

Optimal departure aircraft trajectories minimising population annoyance

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Abstract—This paper presents a strategy for designing noise abatement procedures aimed at reducing the global annoyance perceived for the population living around the airports. By using fuzzy logic techniques it is shown how annoyance can be modelled in function of the maximum perceived noise level at a specific noise sensitive location and the time of the day when the departure takes place. Thus, the annoyance is computed for different kinds of sensibility areas, such as residential zones, industrial zones, schools or hospitals and an annoyance figure is obtained for each possible trajectory. Then, a non-linear multi-objective optimal control problem is presented in order to obtain the minimum annoyance trajectory for all sensitive locations. Lexicographic optimisation is used to cope with the difficulties that arise when several criteria appear in the optimisation process. Finally, a practical example is given for an hypothetical scenario where different optimal trajectories are obtained at different day periods.

I. INTRODUCTION

The noise produced by aircrafts during take-off and landing operations around airports is a very serious ecological and social problem. Aircraft noise can be very annoying for people living in the vicinity of the airports. Therefore, the design of noise abatement procedures aimed at reducing the noise exposure of the population around airports is one of the main issues that airport authorities and national navigation services providers have to address. Noise is generally defined as an unwanted sound and its effects can be appreciated physiologically but also psychologically [1]. Annoyance is a concept that is hard to quantify because there is no underlying physically measurable scale. However, it is usually qualitatively assessed with social surveys. It is clear that fuzzy techniques can help to make more accurate predictions by incorporating the vagueness and uncertainty into the modelling and reasoning process. Recently, few research papers based on fuzzy logic in noise pollution area have been reported [2], [3], [4]. In [3], annoyance is considered as a function of noise level, its duration of occurrence, and the socioeconomic status of a person and the results were applicable to the urban areas of India. In [4], a fuzzy model has been developed, on the basis of field surveys conducted by various researchers and reports of World Health Organisation, for predicting the effects of sleep disturbance by noise on humans as a function of noise level, age and duration of its occurrence. Fuzzy set theory is a generalisation of traditional set theory and provides a means for the representation of imprecision and vagueness. Zadeh [5]

further developed the corresponding fuzzy logic to manipulate fuzzy sets.

The International Civil Aviation Organisation (ICAO) publishes two different Noise Abatement Departure Procedures (NADP), defined in [6]. NADP are generic procedures and are far from being the optimum ones regarding noise minimisation. This is due to several factors, such as the impossibility to define a general procedure satisfying the specific problems that may affect each particular airport, air traffic management and airport capacity constraints or even the the limitations of nowadays on-board technology. Nevertheless, some research in theoretical optimal trajectories minimising the noise impact in departure or approaching procedures is also found in the literature. For instance, in [7], [8] and [9] is presented a tool combining a noise computation model, a Geographical Information System (GIS) and a dynamic trajectory optimisation algorithm, aimed at obtaining optimal noise procedures. A similar methodology is proposed in [10], and an adaptative algorithm for noise abatement can be found in [11]. On the other hand, in [12] and [13] it can be found a dynamic programming technique for minimising noise in runway-independent aircraft operations. All the results and conclusions arisen from these works are encouraging and will set the basis for new noise abatement procedures, specially regarding the forthcoming new navigation concepts, such as area navigation (RNAV) or Performance Based Navigation (PBN). These concepts will allow for air navigation procedures to be designed with a higher level of flexibility than conventional radionavigation ones [14].

This paper is organised as follows: in Section 2 the optimisation criteria are presented introducing how annoyance can be modelled by using fuzzy logic. Section 3 is devoted to the optimisation strategy that is proposed to solve this multi-criteria objective problem. Finally, section 4 shows the results obtained for a hypothetical airport scenario containing two residential zones, a school, a hospital and an industrial zone.

II. OPTIMISATION CRITERIA

This section presents two kinds of optimisation criteria. First one deals with the noise annoyance produced when the trajectory is flown. Second criterion takes into account airliner costs, such as time or fuel consumption.

TABLE I
RULE BASE TABLE FOR THE ANNOYANCE IN FUNCTION OF THE PERCEIVED NOISE AND THE HOUR OF THE DAY

	Residential zone			School			Hospital			Industrial zone		
	Mor.	Aft.	Night	Mor.	Aft.	Night	Mor.	Aft.	Night	Mor.	Aft.	Night
No noise	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
Very low noise	NA	NA	SA	SA	SA	NA	SA	SA	MA	NA	NA	NA
Low noise	NA	SA	MA	MA	MA	NA	MA	MA	HA	NA	NA	NA
Medium noise	SA	MA	HA	HA	HA	NA	HA	HA	EA	NA	SA	SA
High noise	MA	HA	EA	HA	HA	NA	EA	EA	EA	SA	MA	MA
Very high noise	HA	EA	EA	EA	EA	NA	EA	EA	EA	MA	HA	HA

A. Noise annoyance

The annoyance or perception of the acoustic noise describes the relation between a given acoustic situation and a given individual or set of persons affected by the noise and how cognitively or emotionally they evaluate this situation. The acoustic annoyance of the aircraft flights around an urban airport depends logically of the acoustic behaviour produced in the sensitive locations, using for example, the L_{max} or Sound Exposure Level (SEL) metrics, but it is not a sufficient measurement to define completely the annoyance behaviour of a noise. An additional list of non acoustic elements to take into account to define the annoyance behaviour could be:

- Types of affected zones (rural zone, residential zone, industrial zone, hospitals, schools, markets,...)
- Time interval during the noise event (day, evening, night)
- Period of time between two consecutive flights
- Personal elements (emotional, apprehension to the noise, personal healthy, age,...)
- Cultural aspects (young or aged people habits, activities, holiday,...)

In conclusion, the annoyance is a subjective and a complex concept which can be studied as a qualitative form using fuzzy logic sets, as previous similar works in this area have been done (see for instance [2], [3], [4] and [15]). In this paper, the annoyance generated by the aircraft trajectories will be represented by fuzzy logic sets from the fuzzification of the maximum sound level (L_{max}) and from the hour of the day where the trajectory is supposed to be flown regarding four typical zones around an urban airport: a residential zone, a hospital, a school and an industrial zone.

1) *Noise model*: The maximum perceived sound level at location i is defined as:

$$L_i(\vec{z}) = \max_t \text{Noise}_i(\vec{z}(t)) \quad (1)$$

where $\text{Noise}(\vec{z}(t))$ is the perceived noise level at location i for a given trajectory $\vec{z}(t)$ (being t the time variable).

In this work, the same methodology employed by the Integrated Noise Model (INM) program [16] is implemented when computing noise functions. INM is developed by the Federal Aviation Administration¹ (FAA) and has been adopted as the standard package for noise studies and assessments

¹<http://www.faa.org>

in many countries. INM deals with several noise metrics and, in particular, noise levels are computed at a given point by selecting and interpolating appropriate noise values from a noise-thrust-distance (NTD) table, which is derived from empirical measurements.

2) *Annoyance model*: In [17] the authors presented a basic methodology for modelling aircraft noise annoyance by using fuzzy logic. Essentially, two membership set functions are defined. The first set introduces five linguistic terms to describe the magnitude of the maximum sound level (L_{max}):

- Very high noise
- High noise
- Medium noise
- Low noise
- Very low noise

A second set is related with the hour of the day introducing the following linguistic terms:

- Morning
- Afternoon
- Night

Afterwards a rule base is established to represent the annoyance of an event defined by the two fuzzy logic sets for each of the 4 zones considered. The annoyance concept has been represented by the following linguistic terms:

- Extreme Annoyance (EA)
- High Annoyance (HA)
- Moderated Annoyance (MA)
- Small Annoyance (SA)
- Null Annoyance (NA)

Table I shows the rule base of the annoyance at all sensitive locations. For each couple of sound level and time of day linguistic terms a rule is established giving a specific annoyance term. By using this kind of fuzzy rule base it would be easy, for example, to model the output of a population survey, asking for the annoyance produced by the airport.

Finally, the fuzzy set of the annoyance is defined as a crisp set to obtain a normalised degree of annoyance. Extreme annoyance corresponds to a normalised value of 1, high annoyance takes 0.75 value, medium annoyance takes 0.5, small annoyance takes 0.25 and finally null annoyance corresponds to 0. Figures 1, 2, 3 and 4 show in a plot this normalised annoyance in function of the two input variables (L_{max} and the hour of the day) for each noise sensitive location.

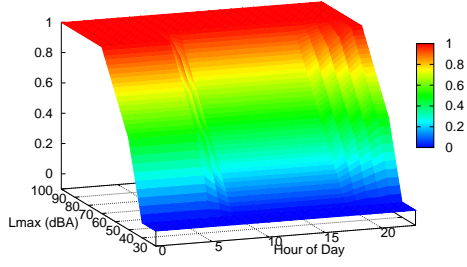


Fig. 1. Normalised Annoyance. **Hospital**

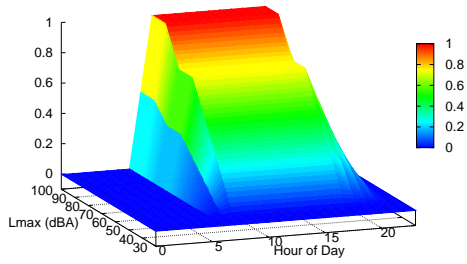


Fig. 2. Normalised Annoyance. **School**

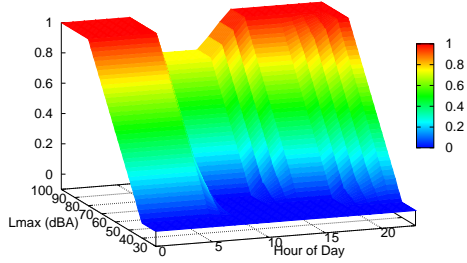


Fig. 3. Normalised Annoyance. **Residential Zone**

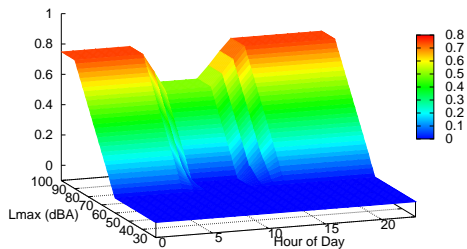


Fig. 4. Normalised Annoyance. **Industrial Zone**

B. Airliner costs

Airliner cost and Air Traffic Management (ATM) efficiency should also be taken into account when designing aircraft

trajectories. In this context, *Fuel* and/or *Time* spent during the trajectory may be considered as optimisation objectives too.

Being t_0 and t_f the initial and final time of a given trajectory, fuel cost C_f associated to this trajectory can be computed as:

$$C_f = \pi_c \cdot Fuel = \pi_c \int_{t_0}^{t_f} FF(t) dt \quad (2)$$

where π_c is the fuel price and $FF(t)$ is the total fuel flow, which in turn can be expressed in function of the current thrust setting.

On the other hand, time cost represents the different constant rate costs associated with aircraft operations (insurances, traffic control fees, crew salaries, etc). This can be easily written as:

$$C_t = \pi_t \cdot Time = \pi_t(t_f - t_0) \quad (3)$$

where π_t is the cost attached to one unit of time of delay.

Current Flight Management and Guidance Systems (FMGS) equipping a wide number of aircraft deal with a compound cost function which involves fuel and time consumption during the flight. A cost index parameter (*CI*) relates the cost of time delay to the price of the fuel and its value is carefully chosen by the operator prior to each flight. Cost index (*CI*) is defined as:

$$CI = \frac{\pi_t}{\pi_c} \quad (4)$$

Fuel saving flights are associated with low values of the cost index while more direct and faster flights are associated with high values of this index. As mentioned above, this strategy is currently used in civil aircraft operations giving optimal flight levels and speed settings for all phases of flight.

In addition, for this study it would be incomplete to consider only these magnitudes regardless of the altitude achieved at the end of the procedure. Reaching a low final altitude $h(t_f)$ would lead to small time or fuel consumption figures during the departure but the consumption would increase in the following phase, when trying to gain the altitude required to reach the optimal cruise flight level. Therefore, the final altitude must be also taken into account as an optimisation criterion to be maximised. Following the same philosophy, an *Height Index (HI)* is proposed in this work. Finally, the airliner cost compound function is defined as:

$$C_a = Fuel + CI \cdot Time - HI \cdot h(t_f) \quad (5)$$

where, by definition, $CI > 0$ and $HI > 0$.

III. THE OPTIMISATION STRATEGY

In [18], the authors presented a framework to optimise departing or approaching trajectories which can be summarised in figure 5. The involved airport, with its surrounding cartography, geography and meteorological data, will define a *scenario* which will be used to compute a given *noise nuisance* in

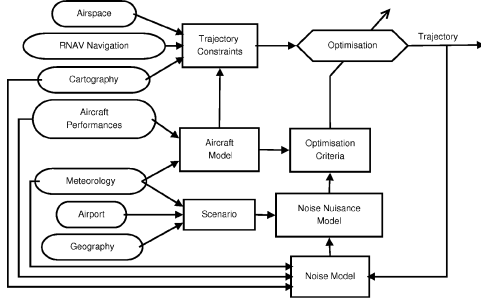


Fig. 5. Framework for the noise abatement optimisation strategy

function of the emitted aircraft noise along its trajectory. This value, together with some airliner economic considerations, will define one or several *optimisation criteria*. Then, an *optimisation algorithm* will compute the best departing or approaching trajectory minimising these criteria and satisfying a set of *trajectory constraints* which, in turn, will depend on the dynamics of the aircraft, navigation constraints and specific airspace configurations.

A. Statement of the problem

This optimisation process can be formally written as a *constrained multi-objective optimal control problem* in a given time interval $[t_0, t_f]$. In this case, the value of t_f is let *free* during the optimisation, meaning that this value is a decision variable itself and will be fixed by the optimisation algorithm. Let $\vec{x}(t) \in \mathbb{R}^{n_x}$ be the state vector describing the trajectory of the aircraft over the time t , $\vec{u}(t) \in \mathbb{R}^{n_u}$ the control vector that leads to a specific trajectory and $\vec{p} \in \mathbb{R}^{n_p}$ a set of control parameters not dependent on t . The goal is to find the best trajectory that minimises a given set of optimisation objectives (or criteria) $\vec{J} \in \mathbb{R}^{n_j}$. Namely:

$$\min_{\vec{z} \in \mathcal{Z}} \vec{J}(\vec{z}) = \min_{\vec{z} \in \mathcal{Z}} [J_1(\vec{z}), J_2(\vec{z}), \dots, J_{n_j}(\vec{z})] \quad (6)$$

where $\mathcal{Z} \subseteq \mathbb{R}^{n_x + n_u + n_p + 1}$, is the admissible set of decision variables $\vec{z} = [\vec{x}(t), \vec{u}(t), \vec{p}, t_f]^T$, and $J_i(\vec{z})$ are scalar valued functions representing each individual criterion or objective.

In order to guarantee a feasible and acceptable trajectory as a result of the optimisation process presented above, several constraints must be taken into account and are summarised as:

- *dynamic constraints* describing the trajectory of the aircraft:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \quad (7)$$

- *end point or event constraints* fixing the initial and final boundary conditions:

$$\mathbf{e}_L \leq \mathbf{e}(\mathbf{x}(t_0), \mathbf{x}(t_f), t_0, t_f) \leq \mathbf{e}_U \quad (8)$$

- *mixed state-control path constraints* allowing to restrict the behaviour of some variables:

$$\mathbf{h}_L \leq \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t), t) \leq \mathbf{h}_U \quad (9)$$

- *box constraints on the state and control variables* allowing to bound them:

$$\begin{aligned} \mathbf{x}_L &\leq \mathbf{x}(t) \leq \mathbf{x}_U \\ \mathbf{u}_L &\leq \mathbf{u}(t) \leq \mathbf{u}_U \end{aligned} \quad (10)$$

Function f is a non linear function that contains the dynamical model of the aircraft trajectory. Vectorial functions \mathbf{e} and \mathbf{h} define the *event* and *path* constraints respectively and vectors \mathbf{e}_L , \mathbf{e}_U , \mathbf{h}_L , \mathbf{h}_U , \mathbf{x}_L , \mathbf{x}_U , \mathbf{u}_L and \mathbf{u}_U are respectively the *Lower* and *Upper* values which bound all constraints. For a detailed description of these functions and vectors, please refer to [18].

B. Numerical solution of the optimisation problem

The optimal control problem described in section III, which contains differential and algebraic constraints, is transformed in two steps into a non linear programming (NLP) problem with only algebraic constraints. First, differential equations (7) are written in its equivalent integral form:

$$\vec{x}(t) = \vec{x}(t_0) + \int_{t_0}^t \vec{f}(\vec{x}(\tau), \vec{u}(\tau), \vec{p}) d\tau \quad (11)$$

Then, equation (11) is discretised using a sampling time $\Delta t = t_{n+1} - t_n$ where t_{n+1} and t_n are two consecutive time instants using an explicit numerical integration rule to approximate the above integral, as Euler or Runge-Kutta. For example, in case of using the Euler rule, the following equivalent discrete-time form is obtained:

$$\vec{x}(k+1) = \vec{x}(k) + \Delta t \cdot \vec{f}(\vec{x}(k), \vec{u}(k), \vec{p}) \quad (12)$$

Once the problem is formulated as a NLP, it can be solved using a commercial optimisation software. In this paper, the General Algebraic Modelling System (GAMS)² is the optimisation package used to code and solve the NLP problem. The numerical optimisation method used to solve the problem is a generalised reduced gradient search [19], implemented in the NLP solver CONOPT³ available in the GAMS optimisation package, which can cater for the nonlinearities of the performance index and constraints.

The CONOPT optimisation algorithm starts by finding a feasible solution; then, an iterative procedure follows, which consists of:

- finding a search direction, through the use of the Jacobian of the constraints, the selection of a set of basic variables and the computation of the reduced gradient.
- performing a search in this direction, through a pseudo-Newton process until a convergence criterion is met.

A detailed description of the CONOPT algorithm and its implementation may be found in [20] and in the manuals available at the GAMS web page.

²<http://www.gams.com>

³www.aimms.com/aimms/product/solvers/conopt.html

C. Lexicographic algorithm

A solution \bar{z}^* of the multi-objective optimisation problem, presented in equation (6), is said to be *Pareto optimal* iff there does not exist another $\bar{z} \in \mathcal{Z}$ such that $J_i(\bar{z}) \leq J_i(\bar{z}^*)$ for all $i = 1, \dots, n_j$ and $J_j(\bar{z}) < J_j(\bar{z}^*)$ for at least one index j . In other words, a solution is Pareto optimal if and only if an objective $J_i(\bar{z})$ can be reduced only at the expense of increasing at least one of the other objectives. In general, there may be many Pareto optimal solutions to an optimisation problem.

Lexicographic optimisation establishes a hierarchical order among all the optimisation objectives. If such a priority exists, a unique solution exist on the Pareto hyper-surface (see [21] and the references therein).

Let the objective functions be arranged according to the *lexicographic order* from the most important J_1 to the least important J_{n_j} . A given $\bar{z}^* \in \mathcal{Z}$ is a *lexicographic minimiser* of equation (6) iff there does not exist a $\bar{z} \in \mathcal{Z}$ and a j satisfying $J_j(\bar{z}) < J_j(\bar{z}^*)$ and $J_i(\bar{z}) = J_i(\bar{z}^*)$ for all $i = 1, \dots, j - 1$. An interpretation of this definition is that a solution is a lexicographic minimum iff an objective J_i can be reduced only at the expense of increasing at least one of the higher-prioritised objectives $\{J_1, \dots, J_{(i-1)}\}$. Hence, a lexicographic solution is a special type of Pareto-optimal solution that takes into account the order of the objectives. This hierarchy defines an order on the objective function establishing that a more important objective is infinitely more important than a less important objective.

A standard method for finding a lexicographic solution is to solve a sequential order of single objective constrained optimisation problems. After ordering, the most important objective function is minimised, subject to the original constraints. If this problem has a unique solution, it is the solution of the whole multi-objective optimisation problem. Otherwise, the second most important objective function is minimised. Now, in addition to the original constraints, a new constraint is added to guarantee that the most important objective function preserves its optimal value. If this problem has a unique solution, it is the solution of the original problem. Otherwise, the process goes on iteratively. More formally, the lexicographic minimum of equation (6), $\text{lex min}_{\bar{z} \in \mathcal{Z}} \vec{J}(\bar{z})$, can be found by using the following algorithm:

- 1: $J_1^* = \min_{\bar{z} \in \mathcal{Z}} [J_1(\bar{z})]$
- 2: **for** $i = 2$ to n_j **do**
- 3: $J_i^* = \min_{\bar{z} \in \mathcal{Z}} [J_i(\bar{z}) | J_j(\bar{z}) \leq J_j^*, j = 1, \dots, i - 1]$
- 4: **end for**
- 5: Determine the lexicographic minimiser set as:
 $\bar{z}^* = \arg(J_{n_j}^*)$

Lexicographic optimisation permits to sort *a priori* the different optimisation criteria according to its relative importance. This method has shown several benefits in front of the classical weighting methodology [22], [23] and has been started to be widely used in control engineering applications (see, for instance [24], [21] and [25]).

TABLE II
HYPOTHETICAL SCENARIO DATA

Departing runway heading	70°
Minimum climb gradient	3.3%
Initial point coordinates	[0, 0] km
Final point coordinates	[10, 20] km
Minimum height at final point	4000 ft
Maximum height at final point	10000 ft
Cost Index (CI)	CI = 1
Height Index (HI)	HI = 0.1

We can assume that the procedure designer in charge of publishing such a departure trajectory (i.e. the decision maker of this optimisation process) has a clear idea of what prioritisations should give to each location, maybe influenced by some *political reasons*. In that case, previous algorithm leads to the best trajectory according to the desired hierarchy. In the case where this prioritisation is not clear, or when a more accurate scenario study is necessary, it is possible to run all optimisations by using all possibilities in the prioritisation order. The number of different prioritisations is $n_P = n_j!$, where n_j is the total number of noise sensitive locations. Then a *performance index* is defined aimed at choosing the best trajectory among all the possibilities.

Let J_i^* be the minimum annoyance that can be achieved at sensitive location i (i.e. when location i is in the first priority). Let J_i^P be the annoyance at location i reached with the optimal trajectory corresponding to priority P . For each priority P a performance factor Δ^P can be defined as:

$$\Delta^P = \max_i (J_i^P - J_i^*) \quad (13)$$

Then, the best trajectory, \bar{z}^* corresponds to the priority minimising this performance factor Δ^P :

$$\bar{z}^* = \arg(\min_P (\Delta^P)) \quad (14)$$

IV. APPLICATION EXAMPLE

This section presents a practical example concerning an hypothetical scenario where a departure route should be optimised.

A. Scenario description

Table II summarises the different data that define this scenario. In a departure trajectory it is enforced that speed and altitude may not decrease during all the procedure. In addition, being all trajectories below 10000 ft maximum airspeed becomes $v_{max} = 250$ Kt [6]. The chosen aircraft model corresponds to the Airbus A340-600 equipped with Trent 556 engines and operating at its Maximum Take-off Weight, ($m = 368000$ kg). Take-off is supposed to be performed with CONF3 flaps/slats configuration. The initial take-off phase going from ground level to a height 400 ft will not be considered in the optimisation process since the standard operational regulations almost restrict all degrees of freedom

TABLE III
NOISE SENSITIVE LOCATIONS

Sensitive location	Acronym	East coord.	North coord.
School	S	2000 m	1500 m
Industrial Zone	I	6000 m	2500 m
Residential Zone 1	R1	4000 m	5000 m
Hospital	H	7000 m	8000 m
Residential Zone 2	R2	6000 m	13000 m

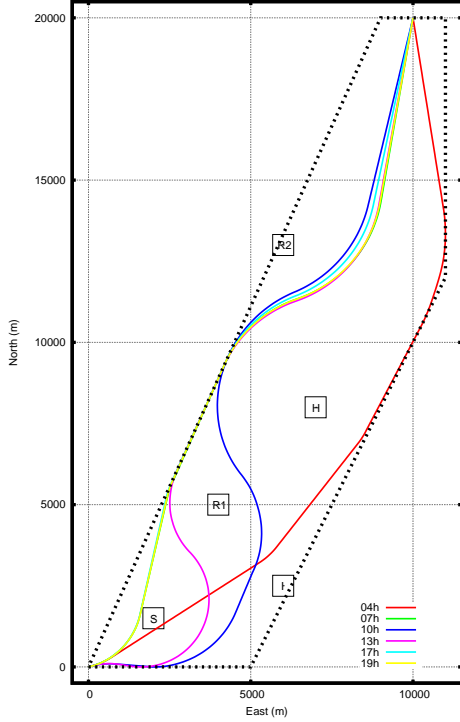


Fig. 6. Optimal trajectories at different hours of the day. **Horizontal tracks**

during this particular phase [6], [26]. In this initial phase the aircraft follows a straight trajectory, following the departing runway heading, at a constant speed (usually v_2), which depends on the aerodynamics and the actual weight of the aircraft. For this problem, and for the sake of simplicity, initial horizontal coordinates are set to zero at the point where the aircraft reaches a height of 400 ft above the runway. Moreover, during a normal take-off, the landing gear has been completely retracted when passing 400 ft so it is not considered in the simulations. Finally, five different noise sensitive locations have been located in the vicinity of the departing runway (see table III).

B. Optimal trajectories

Table IV contains the minimum annoyance values corresponding to the trajectories that minimise only one noise sensitive criterion in function of the hour of the day. In other words, these values are the best annoyance figures that can be achieved with independent single objective optimisations at each sensitive location for this particular scenario. The

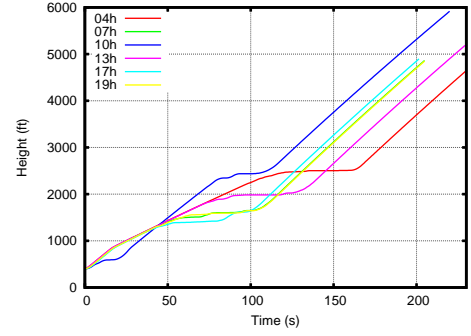


Fig. 7. Optimal trajectories at different hours of the day. **Vertical paths**

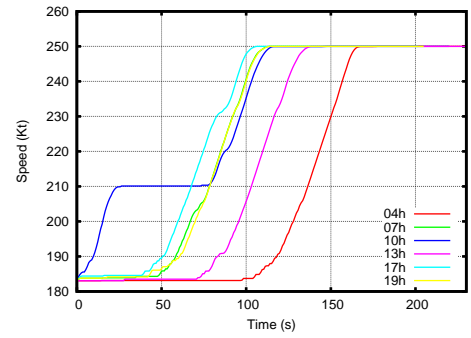


Fig. 8. Optimal trajectories at different hours of the day. **Speed profiles**

corresponding L_{max} values that produce such annoyance values are also given in the table. As it was commented in section III, in a multi-objective optimisation problem there exist multiple *Pareto-optimal* solutions regarding all objectives. Lexicographic optimisation presented in the same section allows to obtain a solution of the *Pareto* front for a given order in the optimisation objectives. Finally, by using equation (14) the “best” *Pareto-optimal* solution is chosen, according to the performance index stated in equation (13). Tables V and VI show, for different hours of the day, the prioritisation P giving the best performance index. These tables contain the annoyance values at each noise sensitive location as well as the corresponding L_{max} values. Finally, it is shown the time and fuel used in the optimal trajectory and the height reached at the end of the procedure.

Figure 6 shows the corresponding 6 optimal trajectories (flown at 04h, 07h, 10h, 13h, 17h and 19h). As it can be seen, optimal night trajectories (04h) start with a straight segment following runway heading and almost over-flying the school location (S). However annoyance in this location is zero since in night periods schools are not annoyed (see table V). This initial path allows to keep a trade-off distance to residential zone 1 (R1) and the industrial zone (I), producing a relatively low value of annoyance (0.38 and 0.21, respectively). The hospital (H) is passed following the east airspace restriction (dotted line in figure 6) producing a relatively high amount of annoyance (0.8) due to the high sensibility of this location during night periods. Finally the annoyance produced in the

TABLE IV
MINIMUM ANNOYANCE VALUES FOR EACH SINGLE OBJECTIVE TRAJECTORY. CORRESPONDING L_{max} VALUES ARE GIVEN IN PARENTHESES

Hour of the day:	04h	07h	10h	13h	17h	19h
$J_S^* = \min_{\vec{z} \in \mathcal{Z}} J_S$	0.00 (87.4 dB)	0.26 (61.1 dB)	0.53 (61.4 dB)	0.53 (61.4 dB)	0.52 (61.1 dB)	0.39 (61.1 dB)
$J_I^* = \min_{\vec{z} \in \mathcal{Z}} J_I$	0.00 (42.6 dB)	0.00 (42.6 dB)	0.00 (42.6 dB)	0.00 (42.6 dB)	0.00 (42.6 dB)	0.00 (42.6 dB)
$J_{R1}^* = \min_{\vec{z} \in \mathcal{Z}} J_{R1}$	0.13 (45.7 dB)	0.06 (45.7 dB)	0.00 (54.9 dB)	0.00 (48.6 dB)	0.00 (45.5 dB)	0.04 (45.6 dB)
$J_H^* = \min_{\vec{z} \in \mathcal{Z}} J_H$	0.32 (47.1 dB)	0.24 (47.1 dB)	0.16 (47.1 dB)	0.16 (47.1 dB)	0.16 (47.1 dB)	0.20 (47.1 dB)
$J_{R2}^* = \min_{\vec{z} \in \mathcal{Z}} J_{R2}$	0.02 (39.1 dB)	0.01 (39.1 dB)	0.00 (54.9 dB)	0.00 (48.6 dB)	0.00 (45.1 dB)	0.00 (41.5 dB)

TABLE V
OPTIMAL ANNOYANCE VALUES FOR THE BEST TRAJECTORY. CORRESPONDING L_{max} VALUES ARE GIVEN IN PARENTHESES

Hour of the day:	04h	07h	10h
Best prioritisation P :	$J_S, J_{R2}, J_H, J_{R1}, J_I, C_a$	$J_I, J_H, J_{R2}, J_S, J_{R1}, C_a$	$J_S, J_H, J_{R1}, J_I, J_{R2}, C_a$
$J_S \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.00 (81.7 dB)	0.42 (79.7 dB)	0.53 (61.4 dB)
$J_I \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.21 (69.0 dB)	0.01 (41.8 dB)	0.03 (68.1 dB)
$J_{R1} \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.38 (55.3 dB)	0.30 (60.9 dB)	0.16 (66.7 dB)
$J_H \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.80 (60.9 dB)	0.24 (47.1 dB)	0.21 (49.1 dB)
$J_{R2} \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.02 (38.9 dB)	0.28 (59.9 dB)	0.07 (61.4 dB)
$t_f \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	230 s	237 s	241 s
$h_f \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	4645 ft	4787 ft	6417 ft
$Fuel \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	1576 kg	1619 kg	1659 kg

TABLE VI
OPTIMAL ANNOYANCE VALUES FOR THE BEST TRAJECTORY. CORRESPONDING L_{max} VALUES ARE GIVEN IN PARENTHESES

Hour of the day:	13h	17h	19h
Best prioritisation P :	$J_I, J_S, J_{R1}, J_H, J_{R2}, C_a$	$J_I, J_H, J_{R1}, J_S, J_{R2}, C_a$	$J_I, J_H, J_{R2}, J_S, J_{R2}, C_a$
$J_S \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.55 (62.2 dB)	0.83 (79.6 dB)	0.62 (79.7 dB)
$J_I \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.00 (53.8 dB)	0.00 (41.6 dB)	0.00 (41.7 dB)
$J_{R1} \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.16 (62.8 dB)	0.24 (60.2 dB)	0.33 (61.0 dB)
$J_H^* \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.18 (47.9 dB)	0.16 (47.1 dB)	0.20 (47.1 dB)
$J_{R2} \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	0.10 (59.5 dB)	0.26 (61.0 dB)	0.30 (59.8 dB)
$t_f \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	201 s	212 s	212 s
$H_f \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	5791 ft	5370 ft	5481 ft
$Fuel \text{lex min}_{\vec{z} \in \mathcal{Z}} [\vec{J}^P(\vec{z})]$	1402 kg	1459 kg	1458 kg

residential zone 2 (R2) is almost null (0.02) due to the high distance kept from this location. In addition, in this trajectory, the initial segment of the trajectory is also used to climb as much as possible (see figures 7 and 8 where the vertical paths and speed profiles are plotted in function of the time). This climb allows to reduce the annoyance in R1, I and H.

Best trajectory for 07h is significantly different. At this time, the school area starts to be annoyed by the over-flying aircraft

so the optimal trajectory for this hour of day starts with an immediate left turn when the aircraft reaches 400 ft above runway threshold. This left turn allows to keep the maximum distance to the residential zone 1 and the hospital locations as long as the west airspace restriction permits, producing a medium amount of annoyance (0.30 for the residential zone and 0.24 for the hospital). When the influence zone of the hospital is passed the aircraft performs a right turn improving

the annoyance at the second residential zone (0.28). At 10h the annoyance produced when over-flying a residential zone is relatively low, being just the opposite when over-flying the school. Therefore, the optimal trajectory at 10h starts with an initial right turn in order to avoid as much as possible the school location (producing a value of 0.53 of annoyance). Then, the aircraft turns left passing in between the industrial zone, the residential zone 1, the hospital and the second residential zone. It should be noted that the initial part of this trajectory is used to accelerate instead of climbing. This acceleration improves further climbing and maximises the aircraft height when approaching all the remaining locations. The best trajectory at 13h is very similar to the previous one but, at this time, the aircraft passes in between the school and the first residential zone because in the afternoon residential zones start to be more annoyed. This influence is noticed in optimal trajectories corresponding to 17 and 19. These trajectories are essentially the same as the optimal trajectory corresponding to 07h.

V. CONCLUSIONS

A technique for designing noise abatement departure procedures is presented in this paper. Noise annoyance produced by over-flying aircraft is modelled by using fuzzy logic in function of the received noise level during the trajectory, the sensibility of the areas being over-flown and the time of the day when the aircraft departure takes place. A non-linear multi-objective optimal control problem is formally written specifying the different objective functions considered. This problem is transformed to a Non Linear Programming (NLP) problem after a suitable discretisation and it is solved by using a lexicographic multi-objective optimisation technique. Finally, an application example is shown considering an hypothetical scenario with an hospital, a school, two residential zones and an industrial zone. Results show how this strategy is valid for solving this kind of multi-criteria optimisation problem, obtaining optimal trajectories that minimise nuisances in the population at different hours of the day. Work is underway extending this study including different aircraft types and different departures (different final points) corresponding to a realistic scenario of an existing airport with its surrounding features. In addition, further work will deal with the model of actual residential or industrial areas, treating them as a surface in the optimisation process and not only as a single point as it is presented in this work.

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