Decimator filter based on B-splines

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Abstract- The cascaded integrator-comb (CIC) filters are widely used as decimators in many applications, such as in delta sigma AD converters to decimate the sampled signal on its output. One problem with these filters is that by increasing its order to improve the signal attenuation in the stop band, worsens the passband response. In this paper we propose a so-called least-squares filters (LSF) based on B-splines to improve that response, compensating for CIC filter drop and flattening the magnitude response of the pass band. We also study the relationship between the CIC filters, otherwise called moving average, and the B-splines expanded by an integer factor. We show that the least-squares filters based on B-splines can be decomposed in CIC filters, with a higher order than the splines used, plus a compensator filter. Another important contribution of this work are the FIR approximations of anti-causal IIR filters needed to implement LSFs.

Keywords: *Decimator filters, cubic B-splines, least squares filters, cascaded integrator-comb filters, AD Sigma delta converters.*

I. Introduction

It is often necessary to change the number of samples we have of a discrete signal. The increase of samples is achieved by interpolating filters and to reduce them one resorts on decimation operations, which simply involves throwing off a given number of samples [1]. For example, signal decimation by 2 implies that only the even samples remain. Before decimation we need to apply a low pass filter to avoid aliasing, due to the decrease in the sampling frequency. In this paper we explore two alternative filters: the cascades integrator-comb CIC [2, 3] also called moving average filters, and a filter based on least-squares B-splines [4, 5]. We also show the relationship of these two types of filter.

Along this paper we will use the digital frequency concept, defined between 0 and 1, 0.5 being the Nyquist frequency.

There are many applications that need decimation usually, such as the delta/sigma AD converter, which over samples the signal and therefore has to decrease the frequency of the signal obtained. Another case is when the electrical power signal is sampled at frequencies beyond the Nyquist rate, such as 10 kHz, in order to study its harmonics; in this case the study of the characteristics of the fundamental frequency, 50 Hz, could be done with far fewer samples, lowering the computational cost required; thus a good solution is to decimate the signal.

This work is divided into the following sections. Section II is a short introduction to the CIC filters. Section III shows the development LSF based on linear B-splines. Section IV shows the design of LSF based on cubic B-splines. Finally we present the conclusions of this work.

II. Introduction to CIC filters

CIC or moving average filters, are based simply on averaging a number of samples; it's a FIR filter, but can be very easily implemented recursively according to equation (1), where *m* is the number of samples used for averaging and that is decimating, and *n* is the filter order.

$$CIC_m^n(z) = \frac{1}{m^n}\cdot\left(1 + z^{-1} + \dots + z^{-(m-1)}\right)^n = \left(\frac{1}{m}\cdot\frac{\left(1-z^{-m}\right)}{\left(1-z^{-1}\right)}\right)^n \qquad (1)$$

The increase of *n* is increases the stop band attenuation but also worsens the response in the pass band. To increase the decimation factor, *m* needs to be increased too.

To show its relationship with the discrete B-splines we use the Z-transform domain. The Z-transform of the signal obtained by sampling a linear B-spline enlarged by a factor of 2, is equal to the Z transform of a CIC filter (1) with *m* and *n* equals 2 multiplied by a gain of 2 and a one shift factor (z), equation (2).

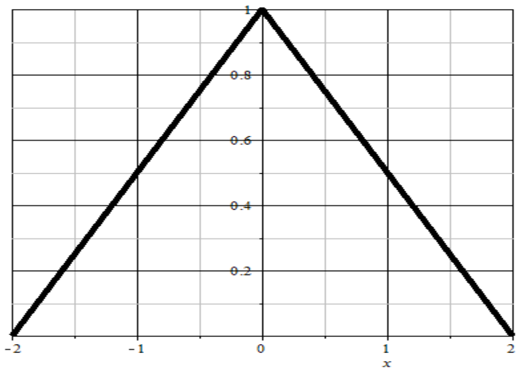
In the case of sampling a cubic B-spline expanded by a factor of 2 the Z transform can be obtained as the multiplication of the Z transform of a cubic B-spline without expanding, a gain of 2, a shift factor of 2 samples (z^2) and the Z transform of a CIC filter with *m* equals 2 and *n* equals 4, equations (3) and (4).

$$B_2^1(z) = B_1^1(z) \cdot 2 \cdot z \cdot \left(\frac{1}{2^2} \cdot (1+z^{-1})^2 \right) = 1 \cdot 2 \cdot \left[\frac{1}{4} \cdot \left(z^{-1} + 2 + z \right) \right] = \frac{1}{2} \cdot z^{-1} + 1 + \frac{1}{2} \cdot z \qquad (2)$$

$$B_2^3(z) = B_1^3(z) \cdot 2 \cdot z^2 \cdot \left(\frac{1}{2^4} \cdot (1+z^{-1})^4 \right) = \left(\frac{z^{-1} + 4 + z}{6} \right) \cdot 2 \cdot z^2 \cdot \left(\frac{1}{2^4} \cdot (1+z^{-1})^4 \right) \qquad (3)$$

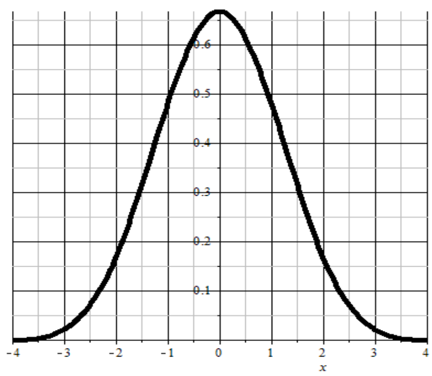
$$B_2^3(z) = \frac{32 + 23 \cdot \left[z^1 + z^{-1} \right] + 8 \cdot \left[z^2 + z^{-2} \right] + 1 \cdot \left[z^3 + z^{-3} \right]}{48} \qquad (4)$$

In Figure 1 and 2 we show a linear and cubic B-spline, both expanded by a factor of 2.
In Table 1 there are the values obtained by sampling these B-splines at the integer values of *x*. One can check matching equations (2) and (4).



a

Figure 1. Linear B-spline expanded by 2.



b

Figure 2. Cubic B-spline expanded by 2.

Table 1. B-splines sampled at Integer values

Sample	linear B-Spline, expanded by 2	Cubic B-spline, expanded by 2
-3	0	1/48 = 0.02083
-2	0	8/48 = 0.16666
-1	0.5	23/48 = 0.47916
0	1	32/48 = 0.6666
1	0.5	23/48 = 0.47916
2	0	8/48 = 0.16666
3	0	1/48 = 0.02083

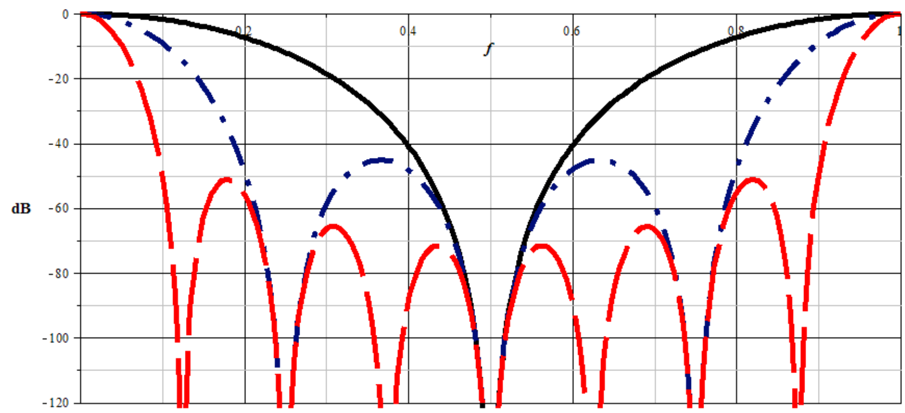


Figure 3. Magnitude of frequency response in dB of a CIC filter, of order 4, with different decimator valor. Solid line *m*=2; dash-dot line *m*=4; dashed line *m*=8.

Although this work only studies the moving average filter of 2 samples, in Figure 3 we can see the frequency response of three filters of fourth order and averaging 2, 4 and 8 samples. By increasing the number of samples to average the cutoff frequency decreases and attenuation increases slightly in the stop band. Table 2 shows the attenuation in the cutoff frequency of each filter, one can see that attenuation increases slightly with increasing decimation factor.

Table 2. Attenuation of cutoff frequency of CIC filters of 4 order and different number of averaging samples.

	Attenuation in dB on cutoff frequency of CIC order=4		
Number of averaging samples	2	4	8
dB attenuation	-12.04 dB	-14.79 dB	-15.46 dB

III. Least square filter with linear B-splines

The least squares filter proposed by Unser in [5] can be decomposed as a CIC filter with *m* equal to 2 and *n* equal to 4, plus a compensating filter and a shift factor of 2. The resulting filter can be seen in equation (5).

$$LBlinear(z) = z^2 \cdot CIC_2^4(z) \cdot Slinear(z) = \left(\frac{z^{-1}+2+z}{4}\right)^2 \cdot \left(\frac{8}{z^{-2}+6+z^2}\right) \quad (5)$$

Slinear, the compensating filter, is an IIR anti-causal filter. Unser proposed in [4, 5] one way to implement it, that has the disadvantage of having to work off-line and needs floating point arithmetic; additionally it is an anti-causal IIR filter. In this paper we propose instead an FIR approximation with only 10 coefficients (Figure 4). Although in Figure 4 there are 20 coefficients, half of them are zero. Its frequency response is like an all pass filter over a bandpass filter (Figure 5). The method used to get the FIR approximation is based on the work presented in [6, 7], and a variation of the method of convolution inverse matrix proposed in [8].

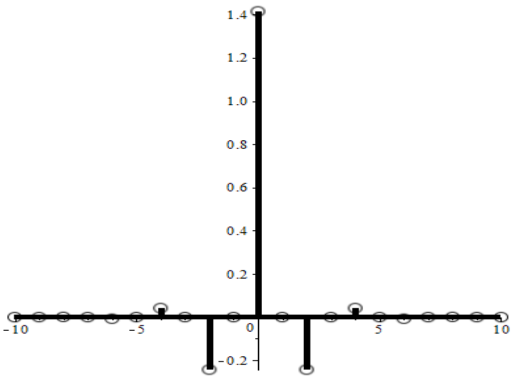


Figure 4. Time response of the filter Slinear.

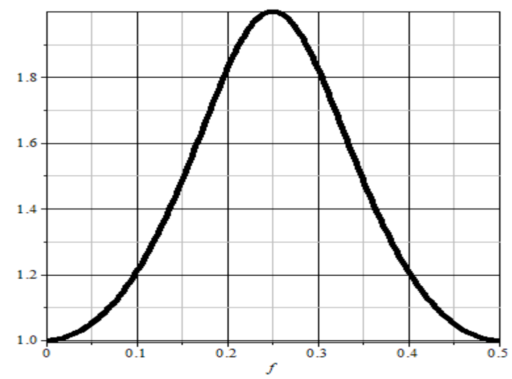
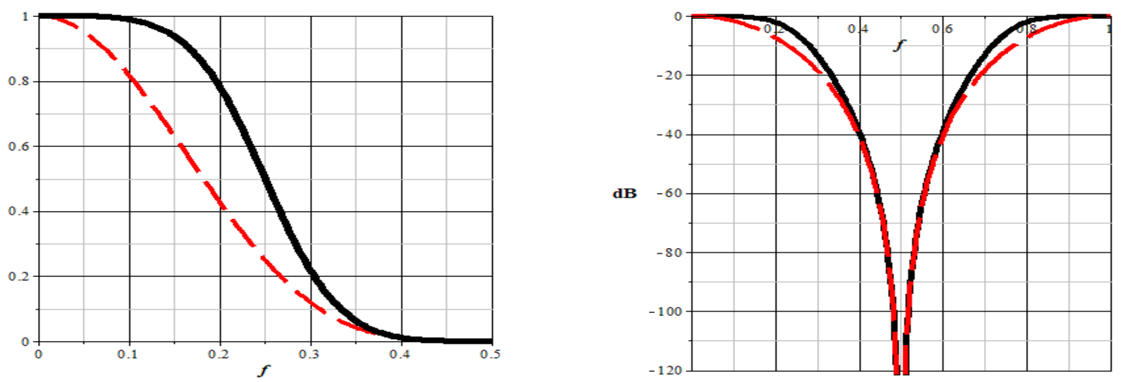


Figure 5. Magnitude of the frequency response of the Sliner filter.



(a) (b)

Figure 6. Magnitude of the frequency response of the filter CIC of order 4(dashed line) and the same filter after compensating with Slinear (Solid line). (a) Linear magnitude. (b) Magnitude in dB.

Figure 6 shows the magnitude of the frequency response of the moving average filter of 2 samples, and 4-order, dashed line, and the same filter compensated by Slinear, solid line, equation (5). We want to emphasize that joining two separate filters that do not have a good response, Slinear and CIC of order 4, you get a filter with a better response, solid line of Figure 6.

IV. Least square filter with cubic B-splines

The least squares filter based on cubic B-splines is decomposed into two cascaded CIC filters with *m* equals 2, *n* equals 4 and a shift of 2 samples, plus a compensating filter called Scubic (equation (6)). This filter compensator (7) consists of three filters, two discrete cubic B-splines and one anti-causal IIR filter (8), for which we also propose a linear phase FIR approximation with 30 coefficients. The final Scubic compensating filter is implemented with only 40 coefficients. Its temporal response is shown in Figure 7, and the magnitude of its frequency response in Figure 8. In equation (10) shows that the least squares filter LBcubic, consists of an 8-order CIC filter, shifted 4 samples, and a compensating filter.

$$LBcubic(z) = z^2 \cdot CIC_2^4(z) \cdot B_1^3(z) \cdot S_2^3(z^2) \cdot z^2 \cdot CIC_2^4(z) \cdot B_1^3(z) \quad (6)$$

$$Scubic(z) = B_1^3(z) \cdot S_2^3(z^2) \cdot B_1^3(z) \quad (7)$$

$$S_2^3(z^2) = \frac{2304}{\left(2212 + 1087 \cdot \left[z^2 + z^{-2}\right] + 110 \cdot \left[z^4 + z^{-4}\right] + 1 \cdot \left[z^6 + z^{-6}\right]\right)} \quad (8)$$

$$B_1^3(z) = \frac{z^{-1} + 4 + z}{6} \quad (9)$$

$$LBcubic(z) = z^4 \cdot CIC_2^8(z) \cdot Scubic(z) \quad (10)$$

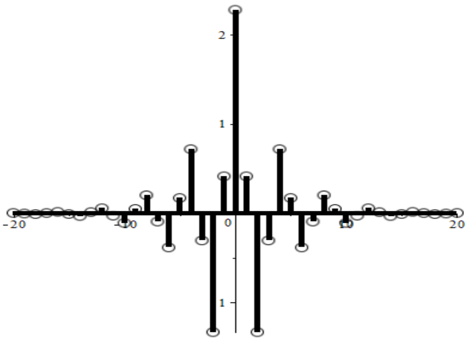


Figure 7. Time response of the filter Scubic, equation (7).

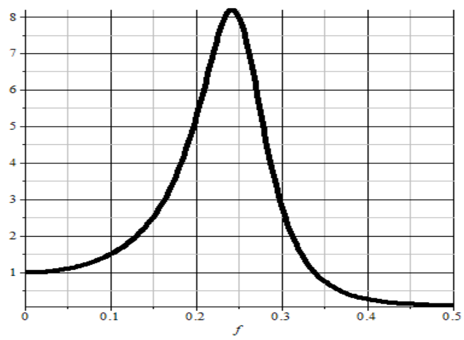
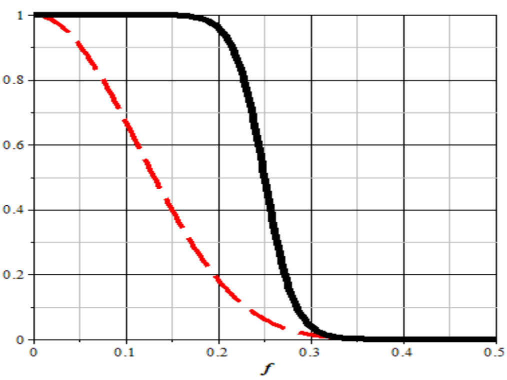
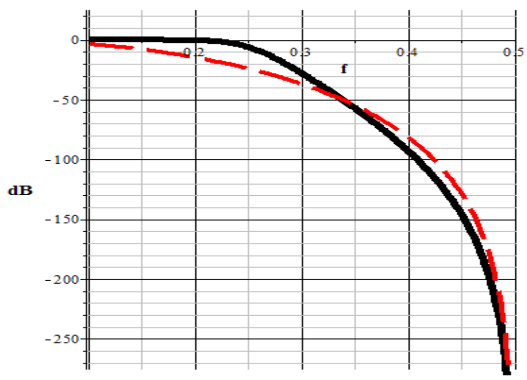


Figure 8. Magnitude of the frequency response of the filter Scubic, (7).



(a)



(b)

Figure 9. Magnitude of the frequency response of the filter CIC of order 8(dashed line) and the same filter after compensating with Scubic (Solid line). (a) Linear magnitude. (b) Magnitude in dB.

Figure 9 shows the poor response on CIC filter passband of 8-order. Before already said that improving the attenuation in the stop band also worsens the response in the passband.
Comparing the final filter frequency response based on linear B-splines and cubic B-splines, one can observe the improved response of the latter. LBcubic (10) is obtained with almost linear response up to the cutoff frequency, digital frequency of 0.25, Figure 9b, and a large attenuation of about 150 dB per octave between frequency 0.25 and 0.45. We want to note that can be achieved filters for decimation with a much more sharp transition bands, but the filters presented in this paper have the ability to compensate for the passband response of the CIC filter.
The frequency response of the filter LBlinear (5), based on linear B-splines, is linear only in the frequency range between 0 and 0.1, and from a frequency of 0.2 to 0.4 the drop is only about 40 dB per octave (Figure 6b).
In favour of the filter based on linear B-splines we have its computational cost, lower than that based on cubic B-splines. The linear equalizer filter needs only 11 non-zero coefficients and the filter for the case of cubic splines requires 40 non-zero coefficients. Additionally, in the first case the CIC filter is 4th-order, and second 8th-order. The first requires only 5 coefficients for its FIR implementation and the second needs 9 coefficients.

V. Conclusions

In this paper, we study well known decimation filters and propose new implementations based on piecewise polynomial descriptions of the signals. Special attention has been given to 8th and 4th-order CIC filters with a decimating factor of 2, and least square filters based on B-splines. It has been demonstrated that least squares filters can be decomposed into a CIC filter in cascade with another filter that compensates the frequency response in the passband. In this case, the CIC filter order is higher of that the B-splines used. The frequency responses of the uncompensated and compensated CIC filter have been studied separately, and great improvement was obtained after applying the compensation filter. Another important contribution of this work are the FIR approximations of anti-causal IIR filters that are part of the least squares filter based on B-splines. For the case of linear B-splines with only 10 non-zero coefficients it results in a good approximation. For the case of cubic B-splines 40 coefficients are required, only 20 different because the filter has even symmetry and therefore linear phase as all filters presented in this paper. Currently we continue studying the same filters for different decimation factors, and different order of filters.

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