5	12393.5*	2762.2
#16	4, 9, 10, 16, 17, 18	227.0	12363.0	12363.0*	5346.3
#17	4, 9, 10, 16, 17, 18, 21	478.0	12597.5	12597.5*	2535.5
#18	4, 9, 10, 16, 17, 18	605.0	13208.0	13208.0*	2083.1
#19	4, 9, 10, 11, 16, 17, 18, 21	945.0*	12810.0	12810.0*	1255.6
#20	4, 9, 10, 16, 17, 18	139.0	11875.0	11875.0*	8443.2
#21	4, 9, 10, 16, 17, 18	560.0	13065.0	13065.0*	2233.0
#22	4, 9, 10, 16, 17, 18, 21	987.0	13062.5	13062.5*	1223.5
#23	4, 9, 10, 11, 16, 17, 18	140.0	11902.5	11902.5*	8401.8

Table 2. Workstations $(k\in K)$ oversaturated, work overload ($W_{4\cup3}, W_{0\_\eta}, W_{4\cup3\_\eta}$), and percentage increase of the dynamic work overload created by limiting saturation for 23 demand plans $(\varepsilon\in\mathrm{E})$.

In Table 2, the first column of the table represents the number of the production plan ($\varepsilon\in E$). The second column shows the set of workstations that contain oversaturated processors. The third column shows the values of the global work overload $(W_{4\cup3})$ achieved by the $M4\cup3$ model from Bautista et al. (2012b). The fourth column shows the inevitable overload of the line due to the limit of the medium static saturation (W_{0_η}) calculated from equation (10). The fifth column presents the values of the dynamic work overload of the line obtained by the $M4\cup3\_\eta$ model $(W_{4\cup3\_\eta})$, also calculated from the equation (12). Finally, the sixth column shows the increase of the work overload that causes the incorporation of the saturation constraints (RPD_1); in each, the data that have been analyzed and calculated by the following equation:

$$RPD_1(\varepsilon)=\frac{W_{4\cup 3\_\eta}(\varepsilon)-W_{4\cup 3}(\varepsilon)}{W_{4\cup 3}(\varepsilon)}\cdot 100 \qquad (\varepsilon=\#1,\ldots,\#|E|) \qquad (25)$$

Based on Table 2, we observe the following:

a. None of the 23 demand plans violate the limitation of maximum dynamic saturation ($\eta_{\max}^{\infty}=1.2$) imposed on all of the workstations. The $M4\cup 3\_\eta$ model always finds a solution.
b. Workstations 4, 9, 10, 16, 17 and 18 appear in the 23 production plans as work overload receivers or uncompleted work. Although stations 1, 2, 3, 5, 6, 7, 8, 12, 13, 14, 15, 19, and 20 are never overloaded, it also means that none of them increases the limit imposed on the medium static saturation ($\eta_{med}^{\infty}=0.95$).
c. The static work overload of the line (W_{0_η}) coincides with the dynamic overload of the line $(W_{4\cup 3\_\eta})$ in the 23 plans, which indicates two observations: (1) the optimal values (maximums) for the completed work are reached in the 23 plans and (2) none of the 23 sequences of manufacture generate idle time.
d. The incorporation of the medium saturation limit in the workstations ($\eta_{med}^{\infty}=0.95$) generates a relevant increase in the dynamic work overload for all of the demand plans (see column $W_{4\cup 3\_\eta}$) with respect to the dynamic overload values obtained without considering such limitations (see column $W_{4\cup 3}$).
e. The increase in the dynamic work overload (see column RPD_1), by limiting the medium saturation, ranges from 986.3% $(\varepsilon=\#10)$ to 15018.6% $(\varepsilon=\#11)$ and has an average value of 4,143%.
f. The improvement in working conditions organized through the limitation of the medium saturation at the workstations can involve a daily loss of non-completed work whose average value is equivalent to 72 engines with a range of 9 engines. Without this limitation, the daily loss of non-completed work is equivalent to an average of 3 engines and a range of 7 engines.
g. Taking into account that the reduction of the work overload by 175 seconds means the manufacture of one engine and that the production line of engines supposes a consolidated operation profit of 10% over the total value of a motor (4,000 €), i.e., the loss of an engine means a cost for the line of 400 €; the economic losses due to the limitation of the medium saturation are equivalent to an average of 28,731.8 euros/day with a range of 3,589.7 euros/day; without the limitation, their respective values are 1,064.7 euros/day and 2,582.9 euros/day.

Obviously, the reduction of the non-completed working losses derived from the improved working conditions can be achieved by means of agreements that benefit the company and its workers.

6 Measures used to Reduce the Work Overload

We have shown that limitation of processor saturation may result in an increase in workload and an increase in economic losses.
Therefore, keeping in mind the main objective of *MMSP-W* and the obligation to respect the limits of saturation, the following alternatives are proposed to counter the violation of the saturation limits:

1. Consult with the times and methods department to search for alternatives in the assembly process that will reduce processing times. This action is not immediate because it requires an intervention via the engineering of product and process.
2. Increase the activation level of the processors (based on activity factor). Therefore, the assigned work of each of processor will be carried out in less time, and thus, the ratio between the total available time and the real work time will be reduced. In other words, this action would reduce the medium saturation. The medium saturation associated with a processor for the same production plan will be higher or lower depending on whether the activity factor is lower or higher, respectively. This activation may not exceed the optimal work pace factor ($\alpha^*_{i,k} = 1.2 : i \in I, k \in K$) set by the company at any time of the workday.
3. Strengthen the production line to increase its capacity by incorporating auxiliary processors, either polyvalent or not in the system. Thus, the work will focus on assisting all of the oversaturated workstations.
4. Resort to rotation among consecutive stations. Using this approach, the oversaturation of a processor at a workstation may be offset in the medium term with lower saturations of another workstation with higher ergonomic quality. This measure against the excess of medium saturation in the medium or long-term is not permitted in the companies that belong to the Organization for Economic Cooperation and Development (*OCDE*) because they are forbidden from exceeding the medium saturation during the workday. However, this measure could be considered for those countries where no such limit exists.

In this work, we focus on the second measure used to reduce the overload. Therefore, we study the impact produced by the increased activation of the processors in the line with the purpose of reducing the economic losses.

7 Incorporating the Processors Activation into $M4 \cup 3\_\eta$

We have already evaluated the negative effect that the limitation of the saturation has on the productivity of the line. Next, we use the $M4 \cup 3\_\eta$ model to recuperate a portion of the production by means of a slight increase in the work pace (activity) of the operators throughout the workday.

If we increase the activity above the normal value $\left(\alpha^N\right)$, the processing times of the operations will be reduced. Therefore, the dynamic overload and the level of saturation will be reduced.
Large companies set the standard activity with pre-set times in the MTM_100 scale. Moreover, a work pace corresponding to the times in a MTM_110 scale is accepted as normal

activity $\left(\alpha^N = 1\right)$, which means that changing the scale of the processing times $\left(p_{i,k} : i \in I, k \in K\right)$ from MTM_100 to MTM_110 will be sufficient:

$$p_{i,k}(MTM\_110) = p_{i,k}(MTM\_100) \cdot \frac{100}{110} \qquad \forall i \in I,\ \forall k \in K \qquad (26)$$

We simplify the notation in the following manner: $p_{i,k} = p_{i,k}\left(MTM\_110\right) : i \in I, k \in K$. These values correspond to the processing times (pre-set) that require the products in a processor in a workstation when it operates at a rhythm corresponding to an activity factor of $\alpha^N = 1$, i.e., when it operates at normal activity.

In such conditions, if we begin from a set of processing times $\left(\hat{p}_{i,k} : i \in I,\ k \in K\right)$ achieved by observation and timekeeping of the operations in the line workstations, it is easy to determine the work pace of each processor that is necessary to perform the associated job. We represent these work paces (activities) via the activity factors according to the product and the workstation, and we calculate them by imposing the following conditions: $\alpha_{i,k}\,\hat{p}_{i,k} = \alpha^N p_{i,k} : i \in I,\ k \in K$. In conclusion: $\alpha_{i,k} = p_{i,k} \big/ \hat{p}_{i,k} : i \in I,\ k \in K$.

Thus, we associate the activities known as standard, normal, and optimal with the time scales MTM_100, MTM_110, and MTM_132, respectively. These values are, respectively, associated with the activity factors $\alpha^{\circ}_{i,k} = 0.\widehat{9}\widehat{0}$, $\alpha^N_{i,k} = 1.0$ and $\alpha^*_{i,k} = 1.2$ for the $i \in I$ product and the $k \in K$ workstation. It should be noted that the optimal activity $\left(\alpha^*_{i,k} = 1.2\right)$ corresponds to the maximum work pace that an operator can support without damaging his/her health.

On the other hand, the Yerkes-Dodson law demonstrates that up to a certain optimum point of stress, when stress increases, efficiency also increases. When the optimum point is reached, the efficiency decreases drastically. Based on this observation, it can be established that the relationship between the performance of an operator and the level of "activation", as reflected in the level of stress, follows a concave function (Muse et al. (2003)). Taking this observation into account as well as the fact that the activity of the workers can change throughout the workday, we can set temporary functions of the activity factor that offer the possibility of completing the largest amount of work. In fact, Bautista et al. (2014) completed additional work by reducing the processing times of the intermediate units using a step function for the activity factor $\left(\dot{\alpha}^S\right)$, defined as follows (see Fig. 2):

$$\dot{\alpha}_t = \alpha^N \qquad (1 \le t \le t_0) \vee (t_\infty + 1 \le t \le T) \qquad (27)$$

$$\dot{\alpha}_t = \alpha^{\max} \qquad (t_0 + 1 \le t \le t_\infty) \qquad (28)$$

$$\dot{\alpha}_t = \dot{\alpha}_{t-T} \qquad (T + 1 \le t \le T + |K| - 1) \qquad (29)$$

$$\overline{\alpha^S} = \frac{1}{T}\left\{\alpha^N (T - t_\infty + t_0) + \alpha^{\max}(t_\infty - t_0)\right\} \qquad (30)$$

Where:

- t_0 : Completion instant of the first interval of hypo-stress, which corresponds to the adjustment period of the beginning of the work shift.
- t_∞ : Finish instant of the activation period, during which the operator works with a higher rhythm than the normal work pace.
- t_{med} : Instant in which the operator reaches his maximum activation. In this paper, this instant corresponds to the half of the activation period included between t_0 and t_∞.

Note that the value of the activity factor defined in each moment will not be able to overcome the maximum value that is established, such as the optimal value defined by the company $\left(\alpha_{i,k}^{*} = 1.2\right)$.

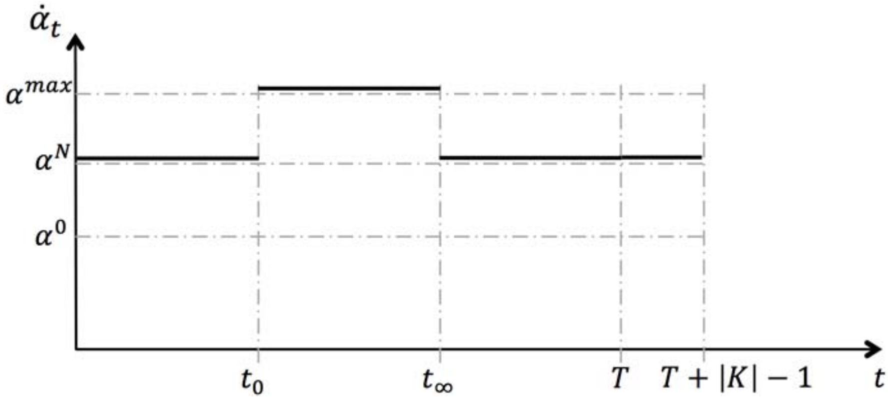

Fig. 2. Step function for the activity factor throughout the workday.

We formulate the $M4 \cup 3\_\dot{\alpha}I\_\eta$ model by taking into consideration the stepped function defined by Bautista et al. (2014) (see Fig. 2). The parameters and the additional variables of the $M4 \cup 3\_\dot{\alpha}I\_\eta$ model are:

Parameters:

$\dot{\alpha}_{k,t}$ Dynamic factor of the work pace or activity associated with the t^{th} operation of the product sequence $(t = 1,\ldots,T)$ at the workstation k $(k = 1,\ldots,|K|)$. Note that $\alpha_{i,k}$ is the activity factor by product and workstation and it not depends on the sequence.

$\dot{\alpha}_t$ Dynamic factor of the work pace or activity associated with the period t $(t = 1,\ldots,T + |K| - 1)$ of the extended workday. This extended workday includes T manufacturing cycles (total demand) and $|K| - 1$ additional cycles, which are required to complete the required work by the production units in all the workstations. If we associate the same dynamic factor with each moment of the workday in all of the workstations, we achieve: $\dot{\alpha}_{k,t} = \dot{\alpha}_{t+k-1}$ $(k = 1,\ldots,|K|;\, t = 1,\ldots,T)$.

Note that if we associate the same dynamic factor to each moment of workday at all workstations, we will have: $\dot{\alpha}_{k,t} = \dot{\alpha}_{t+k-1}$ $(\forall k, \forall t)$.

Variables:

$\hat{v}_{k,t}$	Reduced processing time corresponding to $v_{k,t}$, working with a dynamic factor of activity $\dot{\alpha}_{k,t}$. It is established that $v_{k,t} = \dot{\alpha}_{k,t} \cdot \hat{v}_{k,t} \quad (k = 1,\ldots,\|K\|; t = 1,\ldots,T)$.

$M4 \cup 3\_\dot{\alpha}I\_\eta$ model:

$$\min W = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^{T} w_{k,t} \right) \Leftrightarrow \max V = \sum_{k=1}^{|K|} \left(b_k \sum_{t=1}^{T} v_{k,t} \right) \quad (31)$$

Subjected to:

$$\sum_{t=1}^{T} x_{i,t} = d_i \qquad i = 1,\ldots,|I| \quad (32)$$

$$\sum_{i=1}^{|I|} x_{i,t} = 1 \qquad t = 1,\ldots,T \quad (33)$$

$$v_{k,t} + w_{k,t} = \sum_{i=1}^{|I|} p_{i,k} \cdot x_{i,t} \qquad k = 1,\ldots,|K|;\ t = 1,\ldots,T \quad (34)$$

$$\dot{\alpha}_{t+k-1} \cdot \hat{v}_{k,t} - v_{k,t} = 0 \qquad k = 1,\ldots,|K|;\ t = 1,\ldots,T \quad (35)$$

$$\sum_{t=1}^{|T|} \hat{v}_{k,t} \le \eta_{med}^{\infty} \cdot c \cdot T \qquad k = 1,\ldots,|K| \quad (36)$$

$$\hat{v}_{k,t} \le \eta_{\max}^{\infty} \cdot c \qquad k = 1,\ldots,|K|;\ t = 1,\ldots,T \quad (37)$$

$$\hat{s}_{k,t} \ge \hat{s}_{k,t-1} + \hat{v}_{k,t-1} - c \qquad k = 1,\ldots,|K|;\ t = 2,\ldots,T \quad (38)$$

$$\hat{s}_{k,t} \ge \hat{s}_{k-1,t} + \hat{v}_{k-1,t} - c \qquad k = 2,\ldots,|K|;\ t = 1,\ldots,T \quad (39)$$

$$\hat{s}_{k,t} + \hat{v}_{k,t} \le l_k \qquad k = 1,\ldots,|K|;\ t = 1,\ldots,T \quad (40)$$

$$\hat{s}_{k,t}, v_{k,t}, \hat{v}_{k,t}, w_{k,t} \ge 0 \qquad k = 1,\ldots,|K|;\ t = 1,\ldots,T \quad (41)$$

$$x_{i,t} \in \{0,1\} \qquad i = 1,\ldots,|I|;\ t = 1,\ldots,T \quad (42)$$

$$\hat{s}_{1,1} = 0 \quad (43)$$

The new set of constraints (35) serves to reduce or lengthen the applied processing times depending on the work factor. The remainder of the constraints coincide with the $M4 \cup 3\_\eta$ model, except that, in this case, both the constraints that imitate the saturation of the processors, i.e., (36) and (37) and the ones that determine the start instants of the operations, i.e., (38)-(40), must take into consideration the processing time applied and reduced $\hat{v}_{k,t}$. Obviously, if we consider $\dot{\alpha}_t = 1, \forall t$, the $M4 \cup 3\_\dot{\alpha}I\_\eta$ model is equivalent to the $M4 \cup 3\_\eta$ model.

The terms that determine the medium and maximum statics saturations by taking into consideration the factor matrix of the static activity $\left(\mathrm{A} := (\alpha_{i,k}) : i \in I,\ k \in K\right)$ are:

$$\eta_{med}^{\circ}\left(k, c, \vec{d}, \mathrm{P}, \mathrm{A}\right) = \frac{1}{c \cdot T} \cdot \sum_{i=1}^{|I|} \frac{p_{i,k}}{\alpha_{i,k}} \cdot d_i \qquad \left(k = 1,\ldots,|K|\right) \quad (44)$$

$$\eta^{\circ}_{\max}\left(k,c,\mathrm{P},\mathrm{A}\right)=\frac{1}{c}\cdot\max_{i\in I}\left\{\frac{p_{i,k}}{\alpha_{i,k}}\right\} \qquad \left(k=1,\ldots,|K|\right) \qquad (45)$$

Therefore, the static work overload with activity factors that currently support each one of the processors of the workstation $k \in K$ has the following form:

$$\omega_0\left(k,c,\vec{d},\mathrm{P},\mathrm{A},\eta^{\infty}_{med}\right)=c\cdot T\cdot\max\left\{0,\eta^{\circ}_{med}\left(k,c,\vec{d},\mathrm{P},\mathrm{A}\right)-\eta^{\infty}_{med}\right\} \qquad \left(k=1,\ldots,|K|\right) \qquad (46)$$

And the static work overload of the line is calculated in the following manner:

$$W_0\left(c,\vec{d},\vec{b},\mathrm{P},\mathrm{A},\eta^{\infty}_{med}\right)=\sum_{k=1}^{|K|}b_k\cdot\omega_0\left(k,c,\vec{d},\mathrm{P},\mathrm{A},\eta^{\infty}_{med}\right) \qquad \left(k=1,\ldots,|K|\right) \qquad (47)$$

On the other hand, if we take into consideration the factors of the dynamic activity $\left(\dot{\mathrm{A}}:=(\dot{\alpha}_{k,t}):k\in K,\, t=1,\ldots,T\right)$ and the manufacturing sequence $\pi(T)=\{\pi_1,..,\pi_T\}$, the equations that, respectively, determine the dynamic saturation (medium and maximum) and the dynamic work overload (elemental and global) are:

$$\eta_{med}\left(k,c,\pi(T),\mathrm{V},\dot{\mathrm{A}}\right)=\frac{1}{c\cdot T}\cdot\sum_{t=1}^{|T|}\frac{v_{k,t}}{\dot{\alpha}_{k,t}} \qquad \left(k=1,\ldots|K|\right) \qquad (48)$$

$$\eta_{\max}\left(k,c,\pi(T),\mathrm{V},\dot{\mathrm{A}}\right)=\frac{1}{c}\cdot\max_{1\le t\le T}\left\{\frac{v_{k,t}}{\dot{\alpha}_{k,t}}\right\} \qquad \left(k=1,\ldots|K|\right) \qquad (49)$$

$$\omega(k,c,\pi(T),\mathrm{V},\dot{\mathrm{A}},\eta^{\infty}_{med})=c\cdot T\cdot\max\left\{0,\eta_{med}\left(k,c,\pi(T),\mathrm{V},\dot{\mathrm{A}}\right)-\eta^{\infty}_{med}\right\} \qquad \left(k=1,\ldots|K|\right) \qquad (50)$$

$$W\left(c,\vec{b},\pi(T),\mathrm{V},\dot{\mathrm{A}},\eta^{\infty}_{med}\right)=\sum_{k=1}^{|K|}b_k\cdot\omega(k,c,\pi(T),\mathrm{V},\dot{\mathrm{A}},\eta^{\infty}_{med}) \qquad (51)$$

Therefore, for practical purposes, if we consider that at any time of the working day all processors perform their work at the same work pace, which is set by a temporary function of the activity factor, we can state: $\dot{\alpha}_{k,t}=\dot{\alpha}_{t+k-1}\ (k=1,\ldots,|K|;t=1,\ldots,T)$.

8 Exploitation of the $M4\cup3\_\dot{\alpha}I\_\eta$ Model

Next, we determine the inevitable static work overload of the line and the dynamic work overload according to the $M4\cup3\_\dot{\alpha}I\_\eta$ model. We achieve this goal by working from the same case corresponding to the Nissan engine plant in Barcelona that was used in the exploitation of the $M4\cup3\_\eta$ model. Therefore, we simultaneously consider the limitation of saturation and the activation of the processors. In this experiment, a number of features of the assembly line were taken into account to define the stepped function of the dynamic factor of activity. We state the following:

- Factor of normal activity and maximum allowed: $\alpha^N=1.0$, $\alpha^{\max}=1.1$.
- Shift 1: $t_0=45$, $t_{med}=67$, $t_\infty=90$.

- Shift 2: $t_0 = 180$, $t_{med} = 202$, $t_\infty = 225$.
- Average of the activity factor: $\overline{\alpha^S} = 1.0\hat{3}$.

All workers among the processors will develop their work in a synchronized manner from period to period throughout the workday and will follow the work pace set by the activity factor imposed by the stepped function ($\dot{\alpha}^S$), as shown in Fig. 3.

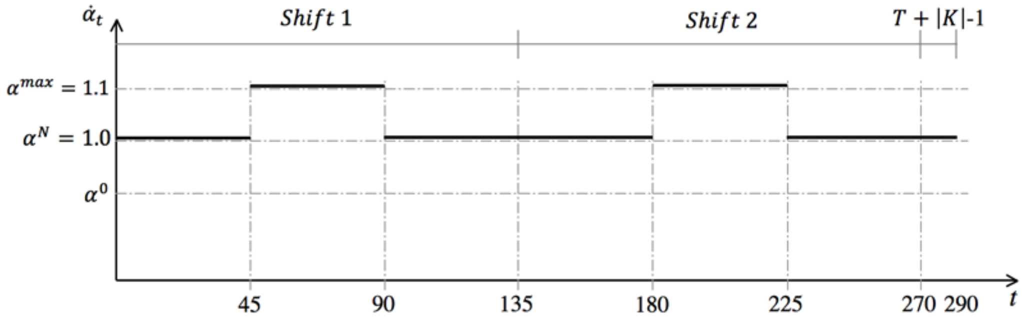

Fig. 3. Stepped function for the work pace factor in the NISSAN case.

The results achieved in this experiment are shown in Table 3. The first column enumerates the 23 plans of production ($\varepsilon = \#1,\ldots,\#23$). The second column determines the statically oversaturated workstations $\left(\eta^{\circ}_{med}(k) \geq \eta^{\infty}_{med} = 0.95\right)$, in each plan. We must take into account that the activity factor follows a step function of which the average value is $\overline{\alpha^S} = 1.0\hat{3}$. Therefore, the third column, $W_0\left(\overline{\alpha^S} = 1.0\hat{3}\right)$ corresponds to the inevitable static work overload (W_0). The $W_{4\cup3\_\dot{\alpha}I\_\eta}$ column contains the values of the dynamic work overload achieved from the $M4\cup3\_\dot{\alpha}I\_\eta$. The columns RPD_2 and RPD_3 correspond to the percentage variances of the work overload given by $M4\cup3\_\dot{\alpha}I\_\eta$ against those given by $M4\cup3$ and $M4\cup3\_\eta$. The RPD_4 column contains the idle times of the demand plans expressed as a percentage of the static work overload when the activity factor follows a step function. Finally, the RPD_5 column contains the reduction in the static work overload achieved by activation of the processors.

$$RPD_2(\varepsilon) = \frac{W_{4\cup3\_\dot{\alpha}I\_\eta}(\varepsilon) - W_{4\cup3}(\varepsilon)}{W_{4\cup3}(\varepsilon)} \cdot 100 \qquad \left(\varepsilon = \#1,\ldots,\#|\mathrm{E}|\right) \tag{52}$$

$$RPD_3(\varepsilon) = \frac{W_{4\cup3\_\dot{\alpha}I\_\eta}(\varepsilon) - W_{4\cup3\_\eta}(\varepsilon)}{W_{4\cup3\_\eta}(\varepsilon)} \cdot 100 \qquad \left(\varepsilon = \#1,\ldots,\#|\mathrm{E}|\right) \tag{53}$$

$$RPD_4(\varepsilon) = \frac{W_{4\cup3\_\dot{\alpha}I\_\eta}(\varepsilon) - W_0\left(\overline{\dot{\alpha}^S}, \varepsilon\right)}{W_0\left(\overline{\dot{\alpha}^S}, \varepsilon\right)} \cdot 100 \qquad \left(\varepsilon = \#1,\ldots,\#|\mathrm{E}|\right) \tag{54}$$

$$RPD_5(\varepsilon) = \frac{W_0\left(\overline{\dot{\alpha}^S}, \varepsilon\right) - W_0(\varepsilon)}{W_0(\varepsilon)} \cdot 100 \qquad \left(\varepsilon = \#1,\ldots,\#|\mathrm{E}|\right) \tag{55}$$

ε	$k \in K : \eta^{\circ}_{med}(k) \geq \eta^{\infty}_{med}$	$W_0\left(\overline{\alpha^S} = 1.0\widehat{3}\right)$	$W_{4\cup 3\_\dot{\alpha}I\_\eta}$	RPD_2	RPD_3	RPD_4	RPD_5
#1	9, 10, 16, 17, 18	4220.6	4601.8*	2360.9	-62.6	9.0	-65.7
#2	9, 10, 16, 17, 18	4312.6	4692.7*	1276.2	-62.3	8.8	-65.4
#3	9, 10, 16, 17, 18	4123.9	4509.2*	956.0	-63.1	9.3	-66.2
#4	9, 10, 16, 17, 18	4269.0	4649.5*	1399.8	-62.7	8.9	-65.8
#5	9, 10, 16, 17, 18	4549.7	4929.8*	678.8	-62.1	8.4	-65.0
#6	9, 10, 16, 17, 18	4506.1	4882.2*	1082.1	-62.2	8.3	-65.1
#7	9, 10, 16, 17, 18	4361.0	4743.9*	539.3	-62.7	8.8	-65.7
#8	9, 10, 16, 17, 18	4075.5	4459.6*	3108.3	-62.9	9.4	-66.1
#9	9, 10, 16, 17, 18	4694.8	5066.8*	592.2	-62.1	7.9	-64.9
#10	9, 10, 16, 17, 18	4404.5	4795.6*	297.0	-63.5	8.9	-66.4
#11	9, 10, 16, 17, 18	3838.4	4231.1*	5324.5	-64.1	10.2	-67.5
#12	9, 10, 16, 17, 18	4209.0	4590.8*	1516.5	-62.5	9.1	-65.6
#13	9, 10, 16, 17, 18	4335.8	4715.1*	1548.6	-62.4	8.7	-65.5
#14	9, 10, 16, 17, 18	4398.7	4777.3*	1037.5	-62.2	8.6	-65.2
#15	9, 10, 16, 17, 18	4271.9	4653.1*	974.6	-62.5	8.9	-65.5
#16	9, 10, 16, 17, 18	4249.7	4630.4*	1939.8	-62.5	9.0	-65.6
#17	9, 10, 16, 17, 18	4375.5	4755.0*	894.8	-62.3	8.7	-65.3
#18	9, 10, 16, 17, 18	4646.4	5020.7*	729.9	-62.0	8.1	-64.8
#19	9, 10, 16, 17, 18	4312.6	4700.6*	397.4	-63.3	9.0	-66.3
#20	9, 10, 16, 17, 18	3983.5	4370.0*	3043.9	-63.2	9.7	-66.5
#21	9, 10, 16, 17, 18	4554.5	4929.1*	780.2	-62.3	8.2	-65.1
#22	9, 10, 16, 17, 18	4452.9	4838.9*	390.3	-63.0	8.7	-65.9
#23	9, 10, 16, 17, 18	3935.2	4323.0*	2987.9	-63.7	9.9	-66.9
Average values				1472.0	-62.7	8.9	-65.7

Table 3. Results achieved by considering the saturation limitation and activation of the processors according to a stepped function. ε

Based on Table 3, we state the following observations:

1. None of the sequences of the 23 plans of production violate the limitation of the maximum dynamic saturation $\left(\eta^{\infty}_{\max} = 1.2\right)$ imposed in all stations. The $M4\cup 3\_\dot{\alpha}I\_\eta$ model always finds a solution.
2. Stations 9, 10, 16, 17, and 18 appear in the 23 production plans as the receptors of the work overload. Stations 1 to 8, 11 to 15, and 19 to 21 are never oversaturated over the limit imposed by the medium static saturation $\left(\eta^{\infty}_{med} = 0.95\right)$.
3. The optimal values (maximum) are achieved for the completed work in the 23 examples (see $W_{4\cup 3\_\dot{\alpha}I\_\eta}$ column)
4. The 23 sequences of manufacture generate idle time (see RPD_4). The idle time is 8.9% on average over the static workload $W_0\left(\overline{\alpha^S} = 1.0\widehat{3}\right)$.
5. The incorporation of the activity factor reduces the inevitable work overload of the line by 65.7%, (see RPD_5 column) which indicates a change from a maximum inevitable work

overload of 13,363 *s* (see $W_{0\_\eta}(\varepsilon = \#9)$ in table 2) to a dynamic work overload of 5,066.8 *s* (see $W_{4\cup 3\_\dot{\alpha}I\_\eta}(\varepsilon = \#9)$ in table 3).

6. The increase of the dynamic work overload (see RPD_2 column) by limiting the medium saturation and activating the processors ranges from 297.0% $(\varepsilon = \#10)$ to 5,324.5% $(\varepsilon = \#11)$ and has an average value of 1,472%.
7. The average improvement of the dynamic work overload by the activation of the processors reaches up to 62.7% with respect to the work overload values achieved by $M4\cup 3\_\eta$. This means an average recovery of the drop in production equivalent to 45 engines per day (range: 4 engines/day).
8. The economic recovery given by the activation of the processors is equivalent on average to 18,012.2 euros/day with a range of 1,679.5 euros/days.

9 Conclusions

We have established a number of basic observations on the working day and the saturation of the workstations. In this proposal, by taking these observations and the *MMSP-W* into consideration, we present a model focused on measuring the impact produced by such labor characteristics on the work overload in a production line.

The proposed model, $M4\cup 3\_\eta$, was applied to a case study of the Nissan engine plant in Barcelona that contains 23 demand plans. This application determines that the improvement in working conditions designed to limit the average saturation of the workstations causes a drastic drop in production. This drop is estimated in terms of 75 daily engines in a line with a capacity equal to 270 engines in two shifts.

It is proposed that the production drop is should be mitigated by activation of the processors, which means that the activity of the operators must be increased (while always respecting the agreed working conditions) under the guidance of a temporary function of the activity factor that unfolds throughout the workday.

Our recommendation to recuperate the production losses is concretized, first in a new model $M4\cup 3\_\dot{\alpha}I\_\eta$ that incorporates the activity factor in the execution times of the operations, and second, in the application of this approach over that of mentioned case study.

If we take into consideration a stepped function for the activity factor as guidance for the activation of the processors, the resultant economic recovery is equivalent to 18,012.2 euros/day on average with a range between 17,283.2 euros/day and 18,962.7 euros/day, according to the demand plan.

Our plans for future work are to: (1) evaluate the recovery of the production given by another temporary function of the activity factor; (2) evaluate from an economic point of view the possibility of incorporating line reinforcement operators who are specialized according to their workstations and who are able to cover several workstations due to their advanced training; and (3) establish rotational programs among the workstations with the purpose of reducing the medium-and long-term saturation.

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References

Battaïa, O., Dolgui, A. 2013. “Taxonomy of line balancing problems and their solution approaches”. *International Journal of Production Economics* 142 (2), 259-277.

Bautista, J., Alfaro, R., Batalla, C., Cano, A. 2014. “Incorporating the work pace concept into the MMSP-W” in C. Hernández et al. (eds).Managing Complexity, Lecture Notes in Management and Industrial Engineering (261-268) Springer International Publishing.

Bautista, J., Cano, A. 2011. “Solving mixed model sequencing problem in assembly lines with serial workstations with work overload minimisation and interruption rules”. *European Journal of Operational Research* 210(3), 495-513.

Bautista, J., Cano, A., Alfaro, R. 2011. “A Bounded Dynamic Programming algorithm for the MMSP-W considering workstation dependencies and unrestricted interruption of the operations” in 11th International Conference on Intelligent Systems Design and Applications (ISDA2011), Córdoba.

Bautista, J., Cano, A., Alfaro, R. 2012a. “Modeling and solving a variant of the mixed-model sequencing problem with work overload minimisation and regularity constraints. An application in Nissan’s Barcelona Plant”. *Expert Systems with Applications* 39(12), 11001–11010.

Bautista, J., Cano, A., Alfaro, R. 2012b. “Models for MMSP-W considering workstation dependencies: A case study of Nissan’s Barcelona Plant”. *European Journal of Operational Research* 223(3), 669–679.

Bautista, J., Cano, A., Companys, R., Ribas, I. 2012. “Solving the Fm|block|Cmax problem using Bounded Dynamic Programming”. *Engineering Applications of Artificial Intelligence* 25(6), 1235–1245.

Bautista, J., Suárez, R. 2009. “Mixed-model sequencing problem with overload minimization considering workstations dependencies” in *Proceedings of 2009 IEEE International Symposium on Assembly and Manufacturing*.

Becker, C., Scholl, A. 2006. “A survey on problems and methods in generalized assembly line balancing”. *European Journal of Operational Research* 168(3), 694-715.

Boysen, N., Fliedner, M., Scholl, A. 2009a.“Sequencing mixed-model assembly lines: survey, classification and model critique”. *European Journal of Operational Research* 192(2), 349-373.

Boysen, N., Fliedner, M., Scholl, A. 2009b.“The product rate variation problem and its relevance in real world mixed-model assembly lines”. *European Journal of Operational Research* 197(2), 818-824.

Elmaghraby, S. 1978. "The Economic Lot Scheduling Problem (ELSP): Review and extensions". *Management Science* 24(6), 587-598.

Lin, D-Y., Chu, Y-M. 2013. "The mixed-product assembly line sequencing problem of a door-lock company in Taiwan". *Computers and Industrial Engineering* 64(1), 492-499.

Lin, D-Y., Chu, Y-M. 2014. "A Lagrangian relaxation approach to the mixed-product assembly line sequencing problem: A case study of a door-lock company in Taiwan". *Applied Mathematical Modelling* 38(17-18), 4493-4511.

Miltenburg, J. 1989. "Level Schedules for mixed-model assembly lines in Just-In-Time production systems". *Management Science* 35(2), 192-207.

Monden,Y. 1983. Toyota Production System. Industrial Engineering and Management Press, Norcross, Georgia.

Muse, L., Harris, S., Field, H. 2003. "Has the inverted-u theory of stress and job performance had a fair test?". *Human Performance* 16(4), 349–364.

Pan, Q.K., Ruíz, R. 2013. "A comprehensive review and evaluation of permutation flow shop heuristics to minimize flow time". *Computer Operation Research* 40(1),117-128.

Parrello, B. D., Kabat, W. C., Wos, L. 1986. "Job-shop scheduling using automated reasoning: A case study of the car-sequencing problem". *Journal of Automated Reasoning* 2(1), 1-42.

Raza, A. S., Akgunduz, A. 2008. "Comparative study of heuristic algorithms on Economic Lot Scheduling Problem". *Computers & Industrial Engineering* 55(1), 94-109.

Salveson, M. E. 1955. "The assembly line balancing problem". *Journal of Industrial Engineering* 6(3), 18-25.

Scholl, A., Klein, R., Domschke, W. 1998. "Pattern based vocabulary building for effectively sequencing mixed-model assembly lines". *Journal of Heuristics* 4(4), 359-381.

Solnon, C., Cung, V. D., Nguyen, A., Artigues, C. 2008. "The car sequencing problem: Overview of state-of-the-art methods and industrial case-study of the ROADEF'2005 challenge problem". *European Journal of Operational Research* 191(3), 912-927.

Yano, C.A., Rachamadugu, R. 1991. "Sequencing to minimize work overload in assembly lines with product options". *Management Science* 37(5), 572-586.