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Channel Holding Time Distribution in Public Cellular Telephony*

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This paper examines the channel holding time of public cellular telephony systems. This is the time that the Mobile Station (MS) remains in the same cell, a fraction of the call holding time. The study is based on actual data taken from a working system. The probability distribution that fits the empirical sample best when applying the Kolmogorov-Smirnov test is a mixture of lognormals. Combinations of memory-less stages are also tested in the paper.

1. INTRODUCTION

The duration of channel holding time (dwell time in the cell) in cellular telephony systems is only a fraction of the total call duration. This is due to the fact that the physical channel is assigned only for the period that the Mobile Station (MS) remains within the same cell. The average channel holding time is equal to the average call holding time divided by the average number of handoffs per call plus one: the number of cells crossed by an average call. But nothing can be said "a priori" about the relationship of further moments of the channel holding time or about its whole probability distribution. Factors such as mobility and cell shape and size cause the dwell time to have a different probability distribution function to that of call duration, this difference being greater for higher mobility and smaller cell sizes. The limit situation occurs for a stopped MS or an extremely large cell size; in these cases the dwell time is equal to the call duration.

Recently, empirical approaches have again been used to look into the probability distribution of call holding time in telephony systems. In his paper in ITC14 [1], Bolotin mentioned up to 8 papers appearing in ITC13 which assumed an exponentially distributed call holding time. The author showed, however, that mixtures of lognormals fit the call holding time better than the exponential

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distribution when applying the Kolmogorov-Smirnov (K-S) goodness-of-fit test to an empirically obtained sample. In ITC15 Chlebus used the Anderson-Darling (A-D) test to show that call duration in mobile telephony follows the same patterns as shown by Bolotin for fixed telephony, as could be expected [2].

In the past, several authors researched into models for channel holding time in cellular systems making use of analytical tools and simulation results; all of them assumed an exponentially distributed call holding time. In [3] the authors obtained analytical results for channel holding time distribution, assuming a specific mobility pattern: uniform speed and direction which changes at the cell borders. The distribution obtained was complex, and the authors used the negative exponential to approximate it and thus further investigate other performance figures. In [4] Guerin simulated a large geographical extension with round cells. When applying the K-S test to the simulation results the exponential distribution gave a satisfactory fit. In the same paper other more complex distributions were analytically obtained for a more restrictive mobility pattern. In [5] Steele and Nofal analytically obtained an exponentially distributed channel holding time for a Manhattan model. In [6] the K-S test was again applied to simulation results and the exponential distribution was again the best fit. In this case the mobility pattern was as presented by the same authors in [7].

It is obvious that channel holding time distribution depends on call holding time, but only partly. Rappaport's words in [8] reveal the complexity involved in finding this distribution: "Clearly, dwell time depends on many factors such as propagation conditions, the path a mobile platform follows, its velocity profile along this path, and especially the definition of the communication range. But even if all of these were known, the dependency is so complex and burdensome that one would eventually have to resort to empirical findings in some way." To the authors' knowledge, the only approach based on real measurements existing in the open literature is the paper presented by Jedrzycki and Leung [9], which accepts the lognormal distribution as being the best fit.

In this paper a field study of the channel occupancy of a cellular telephone system in Barcelona is performed. The results presented in the paper are an extended version of the results presented in [10]. The paper is organised as follows. Section 2 explains in detail how real channel holding time data was collected. Section 3 briefly describes the statistical tools chosen by the authors to investigate the subject and how these are used. The equipment and statistical tools were previously used for the study presented in [11]. The exponential distribution is compared with empirical data in Section 4. Section 5 provides the results of applying the K-S test to other probability distributions. Other statistical results connected with channel holding time and obtained in the same environment studied are presented in Section 6. Section 7 summarises the main points and gives some conclusions.

2. EQUIPMENT USED TO DETECT CHANNEL ACTIVITY

The hardware used to register the necessary data is extremely simple as shown in Figure 1. The equipment is based on a scanner receiver tuned to one of the carrier frequencies of a Base Station (BS). The scanner is connected to a Personal Computer (PC) which registers all the activities of a single channel in a PC file. The monitored system uses the TACS standard (very similar to AMPS except for some minor details) and FM detection of the down-link carrier frequency is sufficient to find out the channel occupancies. The power transmitted in the down-link is higher and more stable than in the up-link. This fact helps to reduce the annoying effects of noise, fading and interference. Each record of the PC file includes fields such as the starting and ending time of every channel occupancy and the carrier strength at the monitored frequency. A very simple process generates a new file containing only the lengths of the channel occupancies.

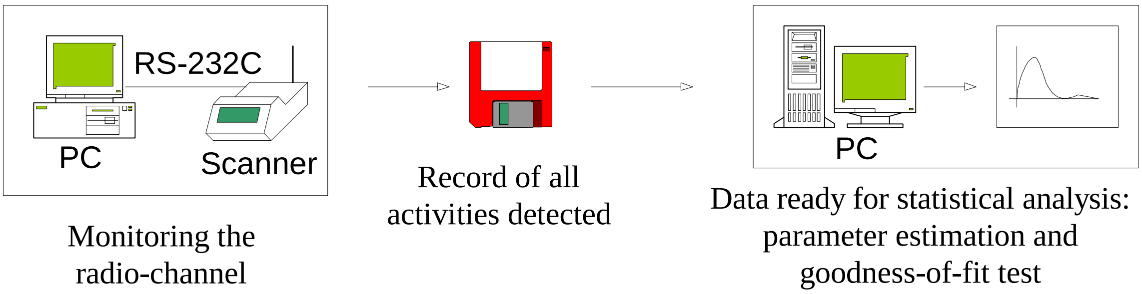

Figure 1. Equipment used to obtain the necessary data.

Before performing the statistical analysis, a ‘cleaner’ file is obtained in which some values of the original sample are eliminated. First, activity values under 2 seconds are considered to be caused by noise or interference and thus suppressed from the data set. Activities separated by a silence of less than 1 second were considered to be short cuts due to signal fading and were therefore joined. In fact, the TACS system is protected against these fading effects and holds the assigned channel for a longer time if a handoff is not required. Both bounds were carefully established by aural monitoring in an attempt to minimise the number of false data. With these bounds, suppression of actual activities or true silences smaller than the bounds were reduced to less than 5%, and more than 95% of false activities or silences were detected.

3. STATISTICAL TOOLS

First of all it must be decided which candidate or theoretical probability distributions are to be statistically tested against the empirically obtained data collection. In all cases the coefficient of variation of the empirical holding time was found to be larger than one, so only distributions which can achieve such coefficients should be tested. The candidate distributions are the same as in Section 5 of [11] but the hyperexponential distribution was excluded because the fit proved to be extremely poor. These probability density functions (p.d.f) can be classified as follows:

- Exponential and shifted exponential. Shifted exponential is the simplest step forward from the exponential when extremely low values of t are not possible to be obtained. For d=0 one has the exponential p.d.f.

$$f(t) = \frac{1}{\beta} e^{-\frac{(t-d)}{\beta}} \qquad \text{for } t \geq d \tag{1}$$

- Combination of memory-less stages are appreciated by researchers to be used along with analytical tools. Erlang-j,k of Equation (2) and erlang-k-2 (also called hyper-erlang-2) of Equation (3) can both achieve coefficients of variation higher than unity:

$$f(t) = p\beta^{-j} \frac{t^{j-1}}{(j-1)!} e^{-\frac{t}{\beta}} + (1-p)\beta^{-k} \frac{t^{k-1}}{(k-1)!} e^{-\frac{t}{\beta}} \qquad \text{for } \geq 0 \tag{2}$$

$$f(t) = p\beta_1^{-k_1} \frac{t^{k_1-1}}{(k_1-1)!} e^{-\frac{t}{\beta_1}} + (1-p)\beta_2^{-k_2} \frac{t^{k_2-1}}{(k_2-1)!} e^{-\frac{t}{\beta_2}} \qquad \text{for } t \geq 0 \tag{3}$$

- Lognormal as in Equation (4) and mixtures of lognormals as in Equation (5) for lognormal-3.

$$f(t) = \frac{1}{t\sqrt{2\pi\sigma^2}} e^{-(\log(t)-\mu)^2 / 2\sigma^2} \qquad \text{for } t > 0 \tag{4}$$

$$\sum_{i=1}^{3} p_i \left[p.d.f.\ lognormal \right]_i \tag{5}$$

Once a candidate distribution has been proposed, its parameters must be estimated according to the empirical data. In this paper the Maximum Likelihood Estimation (MLE) is used [12]. The only reason to use MLE in this work as opposed to other methods is that in our case MLE gives better confidence figures than others when fitting with the empirical distribution.

To select a goodness-of-fit test the authors considered that the observed phenomenon is a non-natural one, and is modulated and distorted by many parameters that depend on very different matters and even on system and network settings. In this situation, simplicity was preferred to extreme accuracy. As in [1, 11] the K-S goodness-of-fit test is used in its 'all parameters known' version in this paper. The K-S test was also applied to analytically model the holding time of mobile telecommunication systems [4, 6] because of its power and simplicity. The K-S test works on the c.d.f. instead of the p.d.f. and avoids the dependency of the significance figures on the selected bin-width found in other tests such as the chi-squared used in [9] or the A-D used in [2].

The modified K-S distance D and the significance level α can be computed as:

$$D = \varepsilon(\sqrt{n} + 0.12 + \frac{0.11}{\sqrt{n}}) \qquad \alpha = 2\sum_{i=1}^{\infty}(-1)^{i-1}e^{-2i^2 n\varepsilon^2} \tag{6}$$

where ε represents the maximum difference between the theoretical and empirical c.d. functions and n is the number of data [12].

It is common practice to establish the allowable level of significance before carrying out a statistical analysis. The proposed theoretical distribution is not rejected if and only if the significance α is higher than the desired level (α=5% in [1, 6] or α=15% in [9]). One of the goals of this paper is to establish the ranking in which reasonable candidate distributions fit the empirical data: a simpler distribution may fit well enough for a particular purpose. The significance level is thus not fixed beforehand, but all the candidate p.d. functions are compared according to their significance α or modified K-S distance D.

4. THE DATA SET AND THE EXPONENTIAL DISTRIBUTION

Many data samples were obtained throughout the busy hour and all of them feature very similar statistical properties. The sample used to illustrate this paper was obtained in June 1996. The sample size is n=2,445, the average holding time m_1=40.6 seconds, and the squared coefficient of variation of the sample is 1.7.

The spikes observed in the empirical histogram of Figure 2 make parameter estimation and fit more difficult. These spikes are due to the hysteresis time that the system requires before retrying the handoff, as was observed in [9]. This time step is necessary to avoid instability and continuous handoffs when the MS is on the border between two cells. It is possible to remove the spikes by estimating the percentage of channel holding times due to immediate handoff, as done in [9]. In this work we prefer to keep the sample unaltered as long as fitting results are satisfactory.

When the negative exponential distribution is fitted with the empirical data the probability of very short occupancies is overestimated, while the area with the highest probability in the empirical histogram is underestimated. This behaviour can be observed in Table 1 and in Figure 2.

When the occupancy remaining time is examined as a function of the occupancy elapsed time, the average remaining time is far from being independent of the elapsed time, as should occur if the empirical data follows an exponential distribution pattern because of the memory-less property of the latter. Figure 3 shows how the average remaining time is greater the longer the elapsed time. This behaviour is similar to what was observed in [1] for the call duration in conventional telephony and in [11] for the transmission duration in PMR systems. The discontinuous shape for long elapsed times is due to the fact that fewer values remain to be averaged for longer elapsed times, leading to fewer and more dispersed values.

Table 1
Percentage of accumulated probability: empirical vs. exponential.

Time (seconds)	3	5	7	10	30	50	70	100	300	500	700
Empirical	0.7	3.1	4.9	13.5	54.4	76.5	87.2	93.6	99.6	99.8	99.9
Exponential	7.1	11.6	15.8	21.8	52.2	70.8	82.2	91.5	99.9	100.0	100.0

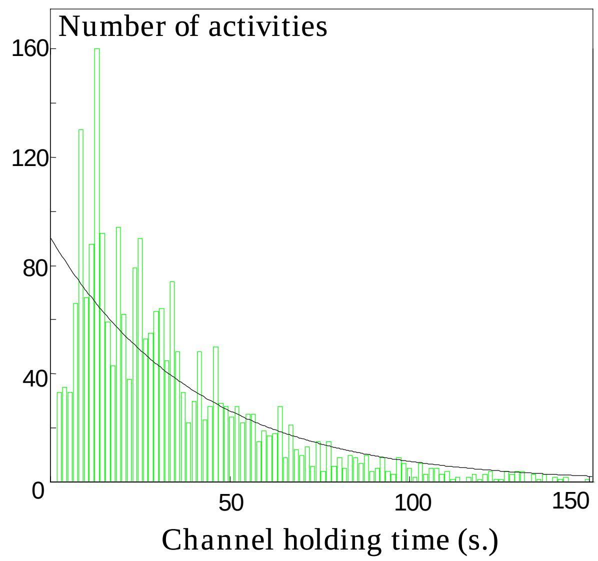

Figure 2. Empirical vs. exponential channel holding time distribution.

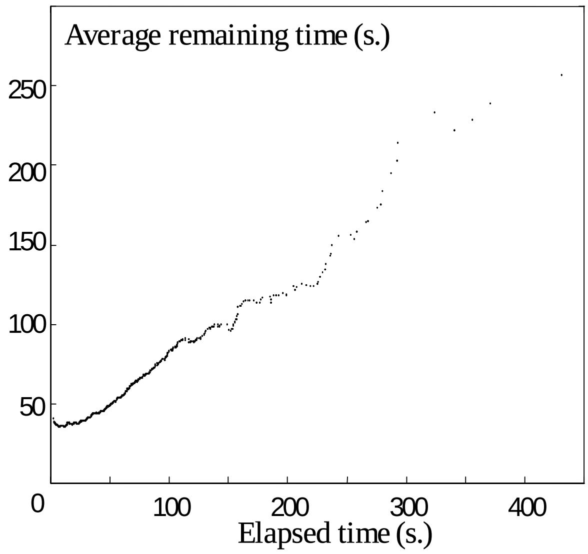

Figure 3. Average remaining vs. elapsed channel holding time

5. NUMERICAL RESULTS

In Table 2 the K-S distance D and the significance α are tabulated for the candidate probability distributions along with the MLE parameters. The best fit is attained by lognormal-3 distribution, with a significance of almost 10%. Note that the K-S test is targeted on continuous functions while in this case there are large spikes in the empirical histogram. Combinations of memory-less stages fit with negligible levels of significance, but the exponential fits much worse than others. When the researcher is to simulate a cellular system, the single lognormal distribution represents a good trade-off between simplicity and accuracy. The lognormal-2 distribution not included in Table 2 gives a significance between that of the lognormal and lognormal-3.

Table 2
Moments and fitting of channel holding time: Sample size=2,445.

Moments of channel holding time:			m_1: 40.60 s.				cv^2: 1.70
Fitting							
Exponential	D: 5.54	α: 0.000	β: 40.60				
Shifted Exp.	D: 2.05	α: 0.000	β: 31.37	d: 4.63			
Erlang-jk	D: 2.27	α: 0.000	β: 15.38	j: 2	k: 17	p: 0.95	
Erlang-k-2	D: 2.59	α: 0.000	β_1: 15.07	k_1: 2	β_2: 64.42	k_2: 4	p: 0.89
Lognormal	D: 1.55	α: 0.016	μ: 3.29	σ: 0.89			
Lognormal-3	D: 1.23	α: 0.097	μ_1: 3.33	σ_1: 1.04	p_1: 0.52		
			μ_2: 3.55	σ_2: 0.50	p_2: 0.33	μ_3: 2.44	σ_3: 0.28

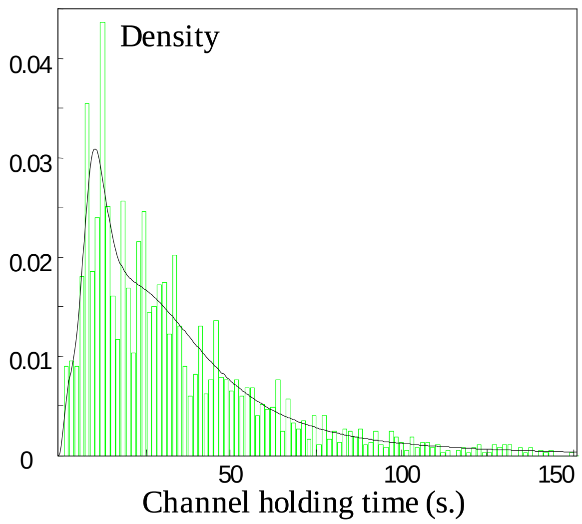

Figure 4. Empirical histogram and best fit (lognormal-3).

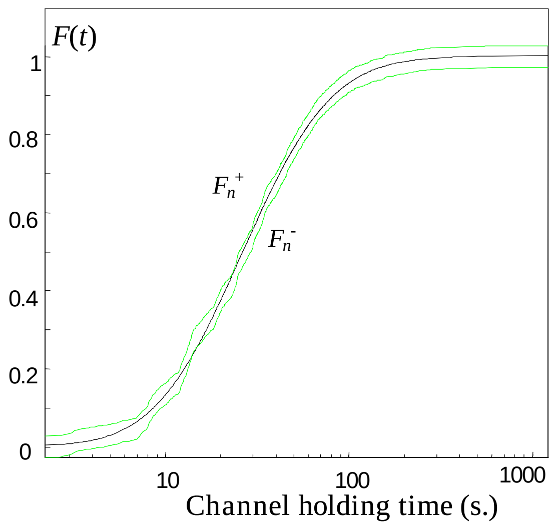

Figure 5. Contours of 5% significance.

In Figure 4 the empirical histogram is plotted against the best fit. The lognormal-3 distribution follows the whole shape of the empirical data much better than the exponential. In Figure 5 the proposed c.d.f. is plotted along with the contours of 5% significance: empirical increased and decreased by the ε

corresponding to 5% in Equation (1). The proposed best fit c.d.f. always remains within the bounds.

The same statistics were obtained for different times of day, loads and cells, and the ranking in Table 2 is always maintained, although the average and coefficient of variation may vary, as explained in Section 6. This gives our work more general scope, as the conclusion that lognormal distributions fit better than the memory-less type relies on statistical analysis for samples from many different situations.

6. OTHER STATISTICAL RESULTS

Although the main goal of this paper is to investigate the probability distribution of dwell time in cellular systems, other statistical results were obtained which can be useful when applying the results of previous sections to the design of cellular systems. In Table 3 the mean and squared coefficient of variation of the dwell time are shown for two different charging periods. As the duration is measured in the same cell the larger average corresponds to a lower mobility and/or to a longer call duration. The mobility is actually lower at night. The average unencumbered call duration is also greater due to the higher proportion of non-professional calls and the lower charging rate.

Table 3
Moments for different charging periods.

Time	Day: 7 to 21	Night: 21 to 7
Average (seconds)	40.6	63.3
cv^2	1.70	2.91

In Table 4 the channel holding times of the same data sample are classified according to the occupancy type: the average and squared coefficient of variation of the channel holding time are tabulated for each type. ‘Start-handoff’ means, for instance, that the call starts in the observed cell and continues in another one. The type ‘with handoff’ includes the first three, while ‘whole call’ means that the call begins and finishes within the cell considered. ‘Not available’ indicates that the called party is not connected and ‘others’ includes calls which can not be assigned, such as erroneous calls, calls blocked by network overload, etc.

From Table 4 it can be concluded that 89%=76/(76+9.6) of the occupancies belonging to answered calls have at least one handoff; this can be considered to be a very high mobility value. If we accept the 113 seconds found in [1] as the average call duration each mobile visits 113/40.6=2.78 cells on average. In the system examined the authors found a slightly longer average call holding time but cannot guarantee that it corresponds to the calls of the same sample. Most of the terminals that generate an occupancy classified as a ‘whole call’ are probably stopped, this being the reason for the longer average holding time.

7. CONCLUSSION

Although the use of analytical tools can lead to the conclusion that channel occupancy in cellular telephony conforms to a negative exponential distribution pattern, field studies show that the exponential distribution is far from matching empirical data. A mixture of lognormal distributions which fits call duration in conventional telephony very well also gives the best result for dwell time in mobile telephony. There are other simpler p.d.f. which fit much better than the exponential distribution: single lognormal, shifted exponential and erlang-*j*,*k*. The latter can be represented as a combination of memory-less stages, this being an advantage when analytical research is performed.

Table 4
Classification of occupancies: average and squared coefficient of variation.

	m_1 (s)	%	cv^2
Start-Handoff	35.9	28%	0.55
Handoff-End	38.8	17%	1.32
Handoff-Handoff	34.8	31%	1.54
With handoff	*37.0*	*76%*	*1.14*
Whole call	86.4	9.6%	1.28
Busy	15.4	6.7%	0.36
No answer	57.0	2.6%	0.03
Not available	37.2	5.1%	0.64
Others	14.4	4.0%	0.70

Our study proves that dwell time follows the same distribution pattern that the unencumbered call duration. The average channel holding time is shorter in our study than in [9], leading to the conclusion that we presumably work with smaller cells or higher MS speed or both. This difference can easily be explained as being caused by the difference between the life-styles of Canada [9] and Europe (ours). The high handoff rate (almost 90% of the occupancies undergo at least one handoff) shows that the agreement between dwell time and the unencumbered call duration distributions does not rely on a low handoff rate.

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