NUMERICAL MODELLING OF SOUND TRANSMISSION IN DOUBLE WALLS

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ABSTRACT

Numerical models for the calculation of sound transmission in double walls are presented. The finite element method (FEM) or analytical solutions (for rectangular domains) are used for the acoustic part of the problem while the wall vibration is solved by means of structural finite elements. The vibroacoustic problem is formulated in the frequency range: the acoustic domains (rooms) are described by the Helmholtz equation, the absorbent materials as an equivalent fluid and the structures by means of dynamic linear elasticity. The acoustic and structural parts of the problem are coupled. The influence of the stiffness and spacing of the studs on the performance of lightweight walls is studied. The studs have been modelled with beam finite elements or by means of mechanical constraints. The effect of the boundary conditions of the walls and its dimensions are also analyzed.
1 INTRODUCTION

Double walls are a common type of construction, very employed in practice. Quite good acoustic performance can be reached by means of a reasonable low waste of material. Their acoustic performance (isolation capacity) has been extensively studied. The sound reduction index can be predicted using different kinds of models [1],[2],[3],[4],[5]. All of them usually assume that both the structure and the acoustic domains are unbounded or that the pressure field is diffuse.

In this contribution the sound transmission through double walls is studied by means of numerical techniques. The vibroacoustic equations are solved and the actual geometries considered without making additional hypothesis during the resolution procedure.

Attention will be focused in the aspects that can be captured using a numerical model but are not usually taken into account in the most usual approaches: finite dimensions of the structure, boundary conditions and modelling of structural details (connection between leaves of the double wall). In section 2 the numerical models are briefly described. Some examples are presented in section 3 before the conclusions of section 4.

2 BRIEF DESCRIPTION OF THE MODELS

The vibroacoustic problem is solved in the frequency domain. The model problem is shown in Fig. 3. Two acoustic domains (rooms) are separated by a structure. This structure can be a single or a double wall. In the latter case, a third acoustic domain is considered (air cavity of the double wall).

The acoustic domains are governed by the Helmholtz equation and the structures by the equations of linear elastics dynamics. When the air cavity is filled with an absorbent material, an equivalent fluid model is considered (Delany and Bazley [8]).

The Helmholtz equation is solved by means of the finite element method or analytically [7],[10] (using modal analysis for rectangular domains). The computational cost of solving a big acoustic domain in the full frequency range is not affordable even for the two-dimensional situations presented in this contribution. In order to study the response of the system for high frequencies, it is useful to enrich the finite element model with the analytical solution of the bigger acoustic domains. Both the modal behaviour and the type of acoustic waves involved in every frequency band are well reproduced with this technique.

Sound is generated in one of the acoustic domains by means of a punctual sound source. As an output from the numerical model, the pressure field in every room is obtained. The sound reduction index $R$ (Eq. 1) can be evaluated from the mean sound level of every room ($L_1$ for the source room and $L_2$ for the receiving room). The surface of the wall ($S$) is known in every case and the equivalent absorption area ($A$) can be calculated by means of the acoustic impedance employed in the contours with Robin boundary condition (in this case, all the acoustic boundaries except the air-wall contact surface). The admittance for all problems is $2.22 \cdot 10^{-4} - 4.14 \cdot 10^{-6} \text{ m}^3/\text{N} \cdot \text{s}$.

\[
R = L_1 - L_2 + 10 \log_{10} \left( \frac{S}{A} \right)
\]  

(1)
The numerical results will be compared with analytical formulations of sound insulation of single and double walls [1],[2]. For the case of the mass law (R(m)), the field incidence version (Eq. 2) which is generally more in accordance with experimental results [2] is considered.

\[ R = 20 \log_{10} \left( h \rho_w f \right) - 47 \] dB \hspace{1cm} (2)

3 NUMERICAL EXAMPLES

3.1 Double walls with connecting elements

An ideal double wall (leave-cavity-leave) is almost impossible to construct in practice since minimum requirements of structural stability must be satisfied. Studs are typically placed between leaves. They give an additional stiffness to the wall but also create a new path of vibration transmission between leaves. They are usually modelled by means of translational and rotational springs connecting leaves in several points [3]. In this section, the effect of the springs in a double wall with a cavity of 0.175 m length and the material properties of Table 1 is studied. Translational and rotational springs are placed every 0.6 m. The wall separates two rooms of 4.4 m × 3 m and 5.1 m × 3 m and is 3 m in length.

| Table 1. Mechanical data for the leaves of the studied double wall (plasterboards). |
|-----------------------------------------------|-----------------|-----------------|
| Young modulus | E (N/m²) | 4.8 · 10⁹ | 3.8 · 10⁹ |
| Density | ρ_w (kg/m³) | 915 | 811 |
| Hysteretic damping | η (%) | 2.0 | 2.0 |
| Thickness | h (m) | 0.013 | 0.009 |

In Fig. 1 the results for the double wall without absorbent material in the cavity are plotted. The value of the translational stiffness influences the slope of the sound reduction curve after the mass-air-mass resonance (f_mam). For values below that 10⁵ N/m², the wall behaves almost like an ideal double wall (R(double wall)). However, for larger values, the response of the wall is comparable to that of a single wall with the same total mass (R(m1+m2)). The value of rotational stiffness is not important in this range of frequencies (below 1000 Hz). It modifies the performance of the double wall in the high frequency range and it is specially important around the critical frequencies of each leave (f_c1 = 2140.5 Hz for the first and f_c2 = 3271.5 Hz for the second one). For higher values of both types of stiffness the dip in sound reduction index due to the critical frequencies is closer to the second critical frequency.
Fig. 1. Effect of translational and rotational stiffness in the sound reduction index (R) of a double wall without absorbent material.

In Fig. 2 the same study has been repeated with the cavity full of absorbent material (resistivity $\sigma = 10000 \text{ Pa s/m}^2$). Tendencies are similar (influence of translational stiffness in the low/mid frequency range and of rotational stiffness for higher frequencies). However, the ranges of variation of both values of stiffness affecting the results and the increase or decrease in sound reduction index are bigger.

The range of both rotational ($K_\theta$) and translational ($K_t$) stiffness affecting the global response of the double wall is in accordance with the values of stiffness of actual connecting elements (for the case of metallic studs the elastic translational stiffness can have values of $10^5$-$10^6 \text{ N/m}^2$ being bigger for high frequencies).

Fig. 2. Effect of translational and rotational stiffness in the sound reduction index (R) of a double wall with absorbent material inside where the leaves are connected with springs.

3.2 Influence of the dimensions of the wall and boundary conditions

In this section, the effect of considering finite dimension walls or different types of boundary conditions are studied. The sound transmission of the lightweight double wall previously described and a massive concrete wall with the mechanical properties of Table 2 are calculated. The dimensions of the rooms in the concrete wall example are $3 \text{ m} \times 3 \text{ m}$ and $4$
m × 3 m. For the case of smaller lightweight double walls, the distance between studs has been kept constant: 0.6 m.

The influence of the size of the wall can be seen in the results of Fig. 3 and 4. Smaller walls always have a bigger isolation capacity. The difference is more important for mid and low frequencies. Analytical formulations unfairly penalize the isolation capacity in the very low frequency range. It is true that low frequencies are critical in terms of sound transmission but the radiations of walls at frequencies causing structural waves with a wave length bigger than half the wave length of the first modes of vibration is very poor. See, for example, the sound reduction index of the small (1.0 m, 1.5 m) concrete wall for frequencies around 100 Hz.

In both situations, the effect of varying the boundary conditions of the wall (c = clamped and s= supported) is not relevant. However, recall that this study is focused in an isolated wall. If a complete structure had been studied, these connections would have been a more influencing parameter since they would control the indirect paths of sound and vibration transmission (flanking transmissions).

Table 2. Mechanical data for the concrete single wall.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (N/m²)</td>
<td>$2.943 \times 10^{10}$</td>
</tr>
<tr>
<td>$\rho_w$ (kg/m³)</td>
<td>2500.0</td>
</tr>
<tr>
<td>$\eta$ (%)</td>
<td>2.0</td>
</tr>
<tr>
<td>h (m)</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Fig. 3. On the left, effect of the finite dimensions of the wall in the sound reduction index (R). On the right, sketch of a model problem of sound transmission.

4 CONCLUSIONS

Numerical models have been employed in order to calculated the sound reduction index of double walls. The effect of mechanical connections (translational and rotational springs) has been studied. The value of translational stiffness is important for frequencies under 1000 Hz (but bigger than the mass-air-mass resonance) while the value of rotational stiffness is important for high frequencies. The response of the double wall with studs is in between of
the response of the double wall without mechanical connections and a single wall with equivalent mass.

The finite length of the walls can be an important aspect for low frequencies. The isolation capacity of shorter walls is higher.

\[ R(f) \]

**Fig. 4.** Effect of the finite dimensions and the boundary conditions of a double wall in the sound reduction index (R). (s-c = first leave supported and second leave clamped) The length of the wall in the right hand side example is 3.0 m

**REFERENCES**


