Pedro Padilla and his Mathematical Course (1753-1756): Views on Mixed Mathematics in eighteenth-century Spain

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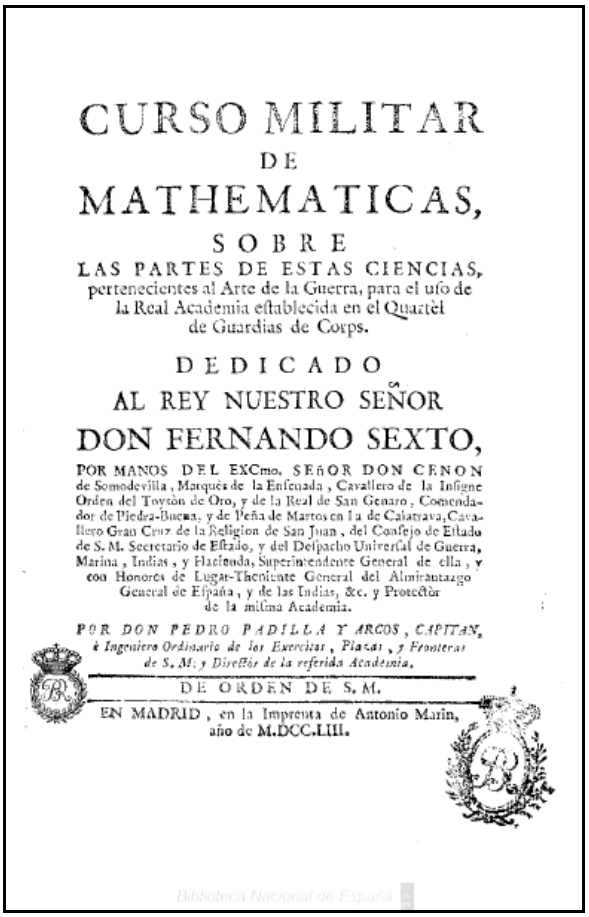
Introduction

In 1717 the King Philip V established the Royal Guards Headquarters (*Cuartel de Guardias de Corps*), mirroring the French *garde du corps du roi*. Intended mainly for noblemen, it was an elitist institution, all its members having the rank of officers and benefitting from huge privileges. Towards the end of 1750 an Academy of Mathematics (*Academia de Matemáticas*) was created within the Royal Guards Headquarters, under the direction of Captain Pedro Padilla (1724-1807?). This academy was ruled by the same regulations as the Military Academy of Mathematics of Barcelona (1720-1803).[1] Attendance was not mandatory; it was only devised for those interested in getting a deeper mathematical knowledge. In fact, rather than its real practical use for the Royal Guards, mathematics was studied as a mark of prestige as Hidalgo (1991) pointed out.[2]

Padilla held the position of Headmaster up to the closure of the Academy of Mathematics in 1760. In 1753 Padilla started publishing his *Curso Militar de Mathematicas, sobre partes de esta ciencia, para uso de la Real Academia establecida en el Cuartel de Guardias de Corps* (1753-1756) [*Military Course of Mathematics, about some parts of this science, for the use of the Royal Academy established in the Military Academy of the Royal Guards*] (Fig. 1). Of the twenty mathematical treatises that Padilla originally intended to develop, only five were finally published: (1) Ordinary arithmetic; (2) Elementary, or Euclidean, geometry; (3) Elementary algebra; (4) Higher geometry, or geometry of curves, and (5) Differential and integral calculus, or the method of fluxions.[3]

[1] For a thorough and recent analysis of the Military Academy of Mathematics of Barcelona see Massa-Esteve *et al.* (2011).
[2] On the creation and organization of the Military Academy of the Royal Guards Ordenanzas of December 21, 1750, September 22, 1751 and November 11, 1755, respectively (Portugues 1765, V, 180–184, 187, 196–199). See also Lafuente and Peset (1982) and Blanco (2013).
[3] Padilla's fifth treatise turned out to be the first Spanish educational book on calculus. See Cuesta Dutari (1985), Ausejo and Medrano-Sánchez (2010) and Blanco (2013).



CURSO MILITAR
DE
MATHEMATICAS,
SOBRE
LAS PARTES DE ESTAS CIENCIAS,
pertenecientes al Arte de la Guerra, para el uso de
la Real Academia establecida en el Quartèl
de Guardias de Corps.

DEDICADO
AL REY NUESTRO SEÑOR
DON FERNANDO SEXTO,

POR MANOS DEL EXCmo. SEñOR DON CENON de Somodevilla, Marquès de la Enfenada, Cavallero de la Insigne Orden del Toyson de Oro, y de la Real de San Genaro, Comendador de Piedra-Buena, y de Peña de Martos en la de Calatrava, Cavallero Gran Cruz de la Religion de San Juan, del Conseio de Estado de S. M. Secretario de Estado, y del Despacho Universal de Guerra, Marina, Indias, y Hacienda, Superintendente General de ella, y con Honores de Lugar-Theniente General del Almirantazgo General de España, y de las Indias, &c. y Protectòr de la misma Academia.

POR DON PEDRO PADILLA Y ARCOS, CAPITAN, è Ingeniero Ordinario de los Exercitos, Plazas, y Fronteras de S. M. y Directòr de la referida Academia.

DE ORDEN DE S. M.

EN MADRID, en la Imprenta de Antonio Marin, año de M.DCC.LIII.

Figure 1: *Title page of the Military Course of Mathematics (Padilla 1753-1756)*

From the preface of his first volume it is evident that Padilla aimed to show the basic principles of each branch of mathematics, useful enough not only for infantry and cavalry, but also for engineers, artillery and navy (Padilla 1753-1756, preface). Padilla's work introduced a significant change in the pedagogical methods used so far (Blanco 2013, 772). Following the royal regulations (*ordenanzas*), the courses taught at the Military Academy of Mathematics of Barcelona were usually dictated by teachers and assistants. This actually was the rule in most teaching institutions in Europe at the time. Students had to copy down the courses and later make a fair copy of their notes that had to be presented to the teacher once a fortnight (Portugues 1765, VI, 867; De Mora and Massa-Esteve 2008). In his dedication to the King, Padilla stated the reason why he undertook his *Curso*:

> … I have composed the current Course, or Compendium of the subjects taught at the mentioned Academy, so that, being printed, the students, relieved from the annoyance of writing, incompatible with their daily duties, can make greater progress in the study (Padilla 1753-1756, dedication to the King).

By getting his *Curso* printed, Padilla seems to have somehow disregarded the *ordenanzas* of 1739 and 1751.

Besides, Padilla's approach to the general division of mathematics, elaborated in the preface, is similar to that of D'Alembert's tree of knowledge in the *Discours préliminaire* of the *Encyclopédie* (1751), including of course the division of

Mathematics into pure and mixed.[4] Therefore Padilla's classification illustrates the reception and circulation of the ideas of the *Encyclopédie* in Spain (Sánchez-Blanco Parody 1991; Puig-Pla 2002; Blanco 2013).
The aim of this contribution is to explore the connection between theory and practice in Padilla's mathematical course and to examine this course to understand what Padilla regarded as useful mathematics for engineers. This is a preliminary study which provides an overview of the volume on geometry and Padilla's views on practical geometry.

On Elementary and Practical Geometry

The abovementioned royal regulations (*ordenanzas*) established for each academic year the topics to be taught at the Military Academy of Mathematics of Barcelona, how to teach them and the staff in charge. Stationed in spots far from Barcelona, not all the officers had the chance to attend the classes at the Academy. To overcome this obstacle, the academies in Oran and Ceuta were founded in 1732 and 1739, respectively, following the regulations established by the one in Barcelona, as the head academy.[5] In 1738, Pedro de Lucuce (1692–1779) was appointed headmaster of the Military Academy of Mathematics of Barcelona. From 1739 he was in charge of the elaboration of a course of mathematics, the orientation of which was stated in the *ordenanza* of 1739 (De Mora and Massa-Esteve 2008). This course (*Curso Mathematico para la Instrucción de los Militares*), completed in 1744, consisted of eight treatises on the main fields of Mathematics, including pure mathematics (arithmetic and geometry) and mixed mathematics (cosmography, statics, hydraulics, architecture, artillery, and fortification) (Massa-Esteve *et al.* 2011).
Given the relevance of the Military Academy of Mathematics of Barcelona, it is only natural to wonder to what extent Lucuce's course could have influenced Padilla's course. In particular, this paper aims to provide an overview of the volumes that deal with geometry (elementary and practical). In the Treatise II (*On Elementary Geometry*), Lucuce provided an eminently practical view, geometrically constructed in the field for students of artillery and fortifications (De Mora and Massa-Esteve 2008). According to Massa-Esteve *et al.* (2011), the topics of this treatise were: Euclid's *Elements*; rectangles formed over a straight line divided into parts; properties of the circle and straight lines both touching it and inside it; ratio and properties of plane figures; the prism and the parallelepiped, the pyramid, the prism, the cylinder and the sphere; conic sections. At the beginning of this treatise Lucuce stated:

> Since the work is extensive and diffuse, we explain in this treatise books 2, 3, 6, 11 and 12, with respect to which book 4 is addressed in Practical Geometry and book 5 in Arithmetic, while the others, being of little use, are omitted. The order I follow in the propositions is the same as that given by Euclid, so that

[4] It is worth mentioning here that in D'Alembert's classification, the branch of elementary geometry (in pure mathematics) included architecture and tactics, whereas in Padilla's course these branches belonged to mixed mathematics.
[5] See Ordenanzas of July 22, 1739, and December 29, 1751 (Portugues 1765, VI, 858–883, 889–925).

they may be cited whenever necessary, the most useful being demonstrated with all possible brevity and clarity in order to save time for the explanation of other subjects that are of concern for the instruction of military personnel (from Lucuce's course, as quoted in De Mora and Massa-Esteve 2008, 874).

Padilla based his Treatise I (*On Ordinary Arithmetic*) on Euclid's book V, as Lucuce did. Likewise, he acknowledged that his Treatise II (*On Elementary, or Euclidean, Geometry*) was mainly based on Euclid's *Elements* books I–VI and XI–XII (Padilla, 1753-1756, II, 8-9). However, from Table 1 it is evident that he did not use exactly the same books as Lucuce did.

Treatise II (Padilla 1753-1756)		Euclid's *Elements*
Section 1	General Principles of Geometry	
Section 2	Straight lines, angles, triangles and parallelograms	Book I
Section 3	Circle and regular figures	Books III and IV
Section 4	Ratio and proportion of plane figures	Book VI
Section 5	Planes (sections and positions)	Book XI
Section 6	Solids	

Table 1: *Content comparison of Treatise II (Padilla 1753-1756) and Euclid's Elements*

It is important to remark that, while Lucuce maintained Euclid's original order, Padilla regarded it as absolutely damaging for the study of the applications of geometry (Padilla 1753-1756, II, 9).
Padilla, as well as Lucuce, favoured algebraic approaches in his use of Euclid's *Elements*. Hence, for instance, in Treatise I Padilla used fractional notation to express proportional quantities and regarded ratios as fractions:

$$\frac{66}{33} = \frac{10}{5}$$

If four numbers 66, 33, 10, 5 would be proportional [that is, if], the product of the extremes 66 · 5 is equal to the product of the means 33·10 (Padilla 1753-1756, I, §131).

Moreover, in Treatise II Padilla used the mathematical symbol *:* when dealing with proportional magnitudes, instead of defining them verbally:[6]

> If *CD* : *DE* = *FD* : *DB* the parallelograms will be equal… (Padilla 1753-1756, II, §2).

Finally, Lucuce addressed the study of practical geometry in his Treatise III, which was based on Euclid's book IV (De Mora and Massa-Esteve 2008, 874). This treatise was divided into eight books.[7] When it comes to Padilla's course, it is true that there was no single treatise devoted explicitly to practical geometry. Yet, the contents of some its treatises could be somehow connected with the contents of Lucuce's Treatise III, as Table 2 shows.

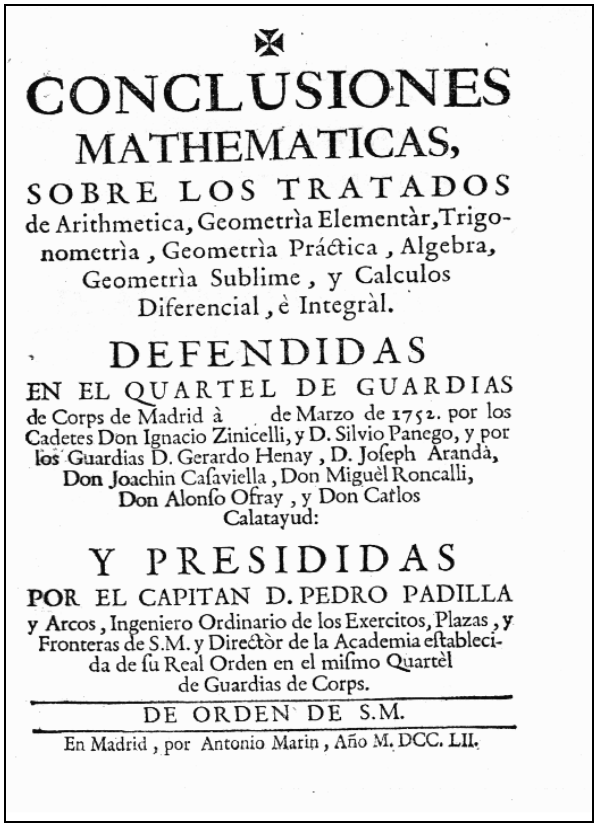
Lucuce (Treatise III)	Padilla (1753-1756)
On plane trigonometry (including a chapter on the nature and use of logarithms)	Treatises VI and VII
On the construction of plane figures	Treatise II
On the inscription and circumscription of plane figures in the circle	
On the proportion, enlargement, reduction and transformation of plane figures	
On the use of some instruments	Not known
On planimetry or euthimetry	Treatise II
On stereometry or the dimension of solids	
On levelling	Treatise XX

Table 2: *Content comparison of Lucuce's Treatise III and Padilla's course*

Table 2 indicates that Padilla's Treatise II shared a number of topics with five of the books of Lucuce's Treatise III. Unfortunately, most of the treatises in Padilla's course

[6] On the algebraic ways to manipulate proportions in Euclid's book V see Goldstein (2000).
[7] For a thorough study of practical geometry in the context of engineering education in Spain see Massa-Esteve *et al*. (2011). In De Mora and Massa-Esteve (2008, 875) there is a full description of the contents of Lucuce's treatise on practical geometry.

that were connected with practical geometry remained unpublished, except Treatise II. Therefore, a thorough analysis of Padilla's views on practical geometry cannot be led. But the fact that practical geometry played an important role in the public examinations held in 1752 at the Military Academy of the Royal Guards proves the importance granted to this subject in this context (Fig. 2).[8] In these examinations, students were asked to measure distances, using the plane table and the protractor, to draw land or city plans, by means of the plane table, or how to pump water from a river up to the highest part of the city for street cleaning.



CONCLUSIONES
MATHEMATICAS,
SOBRE LOS TRATADOS
de Arithmetica, Geometrìa Elementàr, Trigonometrìa, Geometrìa Práctica, Algebra, Geometrìa Sublime, y Calculos Diferencial, è Integràl.

DEFENDIDAS
EN EL QUARTEL DE GUARDIAS
de Corps de Madrid à de Marzo de 1752. por los Cadetes Don Ignacio Zinicelli, y D. Silvio Panego, y por los Guardias D. Gerardo Henay, D. Joſeph Arandà, Don Joachin Caſaviella, Don Miguèl Roncalli, Don Alonſo Ofray, y Don Carlos Calatayud:

Y PRESIDIDAS
POR EL CAPITAN D. PEDRO PADILLA
y Arcos, Ingeniero Ordinario de los Exercitos, Plazas, y Fronteras de S.M. y Directòr de la Academia eſtablecida de ſu Real Orden en el miſmo Quartèl de Guardias de Corps.

DE ORDEN DE S.M.

En Madrid, por Antonio Marin, Año M. DCC. LII.

Figure 2: *Title page of the public examinations held in the Academy of Mathematics of the Royal Guards (1752)*

Final remarks

From this preliminary study, it is very likely that Padilla's volume on geometry was influenced by Lucuce's course. Yet, it is still necessary to carry out a more detailed comparative analysis of the mathematical courses of Padilla and Lucuce on this subject. In particular, it would be worth analysing and comparing not only the process of arithmetization, but also which editions of Euclid's *Elements* they used.

[8] *Conclusiones Mathematicas, sobre los tratados de Arithmetica, Geometrìa Elementàr, Trigonometrìa, Geometrìa Práctica, Algebra, Geometrìa Sublime, y Calculos Diferencial, è Integràl. Defendidas en el Quartel de Guardias de Corps de Madrid*. Madrid: Antonio Marín, 1752 (AGS Guerra Moderna, 3778).

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