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Cr(III) removal from aqueous solutions: A straightforward model approaching of the adsorption in a fixed-bed column

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22 **Abstract**

23

24 Prediction of breakthrough curves for continuous sorption characterization is generally
25 performed by means of simple and simplified equations. These expressions hardly have
26 any physical meaning and, besides, do not allow extrapolation. A novel and simple
27 approach, based on unsteady state mass balances, is presented herein for the simulation
28 of the adsorption of Cr(III) ions from aqueous onto a low-cost adsorbent (leonardite).
29 The proposed model overcomes the limitations of the commonly used analytical
30 solution-based models without the need for complex mathematical methods. A set of
31 experimental breakthrough curves obtained from lab-scale fixed-bed columns was used
32 to calibrate and validate the proposed model with a minimum number of parameters to
33 be adjusted.

34

35 **Keywords:** Adsorption, chromium, continuous, breakthrough curve, modeling

36

37 **Introduction**

38

39 Chromium is usually found in some industrial sewage such as electroplating, leather,
40 tanning and textile industries. Chromium mostly exists in two states, Cr(III) and Cr(VI).
41 Cr(VI) has been classified as a Group I human carcinogen by the International Agency
42 for Research on Cancer (IARC) and as a Group A inhalation carcinogen by the US
43 Environmental Protection Agency (USEPA). Hexavalent chromium is 500 times more
44 toxic than the trivalent form. Moreover, Cr(III) may be readily oxidized to Cr(VI) by
45 certain circumstances of pH changes and redox potential conditions.^[1] Consequently,
46 the removal of chromium from industrial wastewaters is a research topic of great

47 interest. Several procedures have been proposed to remove chromium from industrial
48 wastewater. ^[2-5] These methods include different chemical procedures based on
49 reduction and precipitation processes or even ion exchange processes. Although the
50 effectiveness of these procedures has been sufficiently proved, they present some
51 inconveniences and limitations. **These are mainly two**, the high volume of aqueous
52 solution of colloidal chromium hydroxide handled and their relatively high cost.
53 **Recently, chromium adsorption onto low-cost sorbents has been investigated as an**
54 **alternative for current costly methods.** Among these materials, immature coals (such as
55 leonardite) have already demonstrated their ability to efficiently adsorb pollutants of
56 different nature. ^[6-8]

57 Most studies related to adsorption of **chromium** are performed in batch mode. ^[9] Batch
58 adsorption tests give fundamental information **for a particular sorbent-sorbate pair in**
59 **terms of sorption capacities and kinetics.** However, in the majority of practical
60 (industrial) applications, batch systems are scarcely found. Continuous operation is the
61 most suitable mode from both an economical and process control point of view.

62 The continuous sorption process is usually characterized by the so called breakthrough
63 curves, i.e. a representation of the pollutant effluent concentration versus time profile in
64 a fixed bed column. In addition, breakthrough curves prediction through mathematical
65 models is a useful tool for scale-up and design purposes. **Mechanistic models for fixed-**
66 **bed adsorption columns should include different phenomena such as: axial dispersion,**
67 **film diffusion resistance, intra-particle diffusion resistance (both pore and surface**
68 **diffusion) and sorption equilibrium with the sorbent.** The inclusion of all these
69 **processes involves rigorous but mathematically complex models associated with non-**
70 **linear partial differential equations.** This implies using numerical solutions and, thus, a
71 **time-consuming process framework.** In addition, the use of comprehensive mechanistic

72 models involves the adjustment of several parameters, i.e. the availability of several sets
73 of reliable and well-designed experimental data is required. For this reason, in the last
74 decades a wide variety of semi-empirical models have been stated in order to predict the
75 behavior of the fixed-bed adsorption process through the simulation of the breakthrough
76 curves response.

77 From the above mentioned, the aim of this study is to compare the most commonly used
78 analytical models for the breakthrough curves characterization for heavy metals
79 adsorption in fixed bed columns. Besides, a new approach based on rigorous mass
80 balances and using the minimum number of adjusted parameters is proposed. The new
81 model approach is developed, calibrated and validated for the adsorption of Cr(III) from
82 synthetic solutions by a fixed bed column packed with leonardite.

83

84 **Materials and Methods**

85 *Materials*

86 The leonardite used as adsorbent in this study was supplied by Sociedad Española de
87 Acidos Húmicos, S.A. (SEPHU[®], Zaragoza, Spain). This adsorbent was sieved to a
88 grain size of 0.09–0.2 mm before its use in the sorption experiments. Some of its
89 physical and chemical characteristics are shown in Table 1.

90 Chromium stock solution was prepared by dissolving a known quantity of
91 Cr(NO₃)₃·9H₂O (AR grade) in Milli-Q[®] water. The stock solution was then diluted to
92 obtain standard solutions. All chemicals were of analytical reagent grade (Scharlau[®]).
93 The amount of Cr(III) in the solutions before and after the adsorption tests was
94 determined by flame atomic adsorption spectrophotometry (FAAS), using a Termo
95 Electron Corporation (SOLAAR S2 model) spectrophotometer with an acetylene-

96 nitrous oxide flame. The wavelength was 359 nm. All experiments were performed
97 twice. The repeatability of the results showed a variation coefficient of less than 5%.

98

99 *Continuous Experiments*

100 Fixed-bed column studies were conducted using laboratory-scale polyethylene columns
101 of 1 cm diameter and 70 mm length. The columns were packed with known quantities
102 of Leonardite to obtain a particular bed depth. A glass wood plug was placed at the
103 bottom to support the adsorbent bed as well as to prevent the adsorbent washing out.
104 Another plug was also placed on top of the bed to prevent floating up of the leonardite.
105 To ensure complete purge of the trapped air, leonardite packed columns were fully
106 wetted by filling them with deionized water for 5 h prior starting the experiments. The
107 influent Cr(III) was supplied and maintained in each column throughout the
108 experiments by using a peristaltic pump (ISMATEC mod. MPC). The flow rate was
109 cross-checked at the exit of the column at regular intervals of 200 min to prevent and
110 minimize any flow fluctuations (if occurred) inside the column bed.

111 Breakthrough curve determination experiments were carried out in 6 different columns.
112 The same flow-rate ($0.33 \text{ cm}^3 \text{ min}^{-1}$), amount of adsorbent (2 g) and solution volume (1
113 L) were applied in each column. The chromium (III) concentration varied from 20 to
114 150 mg L^{-1} . Effluent samples were collected with an automated sampler every 200
115 minutes from 0 to 3000 min. Chromium content in each sample was analyzed as
116 described above.

117

118 *Analytical mathematical approaches*

119 *a) Bed Depth Service Time (BDST) (Bohart-Adams model)*

120 Even though Borhart and Adams ^[10] equation was originally developed for gas-solid
 121 systems, it has been widely used in the literature for liquid-solid systems. ^[11, 12] The
 122 BDST model describes the relationship between the bed depth and the column service
 123 time according to the following equation:

124

$$125 \ln\left(\frac{C_0}{C} - 1\right) = \ln\left[\exp\left(kN_0\frac{Z}{U}\right) - 1\right] - kC_0t \quad (1)$$

126

127 Where C_0 is the initial sorbate concentration (mg L^{-1}), C is the sorbate concentration at
 128 time t (mg L^{-1}), k is the sorption rate constant for the column ($\text{L min}^{-1} \text{mg}^{-1}$), Z is the
 129 length of the column bed (m), N_0 is the sorptive capacity of the sorbent (mg L^{-1}) and U
 130 is the linear flow velocity of the liquid phase (m min^{-1}).

131 This expression can be rearranged to:

132

$$133 \frac{C}{C_0} = \frac{\exp(kC_0t)}{\exp\left(kN_0\frac{Z}{U}\right) - 1 + \exp(kC_0t)} \quad (2)$$

134

135 *b) Thomas model*

136 The Thomas ^[13] model considers plug-flow behavior in the bed, coupled to Langmuir
 137 isotherm for equilibrium and second-order reversible reaction kinetics. The most usual
 138 expression for Thomas model is:

139

$$140 \frac{C}{C_0} = \frac{1}{1 + \exp[k_T(q_0m_C - C_0V_{eff})/Q]} \quad (3)$$

141

142 Where K_T is the Thomas model constant ($L \text{ min}^{-1} \text{ mg}^{-1}$), q_0 is the maximum solid-phase
143 concentration of the solute (mg g^{-1}), m_C is the mass of sorbent in the column (mg), V_{eff} is
144 the throughput volume (L) and Q is the volumetric flow rate ($L \text{ min}^{-1}$).

145

146 *c) Yoon and Nelson model*

147 Similarly to the BDST model, the Yoon and Nelson ^[14] model was originally developed
148 for the adsorption of gaseous sorbates in activated carbons by means of a simple
149 equation:

150

$$151 \ln\left(\frac{C}{C_0-C}\right) = k_{YN}t - t_{0.5}k_{YN} \quad (4)$$

152

153 Where K_{YN} is the Yoon-Nelson rate constant (min^{-1}) and $t_{0.5}$ is the time required for 50%
154 sorbate breakthrough (min). This expression can be rearranged to:

$$155 \frac{C}{C_0} = \frac{1}{1+\exp[k_{YN}(t_{0.5}-t)]} \quad (5)$$

156

157 *d) Clark model*

158 Clark ^[15] model was derived from a mass balance on a differential element in a fixed
159 bed coupled with Freundlich isotherm for the liquid-solid equilibrium. One possible
160 expression of the model is the following:

161

$$162 \frac{C}{C_0} = \left(\frac{1}{1+A\exp^{-rt}}\right)^{1/n-1} \quad (6)$$

163

164 Where n is the Freundlich constant (-) and A (-) and r (min^{-1}) are Clark's model
165 parameters.

166

167 *e) Wolborska model*

168 Wolborska ^[16] deduced an equation from mass balances in a column considering
169 external mass transfer and constant kinetic coefficients. The final form of the equation is
170 the following:

171

$$172 \ln \frac{C}{C_0} = \frac{\beta C_0}{N_0} t - \frac{\beta Z}{U} \quad (7)$$

173

174 Where β is the kinetic coefficient of external mass transfer (min^{-1}).

175

176 *f) Dose-Response model*

177 The Dose-Response model ^[17] is based on mathematical issues instead of mechanistic
178 fundamentals. However its final form is similar to Thomas and Yoon-Nelson models.

179

$$180 \frac{C}{C_0} = 1 - \frac{1}{1 + (C_0 V_{eff} / q_0 m C)^a} \quad (8)$$

181

182 Where a is the Dose-Response model parameter (-).

183

184 *New modeling approach*

185 The new modeling approach is based on the well-known diffusion-reaction
186 mathematical model which has been successfully used in a wide range of chemical
187 engineering related systems ^[18].

188 The unsteady state model proposed is based on the following assumptions:

189 1. Isothermal conditions across the column and along the time.

190 2. The aqueous phase circulation regime is modeled as a plug flow pattern. Thus, axial
191 dispersion is not considered.

192 3. Planar geometry and perpendicular diffusion in the adsorbent is used to derive the
193 model equations.

194 4. The aqueous-solid interface resistance is negligible.

195 5. The Aqueous-solid interface equilibrium is described by Langmuir's isotherm.

196 6. An effective interfacial area exists, thus considering the possibility of dry zones.

197 7. There is a maximum penetration depth for the pollutant into the solid phase.

198 These assumptions can be easily justified by means of the experimental conditions and
199 the column dimensions. The Langmuir isotherm equilibrium behavior was derived from
200 previous batch experiments. [19]

201 Considering the above listed assumptions, the following mass balance for the liquid
202 phase can be stated:

$$203 \quad \frac{\partial C}{\partial t} = -\frac{Q}{a_c \cdot \varepsilon} \frac{\partial C}{\partial z} - \frac{D \cdot a_s}{\varepsilon} \left(\frac{\partial N_0}{\partial x} \right) \Big|_{x=0} \quad (9)$$

204 With the following boundary condition: at $z=0$, $C = C_0$.

205 Where C is the Cr(III) liquid concentration (mg L^{-1}), Q is the volumetric liquid flowrate
206 (L min^{-1}), a_c is the cross-sectional area of the column (dm^2), ε is the packed bed porosity,
207 z is the height position across the column (dm), D is the effective diffusion coefficient
208 of the solute ($\text{dm}^2 \text{min}^{-1}$), a_s is the effective sorbent material specific area ($\text{m}^2 \text{dm}^{-3}$), N_0
209 is the sorptive capacity of the sorbent (mg dm^{-3}), x is the depth from the sorbent
210 material surface (dm) and C_0 is the column inlet Cr(III) concentration (mg L^{-1}).

211 Regarding to the solid phase, the following mass balance was formulated:

$$212 \quad \frac{\partial N_0}{\partial t} = D \frac{\partial^2 N_0}{\partial x^2} \quad (10)$$

213 This equation accounts for the time-dependent variation of the solid phase concentration
214 due to **diffusion** fluxes. This equation needs the following boundary conditions
215 (including the Langmuir isotherm expression) to be solved:

$$216 \text{ at } x = 0, N_0 = \rho \frac{q_0 \cdot q_1 \cdot C}{1 + q_1 \cdot C} \text{ and at } x = \delta, \frac{\partial N_0}{\partial x} = 0 \quad (11)$$

217 Where ρ is the sorbent density (mg dm^{-3}), q_0 is the maximum solid-phase concentration
218 of the solute (75.2 mg g^{-1}), q_1 is the Langmuir isotherm parameter ($0.061 \text{ dm}^3 \text{ mg}^{-1}$) and
219 δ is maximum penetration depth for the pollutant into the solid phase (dm).

220

221 ***Models' Solution***

222

223 ***Analytical models***

224 Nonlinear regressions were performed **by** MATLAB software to fit the experimental
225 data to the six different models. Non-linear regression avoids the bias error that might
226 appear from using different linearization procedures. MATLAB package uses nonlinear
227 least-squares data fitting by the Gauss-Newton method, which converged quickly and
228 gave **better** results than other commercial software (e.g. EXCEL™ Solver).

229

230 ***Numerical approach***

231 The set of partial differential equations was discretized in space along the bed height
232 and sorbent thickness. The resulting set of ordinary differential equations was solved
233 using MATLAB in a home-made modeling environment. A variable order method was
234 used for solving stiff differential equations based on the numerical differentiation
235 formulas (NDFs). The maximum time step used in the numerical solution routine was

236 established in 1 hour. The calibration procedure was based on direct experimental
237 determination as well as on curve fitting of experimental data monitored. The effective
238 diffusion coefficient of the pollutant (D) and the effective specific surface area (a_s) were
239 the set of parameters estimated by curve fitting during the model calibration step. To
240 start with the calibration procedure, initial guesses were assigned to the estimated
241 parameters. Predicted Cr(III) liquid concentration by the model was compared with the
242 experimentally measured data (C^*) and the deviations between both were used to obtain
243 updates for the estimated parameters. Parameter values were sought to minimize the
244 objective function (OF) given in equation 12.

$$245 \quad OF = \sqrt{\sum_{i=1}^N [C(D, a_s) - C^*]^2} \quad (12)$$

246 Where N is the number of measurements realized and C^* is the measured concentration
247 of the solute (mg L^{-1}).

248 The parameter estimation was performed using a MATLAB algorithm based on a
249 multidimensional unconstrained nonlinear minimization (Nelder-Mead) algorithm. This
250 is a direct search method that does not use numerical or analytical gradients. Model
251 calibration and validation of the present work was checked by performing a statistical
252 analysis based on a paired t-Student's test at the 5% level of significance.

253

254 **Results and Discussion**

255

256 *Effect of the Initial Chromium Concentration*

257 In order to test the leonardite potential as sorbent for industrial applications, columns
258 filled with leonardite were tested in continuous mode by passing Cr(III) solutions at
259 different concentrations. A total of 6 tests were performed in order to assess the initial

260 concentration effect on the column performance. Figure 1 shows the corresponding
261 breakthrough curves for 6 different initial concentrations. All columns were run for the
262 same period of time (3000 min). As expected, the effect of the initial chromium
263 concentration on breakthrough curves is critical. Breakthrough time, defined as the time
264 required to reach a specific breakthrough outlet concentration, decreases with increasing
265 metal concentration. This means that higher metal loading rates generates lower
266 adsorption zone lengths. In addition, the initial slope of the breakthrough curves became
267 steeper when the initial concentration was increased, showing that the reaction zone
268 advances **faster** at high load conditions. Almost 36 hours were necessary to obtain a
269 measurable chromium outlet concentration for the lowest initial concentrations,
270 demonstrating quite good behavior of the leonardite in continuous operation.

271

272 *Application of the Analytical Models*

273 In order to depict clearly the model predictions for the analytical equations (equations 2-
274 3 and 5-8) only 4 inlet concentrations are shown. Model predictions for the BDSM,
275 Thomas and Yoon and Nelson are summarized in figure 2. In addition model
276 parameters are listed in **table 2**.

277 **The Borhart-Adams** (figure 2a) approach gave fairly good predictions for the
278 experimental behavior (excepting for the highest initial Cr(III) concentration). However,
279 despite the equation was originally derived to describe the initial section of the
280 breakthrough curve, it gave acceptable results for the whole data sets. This might be
281 attributed to the fact that BDSM models imply the use of two parameters to be adjusted,
282 reducing the number of degrees of freedom and, thus, giving a more accurate fitting.
283 **From** the adjustment procedure by **nonlinear** regression, the sorption capacity of the
284 sorbent (N_0) is obtained. The values calculated (between 80-130 mg L^{-1}) are, as

285 expected, clearly lower than those obtained in previous batch tests (data not shown).
286 This also suggests that BDSM model must be regarded as an empirical mathematical
287 expression (or a particular solution to a general problem) capable to fit a set of
288 experimental data. **The assumption considered by this model, that the rate of adsorption**
289 **is controlled by the surface reaction between the sorbate and the remaining active sites,**
290 **could explain the experimental behavior observed.** Further conclusions or information
291 derived for design purposes should be under suspect.

292 The worst predictions were obtained for the Thomas model approach (figure 2b). The
293 main assumptions of this model (no mass transfer limitation and second order kinetics)
294 **seem** to fail in predicting the continuous adsorption of Cr(III) in leonardite. The results
295 suggest some sort of mass transfer resistance as dominating step for the column
296 performance. The use of only one single parameter to fit the equation is another
297 drawback of the approach. The squared sums of errors (ERRSQ) for this model were by
298 far the largest computed among all model approaches.

299 Similar results (in terms of ERRSQ) to the BDST were obtained for the Yoon and
300 Nelson model. Despite this is an empirical equation based on probabilistic aspects of the
301 sorption process, the time to 50% sorbate breakthrough ($t_{0.5}$) are quite realistic as it is
302 shown in figure 2c and table 1. The only important deviation is observed for the lowest
303 inlet concentration (20 mg L^{-1}), although the model predictions **are** acceptable even in
304 this case as shown by the low ERRSQ value.

305 Figure 3a shows the predictions **of** the Clark model. Apparently only slight differences
306 from BDSM and Yoon and Nelson can be observed. The Clark model assumes some
307 sort of mass transfer resistance (external or internal) and Freundlich equilibrium
308 isotherm behavior. Even though the model contains information regarding to mass
309 transfer resistance issues, this phenomenon is lumped to operational parameters.

310 Therefore, it is (again) difficult to draw relevant information from the model itself.
311 From table 2, it is not possible to establish a relationship between the r parameter and
312 the initial concentration, although the Clark model parameter is related to the migration
313 velocity of the concentration front in the bed. Again, the use of a two-parameter
314 approach results in lower ERRSQ values.

315 Similar to the BDSM model, the Wolborska analytical expression was developed for
316 predicting the first part of the breakthrough curve. In this sense, figure 3b shows that
317 effectively the model predictions fail to fit the experimental data except for the lowest
318 concentration. For a better model fitting, only the first part of the experimental
319 breakthrough curve was considered in order to obtain parameters in table 2. N_0 values,
320 despite higher than those predicted by the BDST model, are still much lower than
321 experimental batch values. An increase of N_0 values when increasing the initial
322 concentration was expected but not observed. Again, this places some serious doubts
323 about the usefulness of the analytical solutions approach.

324 As already stated, the Dose-Response model is based on mathematical issues instead of
325 mechanistic information. From the information plotted in figure 3c and listed in table 3,
326 it is clear that model predictions are fairly good or, at least, as good as BDST, Clark and
327 Yoon and Nelson models. This can be also concluded by checking the ERRSUM values
328 listed in table 3. However, **the** values of the chromium concentration in the solid phase
329 (q_0) (table 3) are 100 times lower than those measured in batch experiments.

330 From the discussion above it is clear that analytical solutions models are a simple and
331 useful technique for data fitting, but they hardly supply any information regarding the
332 mechanisms involved in the chromium removal. In addition, from design **purpose** point
333 of view, the suitability of such approaches is also under suspect.

334

335 *Application of the New Modeling Approach*

336 Analytical model solutions must be regarded as simple approaches to the real column
337 behavior. Even though their use implies a fast and easy procedure, the information that
338 can be derived from the equations used in the previous section is hardly useful for
339 scaling-up and real applications design purposes. In this sense, a balance between
340 mathematical complexity and model reliability is needed in order to obtain useful tools
341 capable to come up with design-related information.

342 The model developed, which fulfills the above stated requirements, is a simple and
343 powerful tool able to describe most of the mass-transfer related processes. It is based on
344 a mass balance along the column considering plug flow and the mass transfer from the
345 liquid to the sorbent as a function of the concentration gradient and the equilibrium
346 isotherm. The main objective of the model setup is to characterize the kinetic sorption
347 of the continuous (column) system by adjusting the minimum number of parameters.
348 The value of coefficient diffusion denotes the relative rate of transfer through the
349 material (i.e. sorption rate) and makes possible to quantitatively compare the behavior
350 of different materials by means of a parameter with physical information in contrast to
351 classical empirical models.

352 Figure 4 shows Cr(III) concentrations at the column outlet jointly with the model
353 predictions after the calibration step for an inlet concentration of 100 mg L^{-1} . The
354 effective diffusion coefficient for Cr(III) in leonardite obtained in the procedure is
355 $2.92 \cdot 10^{-6} \text{ m}^2 \text{ h}^{-1}$ and the effective area is $2.9 \cdot 10^5 \text{ m}^2 \text{ m}^{-3}$. This value corresponds only to
356 an 8% of the total area of the material ($3.73 \cdot 10^6 \text{ m}^2 \text{ m}^{-3}$). As it is also shown in Figure 4,
357 the statistical analysis based on a paired t-Student's test at 5% of significance shows a
358 good agreement between results predicted by the model with the optimized parameters
359 and experimental data.

360 Once the kinetic parameters were calibrated for the specific experiment shown above,
361 model validation was performed by comparing the simulation results to experimental
362 data in the same set of experiments shown in Figure 1. The comparative results obtained
363 in the validation step are shown in Figure 5. A satisfactory global agreement is observed
364 between experimental data and model predictions, especially for 20, 40 and 150 mg L^{-1} .
365 In the case of 75 mg L^{-1} , the concentrations predicted are slightly different than those
366 registered experimentally. However, it should be noted that the breakthrough curve at
367 this concentration is less consistent with the experimental behavior of the rest of
368 concentrations and an early material saturation was expected.

369 It is worth mentioning that the new model approach, unlike analytical models, has been
370 calibrated and validated with different experimental data showing a good agreement in
371 both scenarios by means of the same set of parameters. Thus, a good description of the
372 process has been checked for a wide range of cases.

373 Comparing the agreement between experimental and model predictions achieved by
374 analytical models and those obtained by the new model approach, the latter is able to
375 describe more satisfactorily the behavior experimentally observed, according to the
376 ERRSQ values. As shown in Table 1 and 2, the average ERRSQ for each model is
377 between a minimum value of 0.0334 for the Response-Dose and a maximum of 1.3262
378 for the Yoon and Nelson. In the case of the new model approach the average ERRSQ is
379 0.0254 which means an improvement of 24% with respect to the best fitting previously
380 achieved with the analytical models.

381

382 **Conclusion**

383

384 A comparison between classical analytical solutions and a simple unsteady-state mass
385 balances-based approach for breakthrough curves prediction was presented. Analytical
386 solutions showed good capabilities for experimental data fitting despite hardly any
387 conclusion could be derived from their adjustment. In addition, extrapolation of their
388 predictions must be avoided. The numerical approach described offered a good balance
389 between mathematical easiness and rigorous description of the main phenomena
390 occurring in the gas-solid mass transfer process. The model presented herein allowed
391 obtaining a powerful predictive tool together with relevant process parameter values
392 (effective area and effective diffusion coefficient).

393

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395

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398

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450 FIGURES CAPTIONS

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452 **Figure 1.** Breakthrough curves for the adsorption of Cr(III) onto leonardite in a fixed
453 bed column used for process modeling.

454 **Figure 2.** Model predictions (lines) for the BDSM (a), Thomas (b) and Yoon and
455 Nelson (c) models.

456 **Figure 3.** Model predictions (lines) for the Clark (a), Wolborska (b) and Dose-Response
457 (c) models.

458 **Figure 4.** Experimental outlet concentration and numerical approach model predictions
459 in the calibration step corresponding to a Cr(III) inlet concentration of 100 mg L^{-1} .

460 **Figure 5.** Experimental outlet concentration and model predictions in the validation
461 procedure corresponding to a Cr(III) inlet concentration of 20, 40, 75 and 150 mg L^{-1}

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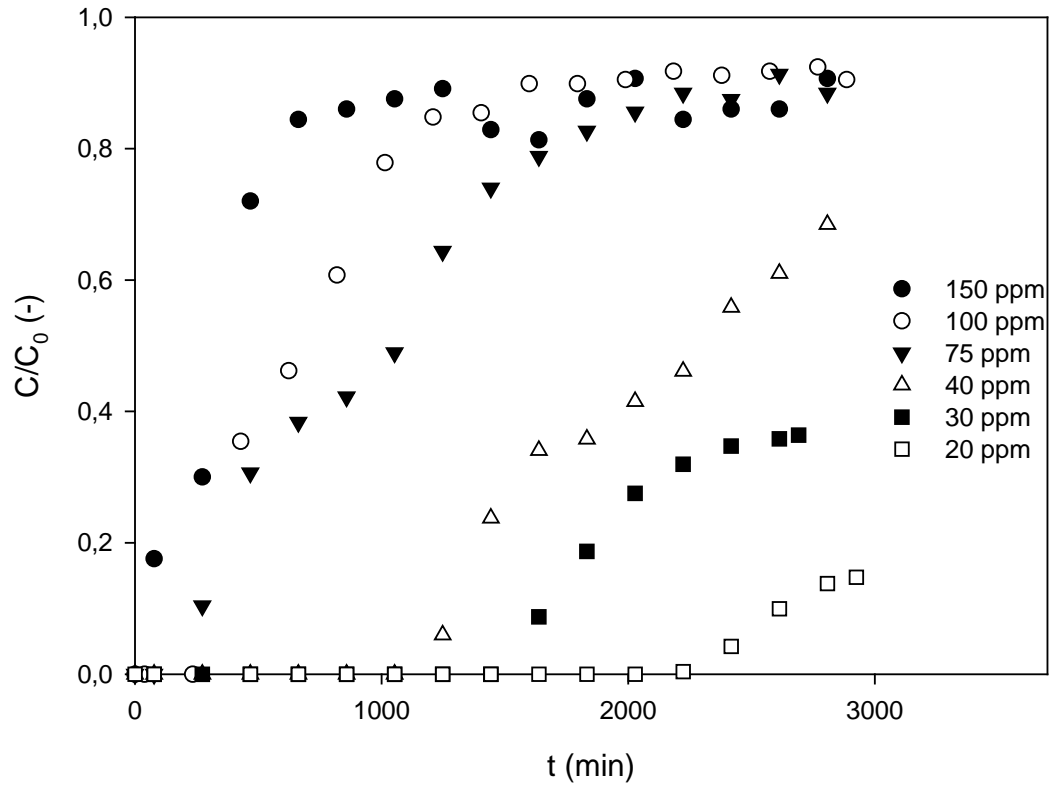
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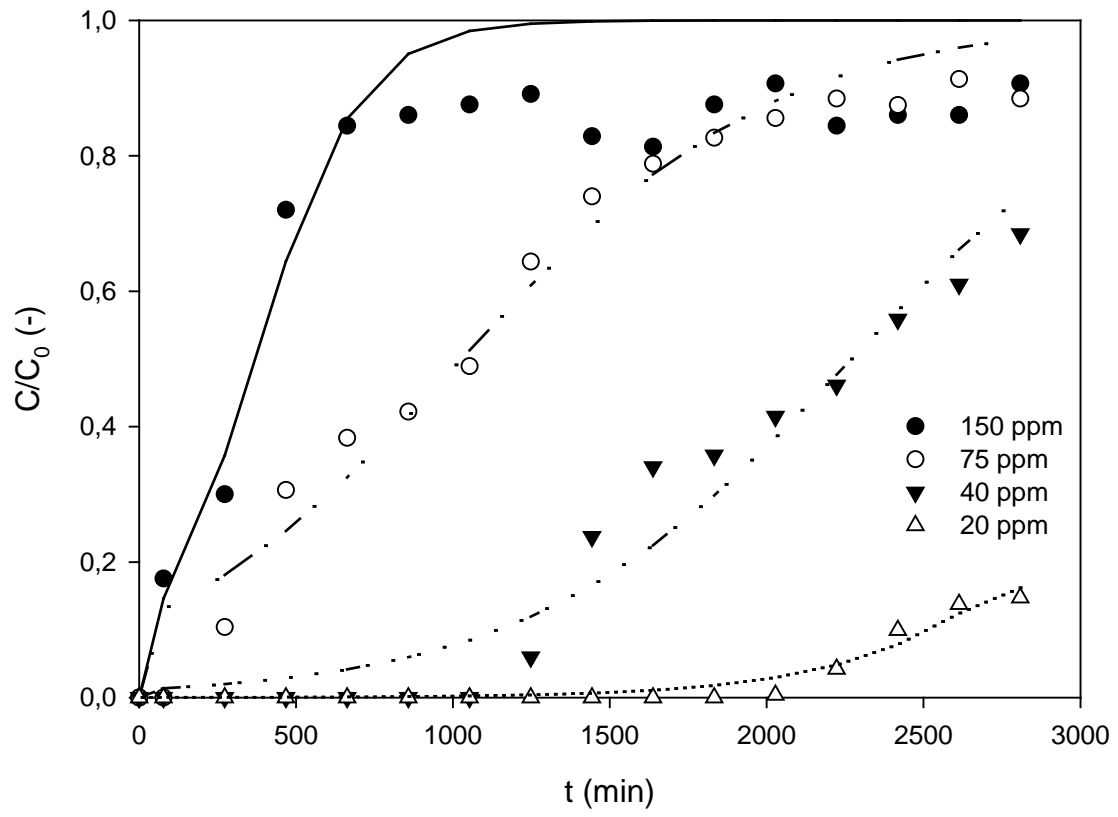
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480 Fig. 1

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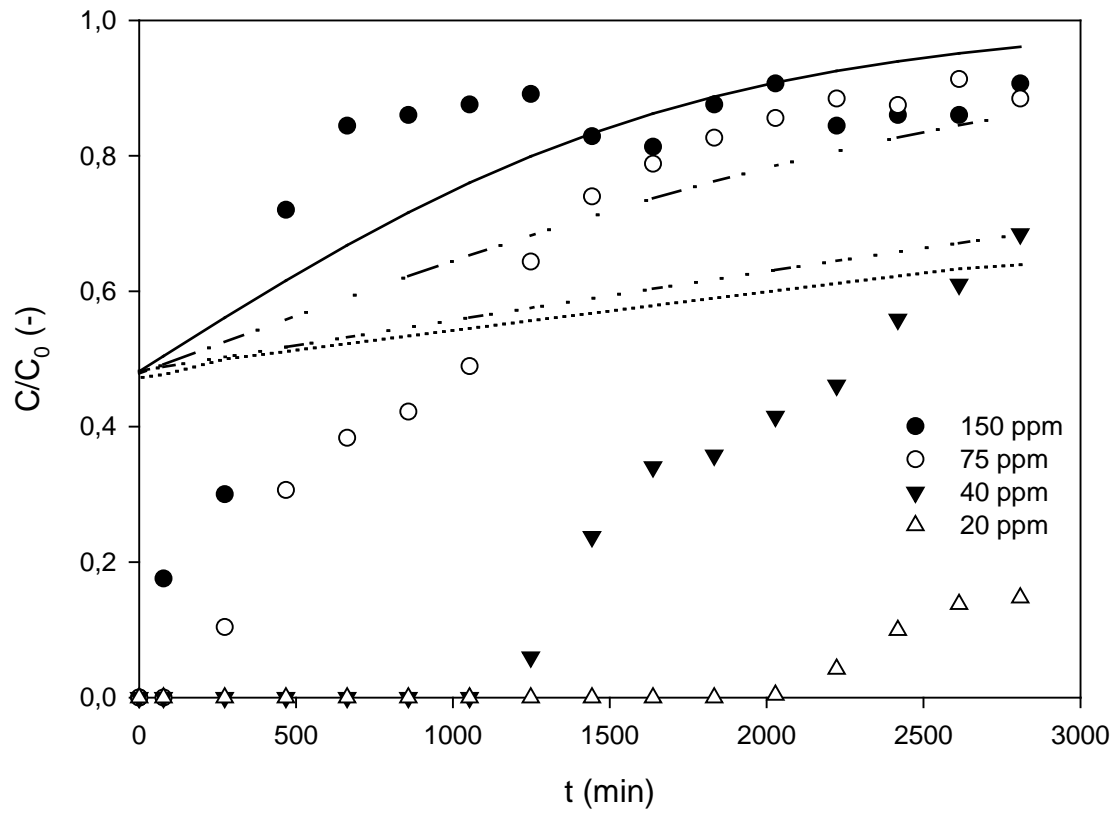
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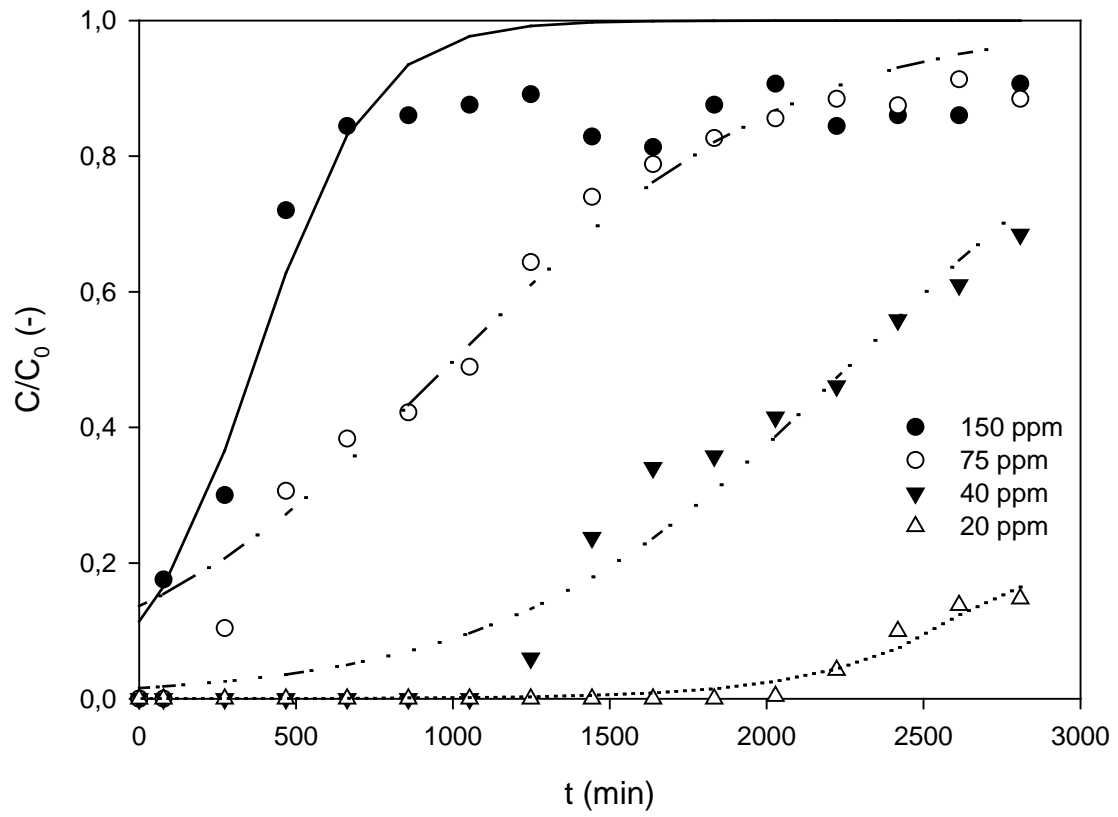
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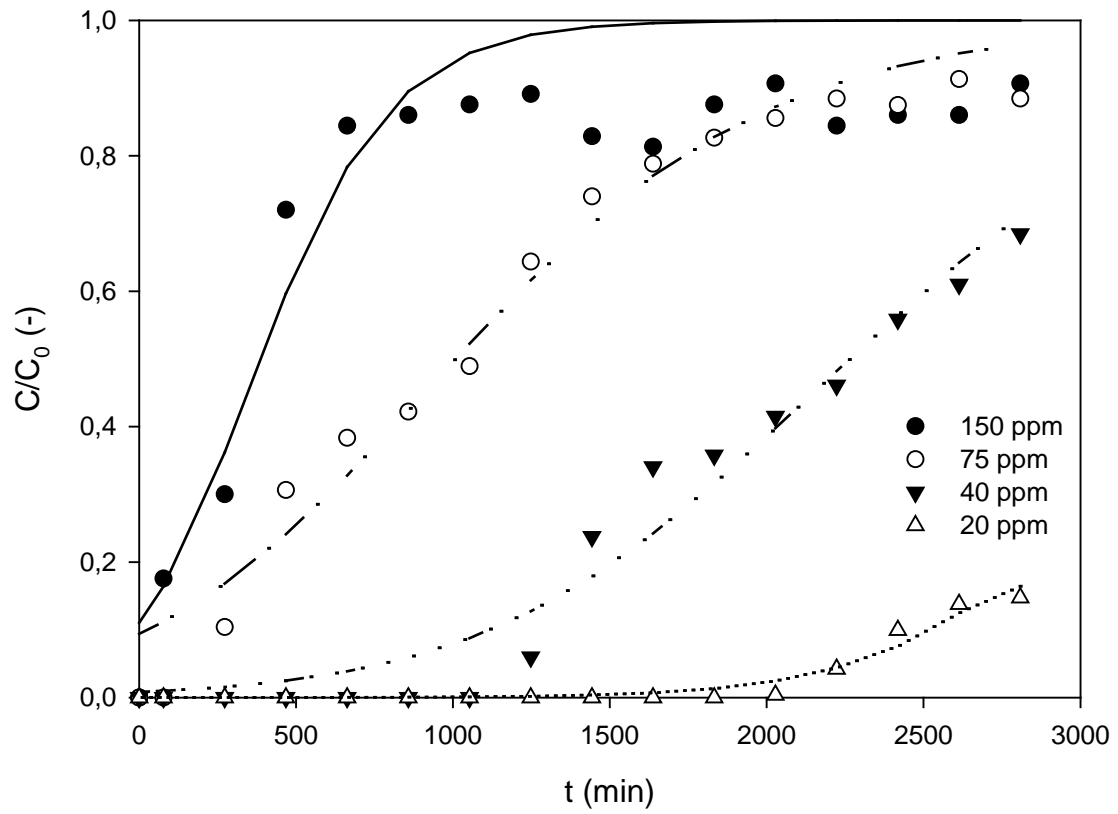
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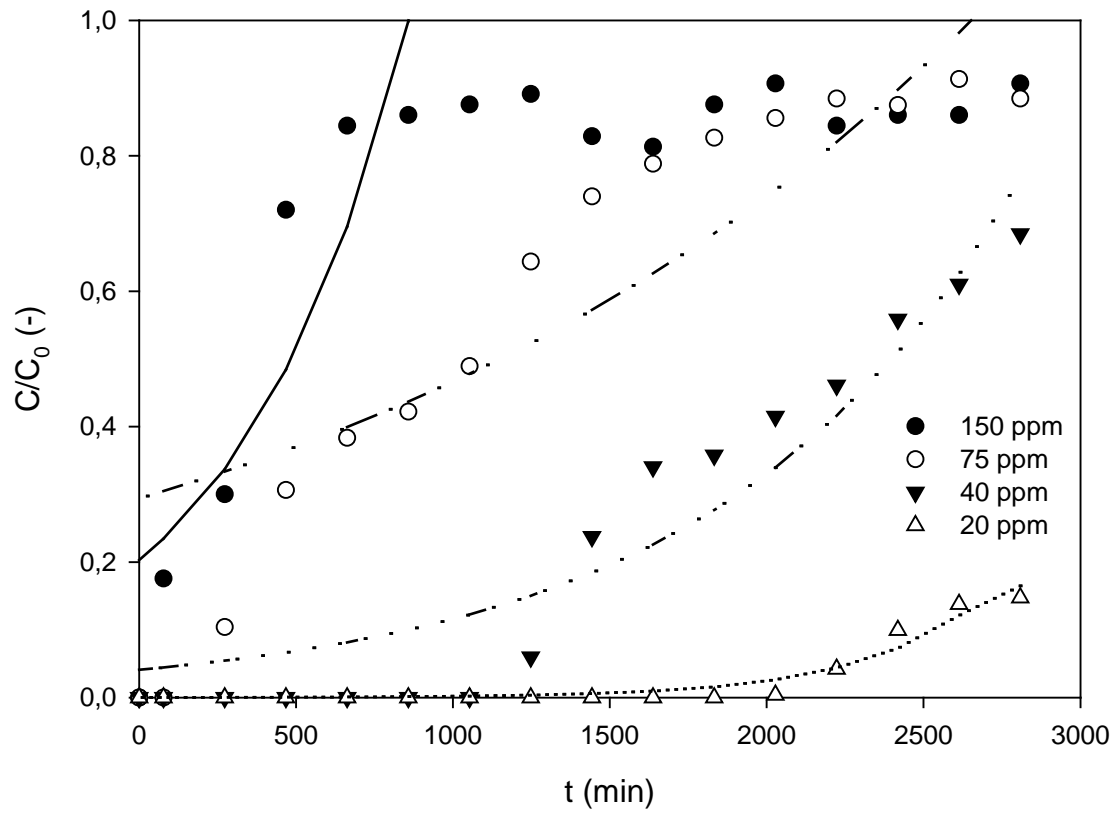
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536 Fig. 3a

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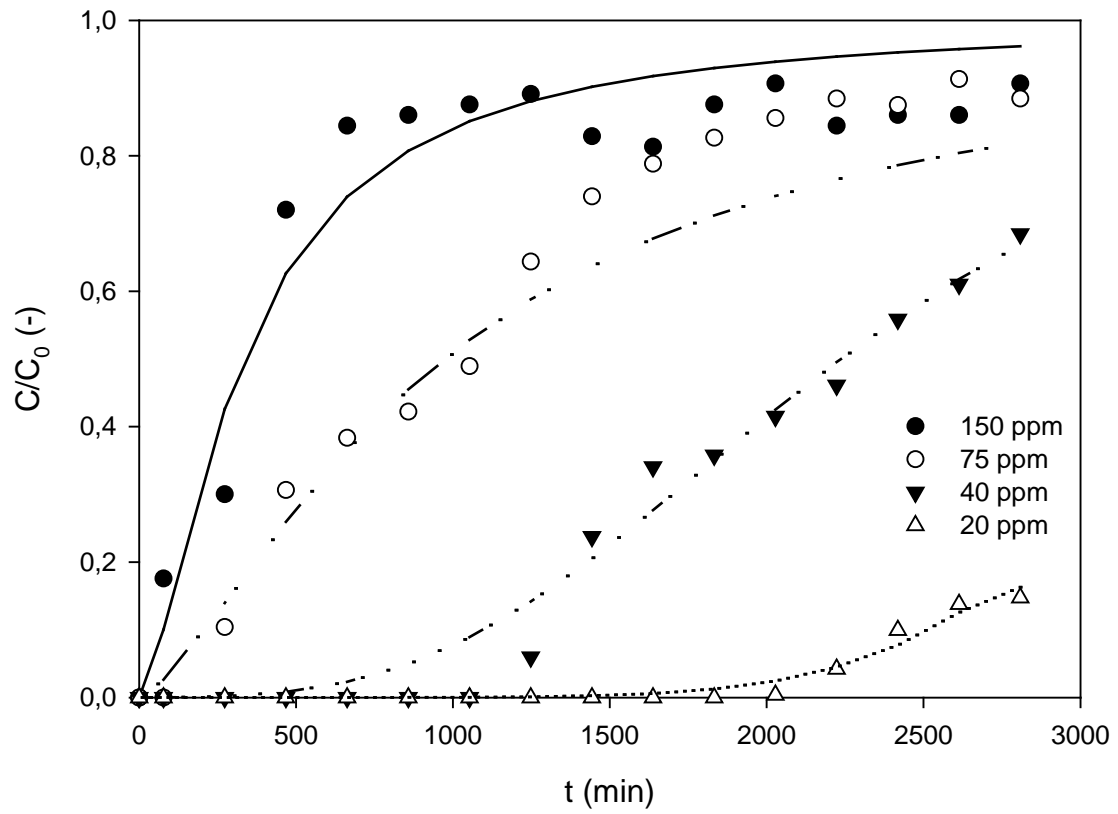
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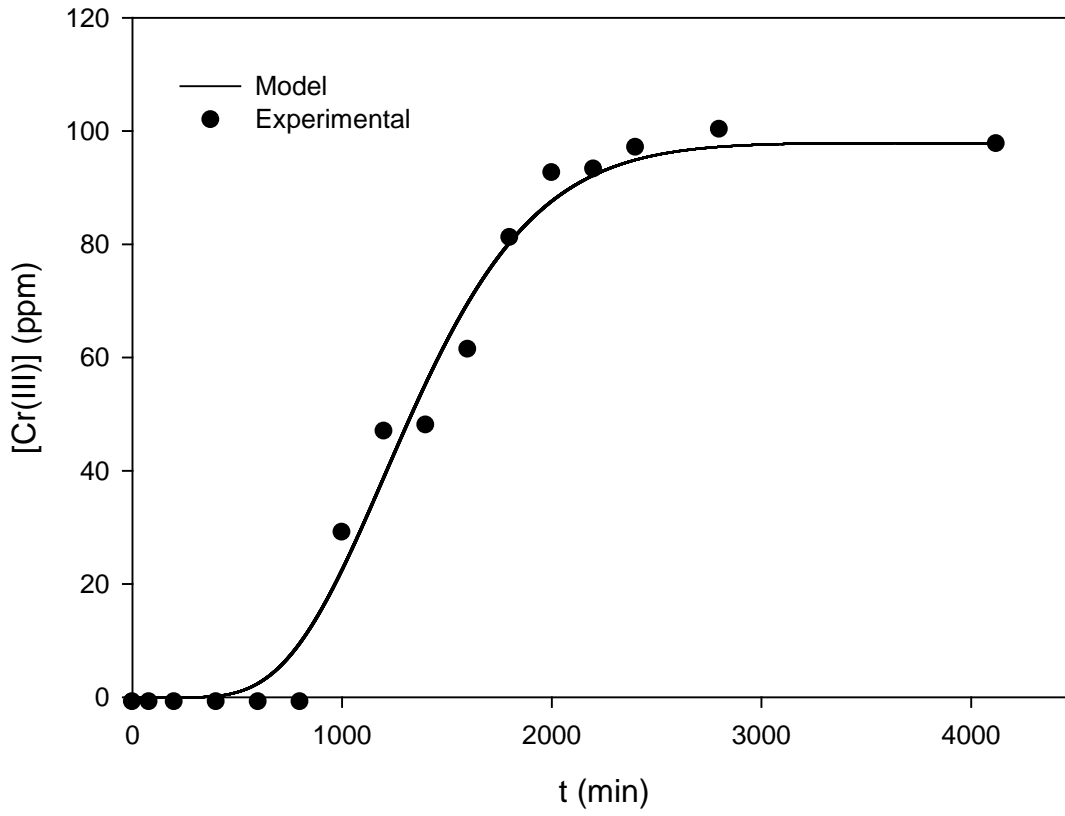
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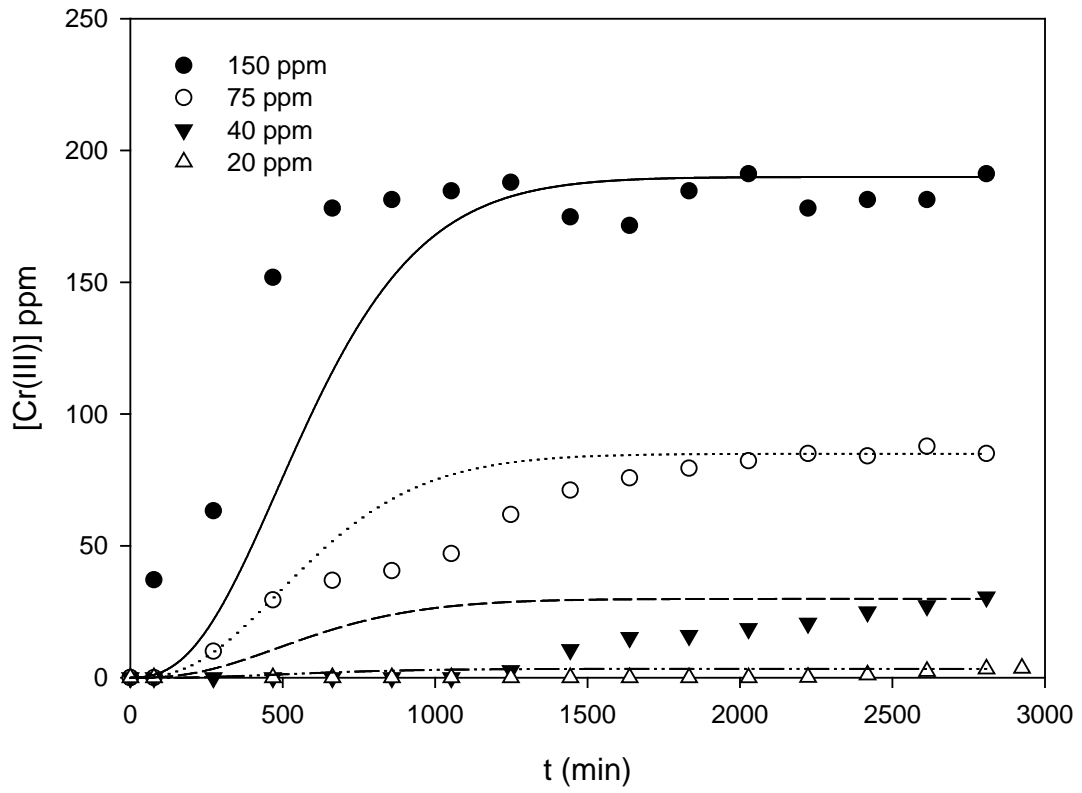
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578 Fig. 4

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592 Fig.5

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594 **Table 1.** Chemical composition and physical properties of leonardite (SEPHU®) used
595 in this study.

| Property | Value |
|---|-------|
| Humic acids (%) | 79.0 |
| C (%) | 55.2 |
| H (%) | 3.4 |
| N (%) | 0.8 |
| O (%) | 38.1 |
| S (%) | 2.4 |
| BET area (m ² g ⁻¹) | 19.9 |
| Moisture (%) | 30.0 |
| Density (g mL ⁻¹) | 1.67 |
| Cationic exchange capacity (meq g ⁻¹) | 2.87 |
| Single bond COOH groups (meq g ⁻¹) | 3.12 |
| Single bond OH groups (meq g ⁻¹) | 2.07 |

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601 **Table 2.** Model parameters and ERRSQ values for the BDST, Thomas and Yoon and Nelson models.

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| C_0 | BDST | | | Thomas | | Yoon and Nelson | | |
|-----------------------|-----------------------|---|--------|---|-------|----------------------|-----------|--------|
| | N_0 | K | ERRSQ | k_T | ERRSQ | k_{YN} | $t_{0.5}$ | ERRSQ |
| (mg L ⁻¹) | (mg L ⁻¹) | (L min ⁻¹ mg ⁻¹) | (-) | (L min ⁻¹ mg ⁻¹) | (-) | (min ⁻¹) | (min) | (-) |
| 20 | 99.33 | 0.00793 | 0.0015 | 0.000700 | 0.972 | 0.00288 | 3488.38 | 0.0019 |
| 40 | 127.53 | 0.00293 | 0.1420 | 0.000450 | 0.617 | 0.00182 | 2281.88 | 0.0445 |
| 75 | 114.28 | 0.00160 | 0.0498 | 0.000546 | 2.275 | 0.00184 | 1004.65 | 0.0299 |
| 150 | 81.24 | 0.00242 | 0.0241 | 0.000466 | 1.441 | 0.00549 | 372.95 | 0.2440 |

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607 **Table 3.** Model parameters and ERRSQ values for the Clark, Wolborska and Dose-Response models.

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| C ₀ (mg L ⁻¹) | Clark | | | Wolborska | | | Response-Dose | | |
|---|----------|---------------------------|--------------|---|---------------------------|--------------|---------------|---|--------------|
| | A (-) | r (min ⁻¹) | ERRSQ (-) | N ₀ (mg L ⁻¹) | B (min ⁻¹) | ERRSQ (-) | a (-) | q ₀ (mg g ⁻¹) | ERRSQ (-) |
| 20 | 591.67 | 0.119 | 0.0016 | 101.20 | 0.795 | 0.0022 | 7.47 | 0.200 | 0.0015 |
| 40 | 14.42 | 0.090 | 0.0355 | 171.53 | 0.269 | 0.0888 | 3.09 | 0.246 | 0.0134 |
| 75 | 2.81 | 0.105 | 0.0397 | 278.66 | 0.103 | 0.2951 | 1.43 | 0.201 | 0.0055 |
| 150 | 2.48 | 0.255 | 0.2380 | 180.40 | 0.134 | 0.1503 | 1.51 | 0.137 | 0.1134 |

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