ROBUSTNESS AND COMPUTATIONAL EFFICENCY OF A KALMAN FILTER ESTIMATOR OF TIME DEPENDENT OD MATRICES EXPLOITING ICT TRAFFIC MEASUREMENTS

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ABSTRACT

Origin-Destination (OD) trip matrices, which describe the patterns of traffic behavior across the network, are the primary data input used in principal traffic models and therefore, a critical requirement in all advanced systems that are supported by Dynamic Traffic Assignment models. However, because OD matrices are not directly observable, the current practice consists of adjusting an initial or seed matrix from link flow counts which are provided by an existing layout of traffic counting stations. The availability of new traffic measurements provided by Information and Communication Technologies (ICT) applications allows more efficient algorithms, namely for the real-time estimation of OD matrices based on modified Kalman Filtering approaches exploiting the new data. The quality of the estimations depends on various factors, like the penetration of the ICT devices, the detection layout and the quality of the initial information. Concerning the feasibility of real-time applications, another key aspect is the computational performance of the proposed algorithms for urban networks of sensitive size. This paper presents the results of a set of computational experiments with a microscopic simulation of a network of the business district of Barcelona, which explore the sensitivity of the Kalman Filter estimates with respect to the values of the design factors, and its computational performance.

Keywords: Dynamic OD Estimation, Dynamic User Equilibrium (DUE), Traffic Detectors Layout, Kalman Filtering

INTRODUCTION

Traffic assignment models have the objective of assigning a trip matrix onto a network, in terms of a route choice mechanism, in order to estimate the traffic flows in the network. Therefore, they all use Origin-Destination (OD) trip matrices as major data input for describing the patterns of traffic behavior across the network. All formulations of static traffic assignment models (Florian and Hearn [1], as well as dynamic, Ben-Akiva et al. [2]) assume that a reliable estimate of an OD is available. However, OD matrices are not yet directly observable, even less so in the case of the time-dependent OD matrices that are necessary for Dynamic Traffic Assignment models; consequently, it has been natural to resort to indirect estimation methods. The main modeling hypothesis of these indirect estimation methods, or matrix adjustment methods, can be stated as follows: if traffic flows in the links of a network are the consequence of the assignment of an OD matrix onto a network, then, for a set of measured link flows, the problem of estimating the OD matrix that generates such flows can be considered as the inverse of the assignment problem (Cascetta [3]). Since the earlier formulation of the problem by Van Zuylen and Willumsen [4], the matrix adjustment problem has been a relevant research and practical problem. The current practices consist of using an initial OD estimate, the OD seed or OD target as input, and adjusting them from the available link counts provided by an existing layout of traffic counting stations and other additional information whenever it is available. Adjustments can be considered as indirect estimation methods, based either on discrete time optimization approaches (Codina and Barceló [5], Lundgren and Peterson [6]) or on adaptations of Kalman Filtering, [7], approaches (Ashok et al. [8], Antoniou et al. [9], Barceló et al. [10]).

In this paper we assume that the usual traffic data collected by inductive loop detectors (i.e. volumes, occupancies and speeds) are complemented by accurate measurements of travel times and speeds between two consecutive sensors, based on new technologies, able to capture the electronic signature of specific on-board devices, such as a Bluetooth device on-board a vehicle. Data captured by each sensor is sent to a central server by wireless telecommunications for processing. This raw data after a suitable filtering and cleaning preprocessing, Barceló et al. [10], [11] and [12], is the main input to a new Kalman Filter approach for estimating time-dependent OD matrices. The proposed approach, which exploits the explicit travel time measurements from Bluetooth (BT) detectors, is based on a reformulation, [11], [12], of the Kalman Filter approach for freeways explored in Barceló et al, [10], that extends the approach to urban networks where *alternative paths are available and route choice is relevant*. This new approach exploits the measurements of travel times in order to reduce the number of state variables as well as to simplify the model.

The computational results reported in [11], and especially in [12] probed in general terms the consistency and robustness of the new approach, with respect to its capability of reconstructing a known OD matrix in synthetic experiments. However, the results raised some questions, that could determine the utility of the procedure in real time applications, with respect to their dependency on the detection layout, in terms of the number and location of the ICT sensors, the % penetration of the ICT technologies, that is Bluetooth on board devices in our case, and the computational performance for networks of significant size. In this paper we explore the consistency of the estimated OD conducting a set of simulation experiments in the network of the Central Business District of the City of Barcelona. The experiment setting has been designed using three main design factors: the detection layout, in terms of number and location of the ICT detectors, the % of vehicles equipped with Bluetooth devices, and the quality of the initial information that is the target OD matrix. In all experiments especial attention has been paid to the computational times.

FACTOR 1: SETTING UP THE DETECTION LAYOUT

In Barceló et al. [13] we dealt with the problem of how to choose a detection layout, for a matrix reconstruction of good quality, when most likely used OD paths and flows can be collected. The link detection layout problem was formulated by Yang and Zhou [14] as a set covering problem on links with additional constraints. An analysis of the advantages and disadvantages of various alternative formulations, in terms of the quality of the adjusted OD matrix can be found in the paper by Larsson et al. [15].

However, when dealing with sensors that capture the electronic signature, as the detectors of Bluetooth devices on board vehicles, the link detector location problem must be formulated in different terms by taking into account that these detectors are more efficiently located at intersections and not at links, where they can capture a higher number of vehicles. Assuming that a Bluetooth sensor is located, as in Figure 1, at the intersection, in such a way that its detection lobule intercepts all equipped vehicles crossing the node on paths (1), (2), (3) and (4), the candidate intersections would be those intercepting a higher number of equipped vehicles.

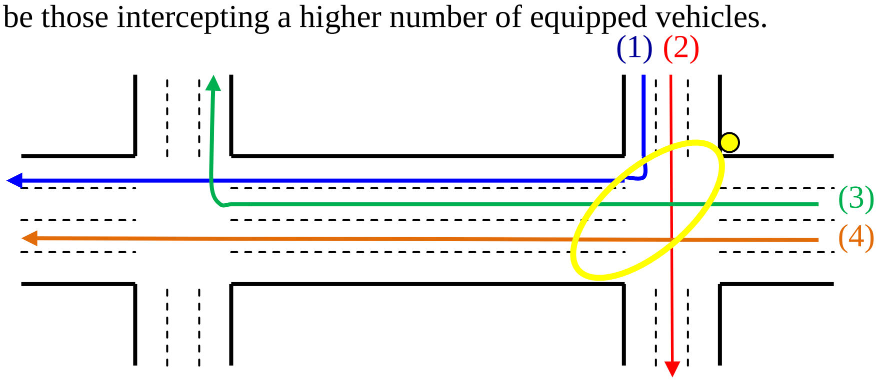

Figure 1 - Flows intercepted from paths crossing a node

The detector location problem becomes then a node covering problem, Bianco et al. [16], instead of a link covering problem. In Barceló et al. [13] we explored a mixed formulation combining network topological aspects, as in [16], with an adaptation of Yang's rules [14]. In this paper we are considering that in practice the location of traditional detectors, e.g. loop detectors, has been set up in the past, determined by the needs of traffic management sytems, therefore the possibility of locating new sensors only affects the new ICT sensors. In consequence the detection layout problem addressed in this paper only concerns the location of new ICT sensors that measure the travel times between two of them. The modified set covering formulation, used to determine the layouts for the computational experiments, is a modification of the model in Barceló et al. [13]. In this case, we propose a new formulation that includes a multiobjective function balancing the total flow captured and the number of intercepted paths in such a way that, if we maximize the multiobjective function, we are indirectly maximizing the set of covered OD pairs. Since our objective is balancing the total flow captured and the number intercepted paths, the model includes two additional sets of constraints in order to:

- Ensure a minimum number of detectors on each path, to achieve the objective of measuring travel time along the likely used paths, identified from a Dynamic User Equilibrium assignment using an historical OD matrix, and
- Impose a condition of minimum linear distance between two detectors, which can be technologically justified to minimize the likelihood of improper detection due to signal overlapping.

The proposed set covering formulation used in this paper has been:

$$MAX \quad \alpha\left(\frac{\sum_{k\in K} h_k \cdot y_k}{\sum_{k\in K} h_k}\right) + \beta\left(\frac{\sum_{k\in K} y_k}{|K|}\right)$$

$$s.t.$$

$$\sum_{n\in N} x_n \leq \hat{l}$$

$$\sum_{n\in N} \delta_{nk} \cdot x_n \geq p \cdot y_k \, , \quad \forall k \in K_i, \forall i \in I \qquad (1)$$

$$x_i + x_j \leq 1 \;\; \forall i, \forall j \in V(i)$$

$$x_{n,} y_k \in \{0,1\}$$

Where:

$N \;=\; set\ of\ intersections\ (nodes) of\ the\ network$
$I \;=\; set\ of\ all\ OD\ pairs\ in\ the\ network$
$K \;=\; set\ of\ all\ paths\ between\ all\ OD\ pairs$
$K_i = \; set\ of\ paths\ for\ the\ ith\ OD\ pair; K = \bigcup_{i\in I} K_i$

$$x_n = \begin{cases} 1 & if\ a\ sensor\ is\ located\ at\ intersection\ n \in N \\ 0 & otherwise \end{cases}$$

$$y_k = \begin{cases} 1 & if\ there\ is\ at\ least\ one\ detector\ along\ path\ k \in K \\ 0 & otherwise \end{cases}$$

$$\delta_{nk} = \begin{cases} 1 & if\ intersection\ n \in N\ is\ into\ path\ k \in K \\ 0 & otherwise \end{cases}$$

$h_k = flow\ on\ path\ k \in K$
$\hat{l} \; = maximum\ number\ of\ detectors\ to\ be\ located$
$V(i) = \{j \in I | dist(i,j) \leq m\ meters\}$, $linear\ neighborhood\ of\ intersection\ i.$
$m = minimum\ linear\ distance\ between\ two\ detectors.$
$p = minimum\ number\ of\ detectors\ per\ path;$
($p > 1\ provides\ travel\ times\ between\ detectors\ along\ a\ path.$)

In practice, budgetary limitations can impose a bounding constraint on the maximum number $\hat{l}$ of detection stations that can be located on a network. Formulation (1) always provides a feasible solution although not all OD pairs and paths are covered. The percentages of total flow, OD pairs covered and paths intercepted provide a measure of the quality of the proposed layout.

Experimental Design: Definition of Factor 1 Levels

The first design factor used is the detection layout defined in terms of the number of the Bluetooth detectors inside the selected urban area, and their location. A set of computational experiments has been conducted with various networks. Due to the space limitations we discuss here only the results for the largest considered network, the Barcelona's BCD district (Eixample), consisting of 2111 sections, 1227 nodes (grouped in intersections), 120 generation centroids, 130 destination centroids and a total of 877 non-zero OD pairs.. Computational experiments have been conducted with a calibrated Aimsun model available from other projects it has been used to simulate a set of scenarios, each one corresponding to a level of Factor 1, the detection layout obtained solving model (1) for a value of the design parameter $\hat{l}$. Under the assumption that Bluetooth sensors are located at inner intersections of the urban area, a measure of the quality of the layout, in terms of number and location of the detectors, [13], is determined by the indices: % of the total intercepted flow, % of OD pairs covered and % of DUE paths intercepted. Table 1 summarizes the numerical results for p = 2, m=150 meters and values of $\hat{l}$ that have been defined incrementally.

BT Sensors	QUALITY OF THE LAYOUT					
	%INTERSECTIONS	% TOTAL FLOW	TOTAL OD PAIRS	% OD PAIRS	TOTAL PATHS	% PATHS
10	<1%	46.93	586	66.82	1551	51.07
15	1.25%	59.34	636	72.52	1781	58.64
30	2.5%	80.66%	747	85.18	2319	76.36
45	3.75%	92.12	808	92.47	2672	87.98
60	5%	98.38	857	97.72	2895	95.32
75	6.25%	99.75	875	99.77	2980	98.12
90	7.5%	99.97	877	100	3032	99.84

Table 1 Summary of numerical results of the quality of Bluetooth sensor location in Eixample

Table 1, summarizes the quality of the BT detection layout when it is incrementally increased. The layout covering is characterized in terms of the percentage of equipped intersections in the scenario, percentage of captured OD pairs, historic OD flows and DUE paths intercepted twice by the BT layout. The minimum number of BT sensors at nodes in the interior area, to cover 95% of DUE OD paths at least twice (p=2) in the 877 OD pairs is 60, and the total captured flow is higher than 98%.

SPACE-STATE MODEL FOR DYNAMIC ESTIMATION

The space-state formulation based on Kalman Filtering, whose robustness is tested in this paper, is the recursive linear Kalman-Filter for state variable estimation, discussed in Barceló et al. [11], [12], adapted to exploit the travel times and traffic counts collected respectively by tracking Bluetooth equipped vehicles and conventional detection technologies.

The proposed approach assumes flow counting detectors and ICT sensors located in a cordon at each possible point for flow entry (*centroids* of the study area) and ICT sensors located at intersections in urban networks covering links to/from the intersection as in Figure 1. Flows and travel times are available from ICT sensors for any time interval length higher than 1 second. Trip travel times from origin entry points to sensor locations are measures provided by the detection layout. Therefore, they are no longer state variables but measurements, which simplify the model and make it more reliable.

A basic hypothesis is that equipped and non-equipped vehicles follow common OD patterns. We assume that this holds true in what follows and that it requires a statistical contrast for practical applications. Expansion factors from equipped vehicles to total vehicles, in a given interval, can be estimated by using the inverse of the proportion of ICT counts to total counts at centroids; expansion factors are assumed to be shared by all OD paths and pairs with a common origin centroid and initial interval.

The proposed linear formulation of the Kalman Filtering approach uses deviations of OD path flows as state variables, as suggested by Ashok and Ben-Akiva [8], calculated in respect to DUE-based Historic OD path flows for equipped vehicles. But our approach differs in that we **do not require an assignment matrix**. We use instead the subset of the most likely OD path flows identified from a DUE assignment. The DUE is conducted with the historic OD flows, and the number of paths to take into account is a design parameter. A list of paths going through the sensor is automatically built for each ICT sensor from the OD path description, ICT sensor location and the network topology. In this way, once an equipped car is detected by ICT sensor *j*, the travel time from its entry point to sensor *j* is available and it is used for updating time varying model parameters that affect OD paths (state variables) which are included in the list.

We model the time-varying dependencies between measurements (sensor counts of equipped vehicles) and state variables (deviates of equipped OD path flows), adapting an idea of Lin and Chang [18], for estimating discrete approximations to travel time distributions. Their estimation of these distributions is made on the basis of flow models which induce nonlinear relationships that require extra state variables, leading to a non linear KF approach. Since our approach exploits the travel ICT time measurements from equipped vehicles, we can replace the nonlinear approximations by estimates from a sample of vehicles. The main advantage is that no extra state variables for modeling travel times and traffic dynamics are needed, since sampled travel times are used to estimate discrete travel time distribution (H bins are used for adaptive approximations). Additionally, travel times collected from ICT sensors are incorporated into the proposed model and it is not necessary that vehicles reach their destination, since at any intermediate sensor that they pass by, the travel time measured from the entry point (centroid) to that sensor updates the discrete travel time approximation. No information about trajectories of equipped vehicles is used in this version.

The demand matrix for the period of study is divided into several time-slices, accounting for different proportions of the total number of trips in the time horizon.

The approach assumes an extended state variable for M+1 sequential time intervals of equal length Δt, M is the maximum number of time intervals required for vehicles to traverse the entire network in a congested scenario.

The solution provides estimations of the OD matrices for each time interval up to the k-*th* interval. State variables $\Delta g_{ijc}(k)$ are deviations of OD path flows $g_{ijc}(k)$ relative to historic OD path flows $\tilde{g}_{ijc}(k)$ for equipped vehicles. A MatLab prototype algorithm has been implemented to test the approach (named KFX2).

Let I be the total number of origin centroids, identified by index i, i = 1,..,I; J the total number of destination centroids, identified by index j, j = 1,…,J; Q the total number of ICT sensors, identified by q=1,…Q, where Q = I+P, I ICT sensors located at origins, and P ICT sensors located in the inner network; and K the total number of most likely used paths between origins and destinations. Let $Q_i(k)$ and $q_i(k)$ be respectively the number of vehicles and equipped vehicles entering from centroid *i* at time interval k. Conservation equations from entry points (centroids) are explicitly considered. Without $Q_i(k)$, a generic expansion factor has to be applied.

State Equations

Let $\mathbf{\Delta g}(k)$ be a column vector of dimension IJK containing the state variables $\Delta g_{ijc}(k)$ for each time interval k for all *most likely* OD paths (i,j,c). The state variables $\Delta g_{ijc}(k)$ are assumed to be stochastic in nature, and OD path flow deviates at current time k are related to the OD path flow deviates of previous time intervals by an autoregressive model of order r <<M; the state equations are:

$$\mathbf{\Delta g}(k+1) = \sum_{\mathbf{w=1}}^{\mathbf{r}} \mathbf{D}(w)\mathbf{\Delta g}(k-w+1) + \mathbf{w}(k) \quad (2)$$

Where $\mathbf{w}(k)$ are zero mean with diagonal covariance matrix $\mathbf{W_k}$, and $\mathbf{D(w)}$ are IJKxIJK transition matrices which describe the effects of previous OD path flow deviates $\Delta g_{ijc}(k-w+1)$ on current flows $\Delta g_{\mathbf{ijc}}(k+1)$ for w = 1,…,r . In the implementation tested we assume simple random walks to provide the most flexible framework for state variables, since no convergence problems are detected. Thus r=1 and matrix $\mathbf{D}(w)$ is the identity matrix.

Observation Equations

The relationship between the state variables and the observations involves time-varying model parameters (congestion–dependent, since they are updated from sample travel times provided by equipped vehicles) in a linear transformation that considers:

- The number of equipped vehicles entering from each entry centroid during time intervals k, k-1, k-M, $q_i(k)$.
- H<M *time*-varying *model parameters* in form of *fraction matrices,* $\left[u_{ijcq}^{h}(k)\right]$.

Where the $u_{iq}^{h}(k)$ are the fraction of vehicles that require *h* time intervals to reach sensor *q* at time interval k that entered the system from centroid *i* (during time interval $[(k-h-1)\Delta t \ , \ (k-h)\Delta t]$); and the $u_{ijcq}^{h}(k)$ represent the fraction of equipped vehicles detected at interval k whose trip from centroid *i* to sensor *q* might use OD path (i,j,c) lasting *h* time intervals of length Δt to arrive from centroid *i* to sensor *q*, where *i* = 1,…,I , *j* = 1,…, J , $h = 1 \ldots M$, $q = 1 \ldots Q$. The H adaptive fractions that approximate u_{iq}^{h} and u_{ijcq}^{h} are updated from measures provided by ICT sensors. Direct samples of travel times allow the updating of discrete approximations of travel time distributions, making it unnecessary to incorporate models for traffic dynamics. Time-varying model parameters u_{iq}^{h} to account for temporal traffic dispersion in affected paths u_{ijcq}^{h}, have to satisfy structural constraints, where H<M:

$$\begin{aligned} & u_{ijcq}^{h}(k) \geq 0 \quad i = 1....I, \quad j = 1....J, \quad c = 1....K_{ij}^{\max}, \quad q = 1,...Q, \quad h = 1...H \\ & \sum_{h=1}^{H} u_{ijcq}^{h}(k) = 1 \quad i = 1....I, \quad j = 1....J, \quad c = 1....K_{ij}^{\max}, \quad q = 1,...Q \end{aligned} \qquad (3)$$

At time interval *k*, the values of the observations are determined by those of the state variables at time intervals *k, k-1, …k-M*.

$$\Delta \boldsymbol{z}(\boldsymbol{k}) = \begin{pmatrix} \mathbf{AU(k)^T} \\ \mathbf{E(k)} \end{pmatrix} \Delta \mathbf{g(k)} + \begin{pmatrix} \boldsymbol{v}_1 \boldsymbol{(k)} \\ \boldsymbol{v}_2 \boldsymbol{(k)} \end{pmatrix} = \mathbf{F(k)} \Delta \mathbf{g(k)} + \mathbf{v(k)} \quad (4)$$

Where $\mathbf{v}(k)$ are white Gaussian noises with covariance matrices $\mathbf{R_k}$. **U(k)** consists of diagonal matrices $U(k),\ldots,U(k-M)$ containing $u_{ijcq}^{h}(k)$. For $U(k-h)$ is a matrix with the estimated proportion of equipped vehicles whose travel time from the access point to the network takes *h* intervals and goes through the *q* sensor at interval *k*. **E(k)** is a row matrix of dimension I containing 0 for columns related to state variables in time intervals $k-1,\ldots,k-M$ and defining conservation of flows (sum of OD path flows for each entry) at *k*. And **A** is a matrix that adds up sensor traffic flows from any possible entry, given time-varying model parameters at interval *k*. $\mathbf{F}(\mathrm{k})$ maps the state vector $\Delta\mathbf{g}(\mathrm{k})$ onto the current blocks of measurements at time interval *k*: counts of equipped vehicles by sensors and entries at centroids, accounting for time lags and congestion effects. Deviate counts at *k* mean the observed counts minus the historical demand $\tilde{g}_{\mathbf{ijc}}(k)$ counts, given the current traffic conditions according to *time-varying model parameters*.

EXPERIMENTAL DESIGN: Factors 2 and 3

To test the robustness of the proposed space-state formulation based on Kalman Filtering exploiting the ICT measurements provided by Bluetooth sensors, a set of computational experiments has been conducted for each combination of levels of the three design factors. The computational experiments have been conducted with an Aimsun microscopic simulation model of the test network, including an emulation of loop detectors and Bluetooth sensors in order to reproduce the data collection. The computational experiments assume that loop detectors are located only at the entry centroids of the study area, no additional counts are assumed from loop detectors inside.

The levels of design factor 2 correspond to the assumed values of the market penetration of the Bluetooth technology. As discussed in [10] and [11] the current penetration in many European and eastern countries is in average higher than the 15%, in Barcelona the measured values reported by manufacturers is greater than 30%, and show a steadily increasing trend. Therefore, to be on the safe side, we have considered values in the range 10%-50% and, only for comparison purposes, a hypothetical scenario with 100%

penetration. However, we should not forget that the actual percentage of vehicles equipped with Bluetooth devices is a variable fraction of the total number of vehicles in the area, which depends on many socioeconomic factors and may change with the time of the day, therefore, to expand the sample of Bluetooth data at entry points additional loop detectors are necessary, or a common expansion factor equal to the inverse of the penetration rate in the study area has to be used.

To determine the levels of design Factor 3 a synthetic Historical Origin-Destination matrix has been used. This OD matrix has been estimated to represent the transition to the congested situation according to the MFD (MacroFundamental Diagram, Daganzo and Geroliminis, [19]) which provides an overall picture of the flow-density relationship. The horizon study is 1h and 15 minutes and the OD is sliced into five 15-minute slices with the same OD pattern --each one accounting for 20%, 28%, 20%, 12% and 20% of the total number of trips, 59774 -- to emulate demand variability.

Figure 2 depicts the methodological framework for the definition of simulation experiments. The assumed OD matrix is the result of perturbing the true historical matrix to define the levels of Factor 3.

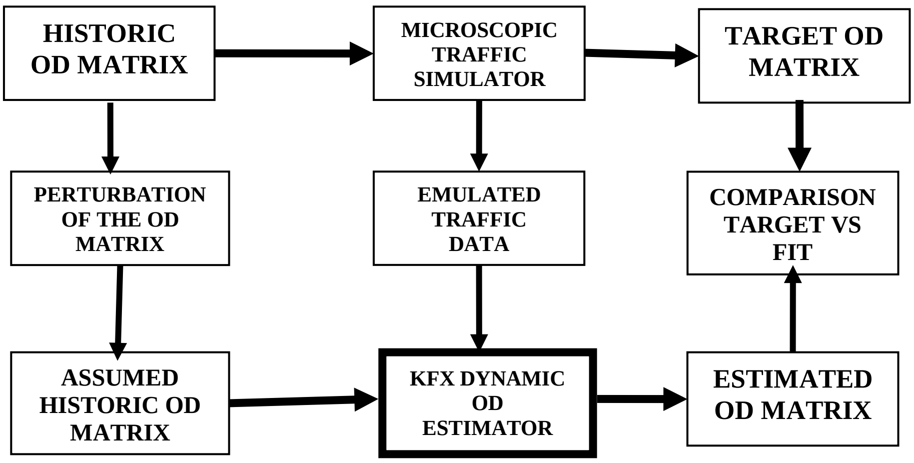

Figure 2. Methodological Design of the Computational Experiments by Simulation

The fixed OD pattern considered in the historical OD matrix provides a simple way to build perturbed matrices, by changing OD pattern in subsets of origin centroids, that will be the assumed ‘false’ historical matrices in the simulation experiments, to test if the KF algorithm is able to converge to the ‘true’ historical OD matrix. We do not impose any restriction related to fixed OD pattern across time-slices in the KF formulation.

Tuning parameters (not considered as design factors) are not discussed in this paper, although the tuning for a network requires a good knowledge of the network, Barceló et al. [12]:

- *Deviate tuning parameter (v_1) is set to 0.5 when deviates are considered or 0 otherwise.*
- *Interval length Δt.* (Δt = 180 seconds has been used)
- *The time-interval for modeling purposes in the KF approach is 3 min*

Design factors in computational experiments are then:

Factor 1. The quality of the BT detection layout in the interior of the study area, defined in Table 1.

Factor 2. The variable percentage of BT equipped vehicles, with levels of 10%, 20%, 30%, 50% and 100%.

Factor 3. The initialization of the state variables $\Delta g_{ijc}(0)$ is set to 0. That is, OD path flows are equal to the *assumed historical OD path flows for all time intervals*, and the OD paths are the most likely used paths from a DUE assignment. The levels of Factor 3 depend on the following situations:

a) Level NoHA: No deviates are computed, no *a priori* information about the historical matrix (reliable or not) is considered.
b) Level HA0: The perturbed matrix is the true historical matrix and deviates of the state variables are computed assuming an equal fraction of use of all the OD paths belonging to an OD pair and true historical OD flow affected by (v_1). This is an excellent initialization point.
c) Level HAx: The perturbed matrix is the maximum entropy without a priori information applied to the marginal generation totals of the true historical matrix and deviates of the state variables are computed assuming an equal fraction of use of all the OD paths belonging to an OD pair and perturbed matrix affected by (v_1). This is an awful initialization point.

Collected Performance Indicators

Target OD flows per interval are compared with estimated OD flows per interval at OD pair level by means of Theil's coefficient, a measure on how close two time series are; bounded between 0 and 1, U=0 representing a perfect fitting, and U=1 an unacceptable discrepancy. Values of U>0.2 recommend rejecting the fit. Theil's coefficient (U) can be computed for each OD pair or for a subset of OD pairs (i.e., defined by the quartiles of hourly historical OD flows). If all feasible OD pairs are considered, then a global Theil measure of fit, GU, is computed as:

$$GU = \frac{\sqrt{\frac{1}{IJ \cdot G}\sum_{k=1:G}\sum_{od=1:IJ}\left(y_{od,k} - \hat{y}_{od,k}\right)^2}}{\sqrt{\frac{1}{IJ \cdot G}\sum_{k=1:G}\sum_{od=1:IJ}\hat{y}_{od,g}{}^2} + \sqrt{\frac{1}{IJ \cdot G}\sum_{k=1:G}\sum_{od=1:IJ}y_{od,k}{}^2}} \quad (5)$$

Other performance indicators used either at OD pair, or set of OD pair levels, are the normalized root mean square error (RMSEN), a weighted indicator for subsets of OD pairs (usually subset of OD pairs whose hourly flow is in 25% of higher flows) and a weighted global indicator for the whole set of OD pairs (GRMSEN, sum of squared differences between target and estimated path flows per interval, relative to total target flows during the simulation horizon):

$$RMSEN = \frac{\sqrt{G\sum\left(y_k - \hat{y}_k\right)^2}}{\sum y_k} \qquad GRMSEN = \frac{\sqrt{IJ \cdot G\sum_{k=1:G}\sum_{od=1:IJ}\left(y_{od,k} - \hat{y}_{od,k}\right)^2}}{\sum_{k=1:G}\sum_{od=1:IJ}y_{od,k}} \quad (6)$$

Where G is the number of time intervals G=25 (since 1h15min is the considered time horizon) and IJ is the number of OD pairs, IJ=877 in our test network.

COMPUTATIONAL RESULTS

Table 2 summarizes the computational results for the set of experiments. Values in the table are for the proposed global performance indicators, U and RMSEN, and the coefficient of determination R^2. Each sub-table corresponds to a level of Factor 3 and all levels of Factors 1 (Rows) and 2 (Columns). Sub-tables (2A), (2B) and (2C) describe respectively the results for experiments with Level HA0, HAx and NoHA of Factor 1, and combinations of all levels of Factors 1 and 2.

Factr 1 # BT Inner Sensors	**(2A) Factor 3 Level HA0**														
	Factor 2 % of BT														
	10%			**20%**			**30%**			**50%**			**100%**		
	GU	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**
10	0.74 (0.48)	230% (135%)	**88.5% (82.6%)**	0.64 (0.38)	213% (92%)	**90.7% (85.7%)**	0.56 (0.32)	193% (78%)	**92.5% (88.3%)**	0.49 (0.25)	147% (53%)	**93.2% (89.3%)**	0.34 (0.15)	86% (32%)	**93.6% (89.8%)**
30	0.74 (0.48)	230% (136%)	**89.9% (85.0%)**	0.64 (0.38)	213% (93%)	**92.2% (88.3%)**	0.56 (0.32)	193% (78%)	**93.5% (90.1%)**	0.49 (0.25)	148% (54%)	**94.1% (90.9%)**	0.35 (0.16)	87% (33%)	**94.5% (91.5%)**
45	0.74 (0.48)	230% (136%)	**90.0% (85.4%)**	0.64 (0.38)	214% (93%)	**91.5% (87.2%)**	0.56 (0.32)	193% (78%)	**93.0% (89.4%)**	0.49 (0.26)	148% (55%)	**93.6% (90.2%)**	0.35 (0.17)	87% (34%)	**94.0% (90.7%)**
60	0.74 (0.48)	231% (136%)	**90.4% (86.1%)**	0.65 (0.39)	214% (94%)	**91.9% (87.9%)**	0.56 (0.33)	193% (79%)	**93.1% (90.3%)**	0.49 (0.26)	149% (55%)	**93.7% (90.3%)**	0.35 (0.17)	88% (35%)	**94.0% (90.8%)**
75	0.74 (0.48)	230% (135%)	**90.8% (86.7%)**	0.65 (0.38)	214% (94%)	**92.2% (88.3%)**	0.56 (0.32)	193% (79%)	**93.3% (89.8%)**	0.49 (0.26)	149% (55%)	**93.9% (90.6%)**	0.35 (0.17)	88% (34%)	**94.1% (91.0%)**
90	0.74 (0.48)	231% (136%)	**90.5% (86.2%)**	0.65 (0.39)	214% (94%)	**92.2% (88.5%)**	0.56 (0.33)	194% (79%)	**93.1% (89.6%)**	0.49 (0.26)	149% (55%)	**93.8% (90.5%)**	0.35 (0.17)	88% (35%)	**94.0% (90.9%)**

Factr 1 # BT Inner Sensors	**(2B) Factor 3 Level HAx**														
	Factor 2 % of BT														
	10%			**20%**			**30%**			**50%**			**100%**		
	GU	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**
10	0.70 (0.53)	247% (136%)	**74.7% (70.1%)**	0.60 (0.43)	264% (95%)	**76.8% (72.6%)**	0.54 (0.37)	295% (78%)	**78.2% (75.3%)**	0.47 (0.31)	265% (59%)	**79.1% (76.3%)**	0.39 (0.23)	206% (39%)	**79.4% (76.7%)**
30	0.70 (0.53)	247% (136%)	**77.2% (74.3%)**	0.60 (0.44)	265% (95%)	**79.4% (77.1%)**	0.54 (0.37)	295% (79%)	**80.2% (78.6%)**	0.48 (0.32)	265% (60%)	**80.9% (79.3%)**	0.39 (0.24)	206% (41%)	**81.3% (80.0%)**
45	0.70 (0.54)	248% (137%)	**77.5% (75.1%)**	0.61 (0.44)	265% (96%)	**78.5% (75.8%)**	0.54 (0.37)	295% (80%)	**79.6% (77.9%)**	0.48 (0.32)	265% (61%)	**80.4% (78.8%)**	0.39 (0.24)	206% (42%)	**80.8% (79.4%)**
60	0.70 (0.53)	247% (137%)	**78.2% (76.1%)**	0.61 (0.44)	264% (97%)	**79.2% (76.8%)**	0.54 (0.38)	294% (80%)	**79.9% (78.2%)**	0.48 (0.32)	264% (61%)	**80.7% (78.9%)**	0.39 (0.24)	205% (43%)	**81.0% (79.5%)**
75	0.70 (0.53)	247% (136%)	**79.1% (77.5%)**	0.60 (0.44)	264% (96%)	**80.0% (78.0%)**	0.54 (0.37)	293% (80%)	**80.6% (79.1%)**	0.48 (0.32)	264% (61%)	**81.4% (79.9%)**	0.39 (0.24)	204% (42%)	**81.6% (80.4%)**
90	0.71 (0.53)	247$ (137%)	**78.8% (77.0%)**	0.60 (0.44)	264% (97%)	**80.2% (78.4%)**	0.54 (0.37)	293% (80%)	**80.5% (79.1%)**	0.48 (0.32)	264% (61%)	**81.4% (79.9%)**	0.39 (0.24)	204% (43%)	**81.4% (80.3%)**

Factr 1 # BT Inner Sensors	**(2C) Factor 3 Level NoHA**														
	Factor 2 % of BT														
	10%			**20%**			**30%**			**50%**			**100%**		
	GU	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**	**GU**	**RMSEN**	**R^2**
10	0.78 (0.53)	234% (143%)	**79.9% (72.2%)**	0.70 (0.45)	219% (103%)	**81.9% (74.7%)**	0.63 (0.40)	202% (91%)	**83.5% (77.1%)**	0.57 (0.34)	159% (68%)	**84.4% (78.2%)**	0.46 (0.27)	103% (51%)	**84.7% (78.8)**
30	0.78 (0.53)	234% (143%)	**81.2% (74.3%)**	0.70 (0.45)	220% (103%)	**83.3% (77.0%)**	0.63 (0.40)	202% (91%)	**84.5% (78.7%)**	0.57 (0.34)	159% (69%)	**85.3% (79.7%)**	0.46 (0.28)	103% (52%)	**85.7% (80.4%)**
45	0.78 (0.53)	234% (144%)	**81.6% (75.1%**	0.70 (0.45)	220% (104%)	**83.0% (76.7%**	0.63 (0.40)	202% (92%)	**84.4% (78.7%)**	0.57 (0.34)	159% (69%)	**85.3% (79.8%)**	0.46 (0.28)	103% (54%)	**85.6% (80.4%)**
60	0.78 (0.53)	234% (144%)	**82.1% (75.8%**	0.70 (0.45)	220% (104%)	**83.5% (77.4%)**	0.63 (0.40)	202% (92%)	**84.7% (78.9%)**	0.57 (0.34)	159% (69%)	**85.5% (80.0%)**	0.46 (0.28)	103% (53%)	**85.9% (80.6%)**
75	0.78 (0.53)	234% (143%)	**82.7% (76.6%**	0.70 (0.45)	220% (104%)	**84.0% (78.1%)**	0.63 (0.40)	202% (91%)	**85.1% (79.5%)**	0.57 (0.34)	159% (69%)	**86.0% (80.7%)**	0.46 (0.27)	103% (52%)	**86.3% (81.2%)**
90	0.78 (0.53)	234% (143%)	**82.7% (76.6%**	0.70 (0.45)	220% (104%)	**84.3% (78.7%**	0.63 (0.40)	202% (91%)	**85.3% (79.8%)**	0.57 (0.34)	159% (69%)	**86.2% (81.0%)**	0.46 (0.28)	104% (52%)	**86.4% (81.5%)**

Table 2. Experimental results, global values of performance indicator GU, RMSEN, R^2 for all combinations of levels of design Factors 1, 2 and 3. Values in parenthesis correspond to 4th Quartile OD flows.

Level HA0 of Factor 3 is the best initialization possible it is recommendable for sites whose available historical matrix is reliable. Level HAx of Factor 3 is the worst possible initialization and could be used in sites when no historical matrix is available. Results for Level NoHA of Factor 3 when the formulation does not include deviates that is no a priori historical information is used show that GU indicator has a poor behavior, but R^2 and RMSEN show a better performance. If historical matrices are not reliable, its use is not recommendable. A deeper statistical analysis of results in Table 2 reveals that:

- When the *a priori* information of the historical OD matrix is reliable (reflected in the perturbed matrix) the quality of the fit is improved as it is reflected in all goodness of fit indicators.
- The coefficient of determination between estimated and observed OD flows increases slightly when the number of BT inner sensors increases and Theil's coefficient and RMSEN (either global or belonging to the fourth quartile OD flows) become stable.
- When the %BT equipped vehicles decreases all performance indicators decrease, showing that the effect of BT rates is stronger than the detection layout or the quality of the *a priori* historical matrix, although non-additive factor effects on RMSEN indicator are appreciated.
- The Kruskall-Wallis nonparametric statistical test for means of the R^2, coefficient of determination of the fit, shows that the gross effect of the number of inner BT sensors is not significant at a confidence level of 95%, but the gross effects of the BT% of equipped vehicles and the quality of the *a priori* historical matrix show statistical significance, either globally or considering the most important OD pairs, for R^2 indicator (see Figure 3).

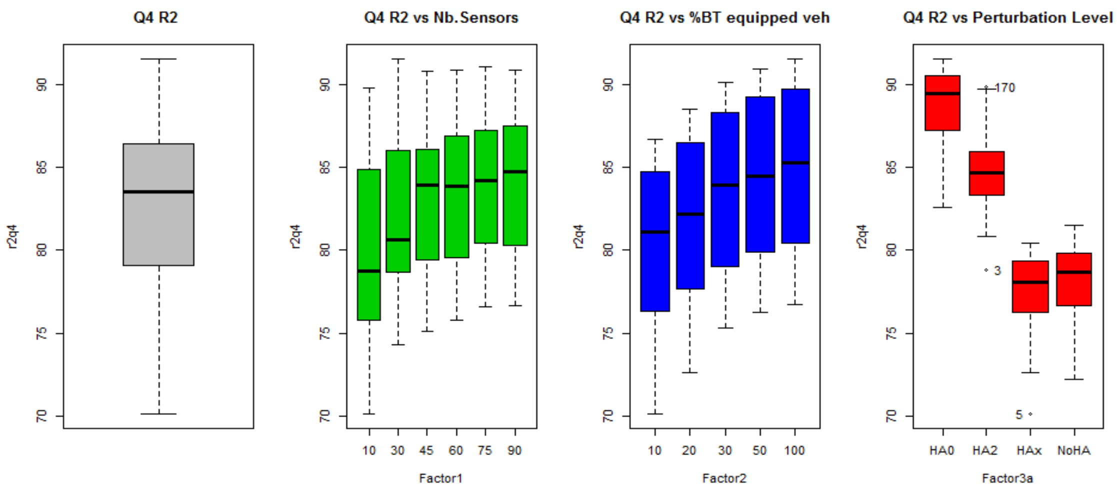

Figure 3. Boxplot for R^2 of OD flows in 4rt quantile vs #BT Inner Sensors (Factor 1), %BT Equipped Vehicles (Factor 2) – And levels of Factor 3.

- The same nonparametric test for the global RMSEN indicator, shows that the gross effects of the number of inner BT sensors is not significant at a confidence level of 95%, but the gross effects of the BT% of equipped vehicles and perturbation level are significant.
- Similar results are obtained for the global GU indicator.
- The general linear model for analysis of variance/covariance for Factors 1 to 3, after validation of the auxiliary regression models, shows that:

- GU for Q4 OD pairs is affected by %BT equipped vehicles and the reliability of the a priori historical matrix (to compute deviates).
- GU is affected by all 3 factors being %BT equipped vehicles the most important and Factor 3 the second.
- RMSEN for OD pairs with flows in the fourth quartile is affected by all 3 factors and %BT equipped vehicles at the conventional 95% level of confidence is the most important. See Figure 4. Similar results can be observed for GU.

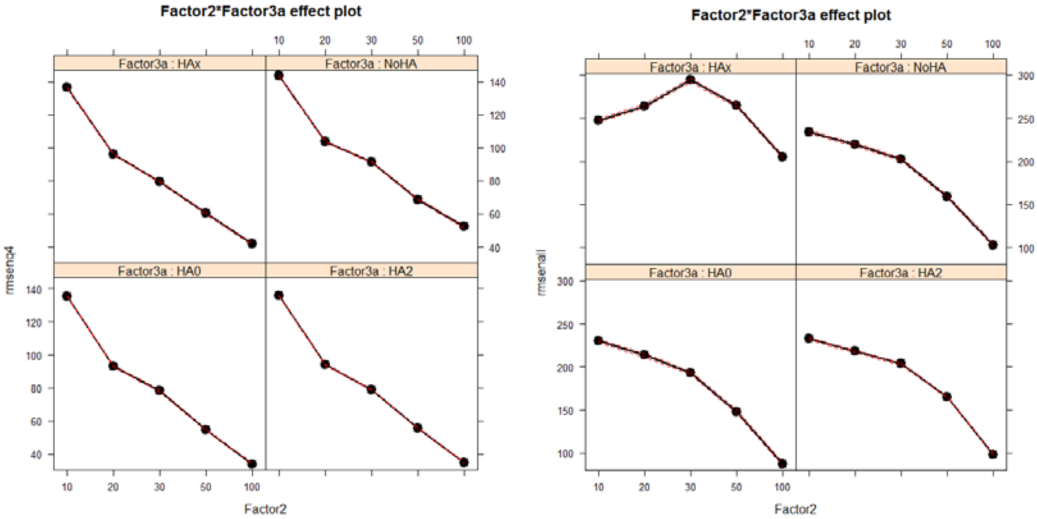

Figure 4. Influence on RMSEN of Factor 2 (%BT Equipped Vehicles) and Factor 3. Fourth quartile OD pairs (left). All OD pairs (right)

- The R^2 coefficient for fitted and observed OD flows in the fourth quartile depends on the 3 factors. A covariate treatment for Factor 1 (Number of BT sensors, NbBTIS) and Factor 2 (% of BT devices, PerEqBT) gives, a validated additive NbBTIS + second degree polynomial (PerEqBT)+Factor3 ancova model. The graphics in Figure 5, for the fourth quartile total OD flows illustrate the dependency of R^2 on the levels of Factor 1 (NbBTIS), Factor 2 (PerEqBT) and Factor 3. The left tridimensional display clearly discriminates the role of the quality of the a priori information as defined in Factor 3.

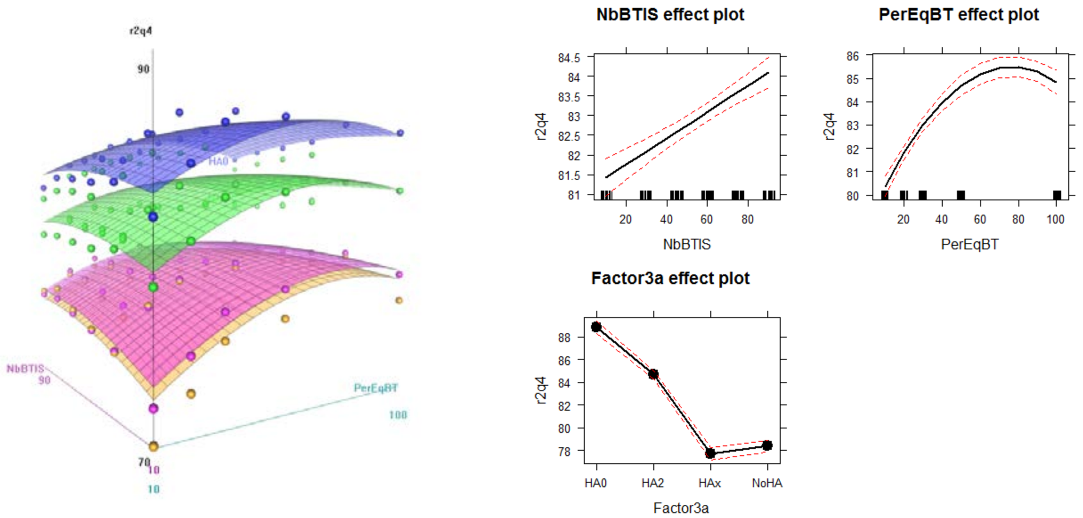

Figure 5. R2 Fitted vs Target OD flows (1h 15min) according to #BT Inner Sensors (Factor 1) and %BT Equipped Vehicles (Factor 2) – A priori Historical Matrix use (Factor 3). Only Q4 OD pairs

COMPUTATIONAL PERFORMANCE

We pointed out in the introduction that one of the critical aspects for the practical application of these techniques, to support traffic management decisions, is the heavy computational loadings that usually require these methods. A review of the literature, see for instance [8], [9], [18] or [20], shows that most of the experimental results have been obtained either for toy networks, or for two widely accepted test beds, the Central Artery/Third Harbor Tunnel network in Boston, a medium-scale network, with 211 links and 183 nodes and 10 OD pairs; and the Irvine, CA traffic network, which consists of three freeway corridors (I-5, I-405, Highway 133) and other main arterials, it includes 61 OD zones, 326 nodes and 626 links. We are not aware of computational experiments, based on similar approaches, in urban networks like the one used in this paper, both in terms of number of links, nodes and OD pairs, and of network complexity, reflected by the number of significant paths between OD pairs. Unfortunately there are no references on the computational times, but a paper by Bierlaire and Critin, [21], allows us to assume that they should be unacceptably high, as they investigate alternative numerical implementations with the purpose of reducing significantly the computational burden. Unfortunately the results measured in terms of mflops do not allow a direct comparison. Therefore a critical question for us was to investigate whether the alternative KF approach, exploiting the ICT measurements, could be fast enough to open the door to actual real-time applications.

We have conducted our experiments in a Windows 7 – 64 bits – 8 GB RAM -Intel Core i7-2600 (8M,3,40 GHz) 4C/8T. In Table 3, the total CPU time for executions consisting on reading the microsimulation emulation of real-time data measurements and, interval by interval, perform a Kalman iteration. There are 25 intervals of 3 min in the defined 1h 15min time-horizon. Each Kalman iteration takes 7.6 to 10.2 CPU sec depending on the number of BT Sensors and the number of BT equipped vehicles. For a 10 intervals forecasting (half an hour), the CPU time ranges from 1 to 2 min and makes the proposal suitable for real-time applications.

CPU time (sec)		Factor 2: % BT Equipped Vehicles				
		10	**20**	**30**	**50**	**100**
Factor 1: Nb BT Sensors	**10**	245	260	265		
	30	290	295	290	295	290
	45	-	330	315	380	350
	60	-	335	350	370	380
	75	340	350	370	395	405
	90	435	375	-	-	-

Table 3. KFX2 CPU Time in seconds for a time-horizon of 1h15min discretized in 3 min subintervals

CONCLUSIONS

The computational experiments presented in this paper probe the robustness and quality of the real-time OD estimates of the proposed KF approach exploiting ICT measurements. Robustness has been tested in terms of the three main factors determining the quality of the results: the number and layout of the ICT sensors, the percentage penetration of the ICT technology and the quality of the initial OD information. The experiments have been conducted with a realistic network of significant size as Barcelona's CBD network. The importance of the results relies in the possibility of applying a linear KF approach for estimation of dynamic OD matrices in a real-time traffic management system, since a half hour forecast is reliable in less than 2 minutes of calculations, using standard software as Matlab. Obviously an ad hoc implementation could significantly reduce these times.

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