A Simple Closed-Form Approximation for the Packet Loss Rate of a TCP Connection Over Wireless Links

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***Abstract*—This letter presents a new and simple model for a TCP flow experiencing random packet losses due to both transmission errors and congestion events. From this model, we will derive a straightforward expression of a unified loss probability (ULP). This ULP gives the opportunity to reuse classical analytic models to analyze the performance of TCP and to size the buffer to optimize the wireless link utilization. Extensive simulations using TCP Reno in ns-2 demonstrate that our model is valid not only for the extreme cases where either transmission errors or congestion losses dominate but also in the situations where both types of losses are significant.**

***Index Terms*—TCP models, TCP performance, congestion control, wireless networks, random packet losses.**

I. Introduction

IN wireless networks, Transmission Control Protocol (TCP) has to cope with transmission error losses and losses due to congestion, but no formula seems to be available to unify these interactions. The analytic models proposed to date for the TCP congestion avoidance phase are mostly stochastic. Some of these models consider the network as a blackbox, in which packet loss is regarded as an independent random process [1], [2]. In these simplified models, the congestion window and throughput depend on the packet loss rate, and no other parameters about the underlying network are considered. Other analytic models only consider wired communications [3] without transmission losses. These models are based on the prediction of the congestion window evolution of the TCP flow over a single bottleneck link. This evolution follows a periodic behavior where the buffer size and the link characteristics determine the average throughput of TCP [4].

In this letter we determine a Unified Loss Probability (ULP) including both congestion and transmission error losses. We derive a closed-form approximation of this probability depending on the transmission error probability, bandwidth, delay and buffer size of the underlying network path. We demonstrate that ULP can be reused in classical analytic models [2] to analyze the performance of TCP, and also to size the buffer to optimize the wireless link utilization. The accuracy of the derived analytic expression is verified by extensive simulations conducted with ns-2 [5]. These simulations demonstrate that our model is valid not only for the extreme cases where either transmission errors or congestion losses dominate, but also in the situations where both types of losses are significant.

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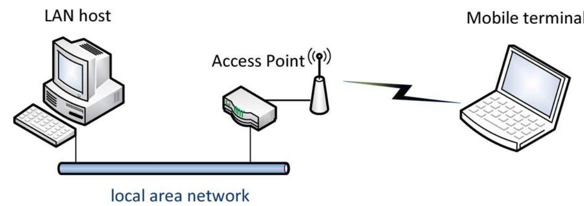

Fig. 1. Network model for our analysis.

II. Network Model

Fig. 1 shows our network model. A mobile host is connected to a base station via a wireless link, which is also the last link in the TCP path. This wireless link introduces transmission random losses. We do not take into consideration other particular characteristics of wireless networks (available bandwidth, channel access scheme, path asymmetry, etc.) that might have an impact over the TCP performance. Though there are two directions of TCP communication, we consider only the case of communication from a fixed host on a wired network to a mobile host for performance modeling, without loss of generality. We assume that the wireless link is the bottleneck of the connection, and packets from wired networks get queued in the access point (AP). During a bulk transfer over a TCP connection the packets are stored-and-forwarded by the AP. The AP employs a dedicated buffer to store packets waiting to be transmitted over the lossy link. We will model the loss process considering buffer overflows at the AP and corruption losses at the wireless link.

III. Modeling TCP Under Transmission Errors and Congestion Losses

A. Hypotheses

The following starting-points, called "long term behavior hypotheses",[1] are strictly necessary for this work:

- The source host has always data to send.
- Only the congestion avoidance phase is considered.
- Retransmission timeouts are not considered.
- Losses are due to transmission errors and congestion.
- Random losses are independent of each other, and follow a geometric probability distribution function.

[1] These hypotheses are similar to others also used in the literature [6].

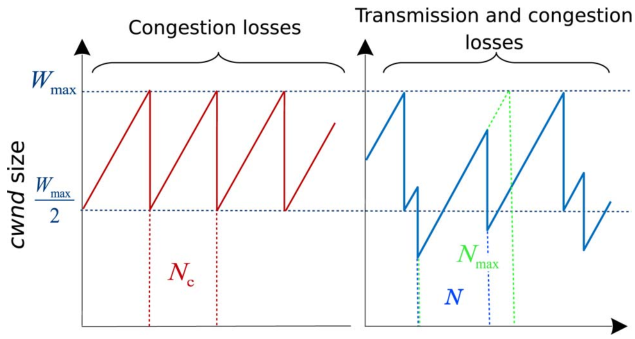

Fig. 2. Congestion window evolution.

- The window size of the TCP sender is not limited by the advertised window of the receiver.
- All TCP segments have the same length.
- For simplicity, no delayed-ack policy is considered but only that every segment is individually acknowledged.
- The buffer follows a drop-tail discipline.
- The network has a large bandwidth-delay product (BDP).

We characterize the behavior of a single TCP-Reno flow by means of the time evolution of its congestion window, denoted as $W(t)$. We consider only the congestion avoidance phase, so the congestion window size increases linearly over time while no losses occur, and it is halved when there is a loss.

B. Loss Probability Under (Only) Congestion Events

The evolution of a cycle in TCP-Reno is determined by the initial window size at the beginning of the cycle. Cycles are separated by one packet loss.[2] In absence of random losses, the initial window size will be the same for all cycles and congestion episodes will appear periodically [2]. In this case, $W(t)$ will reach its maximum value $W_{\max}$ in all cycles. Then, the maximum window size permits an easy calculation of the number of transmitted packets in a congestion cycle, N_c, summing all the consecutive values of $W(t)$ for each round-trip time (RTT) along the congestion cycle. As it is known, during the congestion avoidance phase, the congestion window is incremented by roughly 1 full-sized segment per RTT. So, this leads to the following expression:

$$N_c = \sum_{i=0}^{\frac{W_{\max}}{2}} \left(\frac{W_{\max}}{2} + i \right) \approx \frac{3W_{\max}^2}{8} \quad (1)$$

which is a well-known result that appears in the literature [2]. N_c can also be calculated as the area below the sawtooth in a congestion cycle (see the solid red line in left graph of Fig. 2). Since the only packet loss is the last in the cycle, we can define the congestion loss probability as:

$$p_c \triangleq \frac{1}{N_c} = \frac{8}{3W_{\max}^2} \quad (2)$$

where $W_{\max}$ is approximately the sum of the buffer size B and the BDP when $W_{\max} \gg 1$ (see [4]):

$$W_{\max} \approx B + BDP \quad (3)$$

[2]During the paper we will use the term *packet* to refer to network layer packets. We emphasize this in order not to confuse with layer-2 packets.

C. Loss Probability Under Transmission Errors and Congestions

Corruption losses may give rise to premature terminations of a congestion avoidance cycle, so the highest value reached by $W(t)$ could vary from cycle to cycle. Therefore, the number of transmitted packets in each cycle (N) could also vary according to the behavior of the transmission error process (see the solid blue line of right graph in Fig. 2).

We model the transmission error process by taking into account that any given packet may be lost with probability p_e due to a transmission error, and that random losses are independent. Then, the transmission error model is characterized by the packet burst length probability of the congestion window cycle $P(i)$, which is given by:

$$P(i) = P[N=i] = \begin{cases} (1-p_e)^{i-1} p_e & 1 \le i \le N_{\max}-1 \\ (1-p_e)^{N_{\max}-1} & i = N_{\max} \end{cases} \quad (4)$$

where p_e is the packet transmission loss probability and $N_{\max}$ represents the maximum number of packets that would be sent only in case a congestion loss occurs. In a cycle, $N_{\max}$ can be calculated as the area below the sawtooth until reaching congestion (see the dotted green line of right graph in Fig. 2). We are interested in obtaining an average value of N, which will be used to obtain an average TCP congestion window to assess the random loss effects on TCP behavior. Thus, the average value is calculated as the expectation of N:

$$\overline{N} = \sum_{i=1}^{N_{\max}} i \cdot P(i) = \sum_{i=1}^{N_{\max}-1} i \cdot P(i) + N_{\max} \cdot P(N_{\max})$$
$$= \frac{1-(1-p_e)^{N_{\max}}}{p_e\left[1-p_e+p_e(1-p_e)^{N_{\max}-1}\right]} \quad (5)$$

Taking into account that in practice $p_e \ll 1$, we can carry out a first approximation as:

$$\overline{N} \approx \frac{1-(1-p_e)^{N_{\max}}}{p_e} \quad (6)$$

and by means of the asymptotic expansion of the binomial expression in the numerator:

$$(1-p_e)^{N_{\max}} = e^{N_{\max}\ln(1-p_e)} = e^{-N_{\max}\left(p_e+O\left(p_e^2\right)\right)} \quad (7)$$

we get another approximation of the mean number of packets:

$$\overline{N} \approx \frac{1-e^{-N_{\max}p_e}}{p_e} \quad (8)$$

Using the same error model than Lakshman and Madhow [4], we approximate $N_{\max}$ to a constant value that is equal to N_c:

$$N_{\max} \approx N_c \quad (9)$$

Note that $N_{\max}$ is equal to or greater than N_c (see Fig. 2). We obtain the expression of the Unified Loss Probability (ULP), which includes the effects of congestion and random losses:

$$\hat{p} \triangleq \frac{1}{\overline{N}} \approx \frac{p_e}{1-e^{-N_c p_e}} = \frac{p_e}{1-e^{-\frac{p_e}{p_c}}} \quad (10)$$

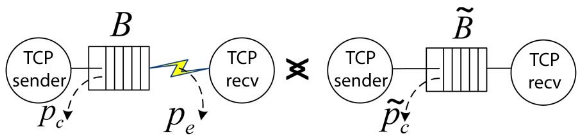

Fig. 3. Actual model and equivalent model.

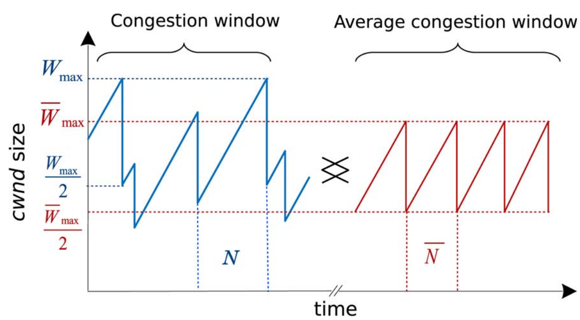

Fig. 4. Congestion window evolution in the actual and equivalent models.

To corroborate the validity of the expression obtained for the mean number of transmitted packets, we check the cases where only either congestion losses or transmission losses occur (asymptotic behavior). When only congestion losses are present, the transmission loss probability is negligible. Thus, the mean number of sent packets is N_c. Mathematically,

$$\lim_{p_e \to 0} \overline{N} = \lim_{p_e \to 0} \frac{1 - e^{-\frac{p_e}{p_c}}}{p_e} = \frac{1}{p_c} = N_c \tag{11}$$

On the other hand, if the transmission loss probability is much higher than the congestion probability ($p_c \ll p_e$), losses due to congestion are negligible, i.e., there is a random loss before a congestion loss occurs. Formally:

$$\lim_{p_c \to 0} \overline{N} = \lim_{p_c \to 0} \frac{1 - e^{-\frac{p_e}{p_c}}}{p_e} = \frac{1}{p_e} \tag{12}$$

D. Buffer Sizing Under Transmission Errors and Congestions

TCP treats all losses equally, no matter if they are due to transmission errors or congestion events. We will design a model that is equivalent to the actual one, but which is based only in congestion losses. The idea behind our equivalent model is shown in Fig. 3. In the left part of this Figure we can see the actual network model, which is characterized by its transmission errors (p_e) and congestion losses (p_c). Likewise, congestion losses mostly depend on the actual buffer size (B) and the BDP. Also, note that we are able to calculate an ULP $\hat{p}$ for this actual model using Eq. (10). In the right part of this Figure we show our equivalent model, characterized only by congestion losses ($\widetilde{p}_c$), or equivalently, by the associated buffer size ($\widetilde{B}$) and the BDP. It is worth noting that the BDP is the same in both models. The TCP in both models will behave in the same manner when they have the same overall packet loss. This equivalency between models is met when:

$$\widehat{p} = \widetilde{p}_c \tag{13}$$

On the other hand, in the left part of Fig. 4, we show the actual evolution of the congestion window, which depends on both transmission errors and congestion losses. For this actual model, we have demonstrated that we are able to calculate an average number of transmitted packets in a congestion cycle ($\overline{N}$), using (8).

In addition, in the right part of Fig. 4, we draw the evolution of our equivalent model, which is only based on congestion losses, so it is perfectly periodical. For this equivalent model, the number of transmitted packets is $\widetilde{N}_c = 1/\widetilde{p}_c$. Both models are equivalent when they transmit the same mean number of packets per cycle. In other words, their performance is the same when the mean value of $W(t)$ (or the same mean area per cycle) is equal in both models:

$$\overline{N} = \frac{1}{\hat{p}} = \widetilde{N}_c = \frac{1}{\widetilde{p}_c} \tag{14}$$

which is perfectly equivalent to (13).

If we are able to optimize our equivalent model, the actual model will be automatically optimized. To achieve 100% bandwidth utilization in case of having only one flow, TCP has to halve its congestion window after a loss detection to a value equal to or greater than BDP. This is a traditional sizing technique for wired networks [7], [8] since this ensures that the bottleneck link is always filled with packets. For a minimum delay and a maximum throughput in our equivalent model, $\widetilde{B} = BDP$. Then, using Eqs. (2) and (3):

$$\widetilde{p_c^{opt}} \triangleq \widetilde{p}_c \Big|_{\widetilde{B}=BDP} = \frac{8}{3}\frac{1}{(\widetilde{W}_{\max})^2} = \frac{8}{3}\frac{1}{(\widetilde{B}+BDP)^2} = \frac{2}{3}\frac{1}{BDP^2} \tag{15}$$

According to Eqs. (10), (13), (15):

$$\hat{p} = \frac{p_e}{1 - e^{-\frac{p_e}{p_c}}} = \widetilde{p_c^{opt}} = \frac{2}{3}\frac{1}{BDP^2} \tag{16}$$

Also, according to Eqs. (2) and (3):

$$p_c = \frac{8}{3(W_{\max})^2} = \frac{8}{3(B + BDP)^2} \tag{17}$$

So, substituting this value of p_c in Eq. (16), and isolating the actual buffer from that expression, we obtain:

$$B^{opt} \triangleq B\Big|_{\widetilde{B}=BDP} \approx \sqrt{-\frac{8}{3p_e}\ln\left(1 - \frac{3p_e}{2}BDP^2\right)} - BDP \tag{18}$$

Note that there are limitations due to the natural logarithm: $1 - (3p_e/2)BDP^2 > 0$. Mathematically:

$$p_e < \frac{2}{3}\frac{1}{BDP^2} = \widetilde{p_c^{opt}} \tag{19}$$

If this is not accomplished, the expression of the optimal buffer size has no real values. Hence, it will not be possible to reach full bandwidth utilization when transmission errors are much more frequent than congestion losses for any arbitrary large buffer. As a final remark, it is possible to compensate the effect of transmission losses and maintain the throughput when there is a solution for Eq. (18) by simply increasing the buffer size. This fact was also stated and tested in [9]. Also, it is possible to calculate the throughput by using the formula of Mathis *et al.* [2] substituting $p = \hat{p}$.

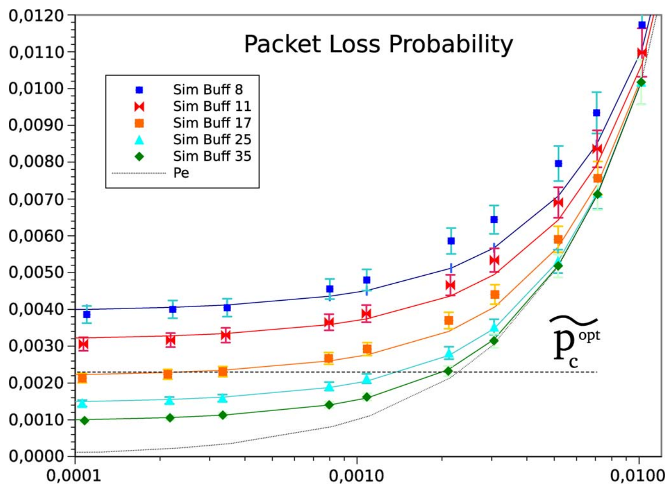

Fig. 5. Packet loss probability (simulated and ULP).

IV. Performance Evaluation

We have carried out a set of extensive simulations using TCP Reno in ns-2. We use a wireless network at 11 Mbps with 8.7 ms of one-way propagation delay; therefore the $BDP = 1 + (11\ \text{Mbps}/1500 * 8\ \text{bits}) * 2 * 8,7\ \text{ms} \simeq 17$ packets, assuming IP packets of 1500 bytes. We will compare our analytic expression of $\hat{p}$ from Eq. (10) to the actual Packet Loss probability (PLP) perceived by the TCP sender via simulation. We have generated simulations for a vast set of transmission loss probabilities and for various buffer sizes ranging from $BDP/2$ to $2 \cdot BDP$. For all the probabilities, we have used a sample size greater than 100 events. The Figure also shows the associate margin of error for these probabilities, which will be always less than 10% for a confidence interval of 95%.

Fig. 5 shows the packet loss probability as a function of the transmission error probability (this one in logarithmic scale). The markers correspond to the packet loss probability perceived by the TCP sender and obtained via simulation. This packet loss probability is the result of both congestion events and random transmission errors. Different colors of the markers have been used for the different selected buffer sizes. The solid lines correspond to the ULP $\hat{p}$ calculated from Eq. (10), also using a different color for each buffer size. Thus, we can compare our ULP with the simulation results by simple measuring the distance from the marker to the solid line of the same color. According to Fig. 5, the approximation is quite accurate for all the buffer cases, but especially when buffers are sized to maximize the link utilization (i.e., $B \geq BDP$).

First, we verify the asymptotic behavior of our model. When there are no transmission errors, packet losses are only due to congestion and equal to $p_c = 1/N_c$. Both simulation and ULP perfectly fit these values, which can be calculated by using Eq. (17) as a function of the buffer size B. As expected, when the transmission error probability is small, the buffer size greatly impacts on the total packet loss probability. So, smaller buffers have a higher probability to be congested and to lose packets. On the other hand, when the transmission error probability is high, losses due to congestion are negligible, and the buffer size does not affect the packet loss probability (the buffer hardly congests as there are many transmission losses). To better show this behavior, we have included in Fig. 5 a black solid line drawing $PLP = p_e$. Both simulations and our ULP have the same asymptotic behavior as this line.

However, the main contribution of our model is not shown in these extreme cases (as previously stated, there were separated models in the literature that deal with them), but in the situations where both types of losses are significant. In these situations, $\hat{p}$ provides an accurate approximation of the actual packet losses that no other model in the literature provides.

In addition, Fig. 5 can help us to size the actual buffer to optimize the system. In this Figure we have drawn a constant dashed line of value 0.0023, which correspond to $\widetilde{p_c^{opt}}$ (calculated with Eq. (15) for a $BDP = 17$). As a design requirement, we stated in Eq. (13) that $\hat{p} = \widetilde{p}_c$. In case of optimizing the equivalent model we also optimize the actual one. So, the actual buffer size that assures maximizing throughput for a particular p_e can be found looking for the intersection of this constant dashed line and one of the solid lines that draw $\hat{p}$. For instance, for $p_e = 0.001$ a buffer of $B = 25$ packets will be enough to assure maximal throughput, because the blue solid line intersects with $\widetilde{p_c^{opt}}$ at this point of p_e. Our model is in line with [9], as it is possible to compensate the effect of transmission losses and to maintain the throughput by increasing the buffer size. But we go further with Eq. (19), as we demonstrate that there is a limit for the transmission error probability (0.002 in this particular case) in which it is impossible to compensate this by increasing the buffer size.

V. Conclusion

We introduce a simple closed-form expression that unifies the dynamical behavior of a long lived TCP flow in front of packet losses due to transmission errors and congestion. The Unified Loss Probability is proven to be accurate thanks to extensive simulations. Our model is not only valid when either transmission error or congestion losses dominate, but also in the situations where both types of losses are significant. Also, we have derived an expression to size the buffer at the AP to achieve a high utilization of the bottleneck wireless link.

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