Averaging of collected-power fluctuations by a multiaperture receiver system

Aniceto M. Belmonte
Adolfo Comerón, MEMBER SPIE
Javier Bará
J. A. Rubio
Universidad Politécnica de Cataluña
ETSI Telecomunicación
Departament of Signal Theory and Communications
P.O. Box 30002
08080 Barcelona, Spain
E-mail: belmonte@voltor.upc.es

Abstract. When a large aperture is used to collect light propagated through the atmosphere, it performs an average operation on the irradiance fluctuations, resulting in collected-power fluctuations that are not as large as those that could be obtained from a point receiver. We present an expression for the aperture-averaging factor of optical scintillation using multiaperture architectures on the receiver configuration. That expression is used to assess the decrease in the total collected power fluctuations for several multiaperture receiver configurations.

Subject terms: atmospheric optics; scintillation; averaging; multiaperture.

Paper 06026 received Feb. 6, 1996; revised manuscript received Apr. 21, 1996; accepted for publication Apr. 21, 1996.

1 Introduction

One of the effects of atmospheric turbulence on propagating electromagnetic waves is intensity scintillation. In atmospheric optical communication systems, this translates into fluctuations of the power collected by the receiving aperture, which impairs the system performance. It is a well-known phenomenon that the collected- power fluctuations decrease when the collecting aperture size increases, and a great deal of work has been devoted to the quantitative assessment of this so-called aperture-averaging effect.[1–3]

All the later work is based on the formulation proposed by Tatarskii[1] and corrected by Fried,[2] which describes the aperture-averaging effect in terms of an averaging factor defined as the ratio between the normalized variance of collected-power fluctuations and the normalized variance of intensity. The averaging factor depends on the aperture form and size, and on the intensity spatial covariance function; its form shows clearly that its value must decrease when the aperture size increases, the specific value depending on the form of the intensity spatial covariance function.

While the averaging factor, thus the collected-power fluctuations, can be indefinitely reduced by increasing the collecting aperture size, this might turn out to be expensive, since the price of a receiving telescope increases more than proportionally to its aperture size. One can envisage obtaining the spatial diversity implied by a large aperture by employing instead several smaller apertures. If an effective method of conveying the collected power to the optical detector can be devised—or if the outputs of individual detectors on each of the apertures are electronically combined[4–7]—the apertures can be set at large distances from each other to obtain an enhanced averaging effect through the decorrelation of the intensity fluctuations on the wavefront portions arriving at the different apertures.

This paper deals with the decrease of the total collected-power fluctuation in a multiaperture receiving optical terminal. To this end, we present a generalization of the Fried formula for the averaging factor to a multiaperture receiver. If the intensity spatial covariance function is known, the averaging factor of the multiaperture collecting system can be readily computed. Examples assuming the intensity covariance function for weak turbulence are presented. However, the expression giving the averaging factor can be used for any turbulence regime if the intensity spatial covariance function is known. It can thus be applied to the power fluctuation assessment of systems such as multiaperture astronomical instruments, optical ground stations aimed at checking the operation of optical payloads of intersatellite optical links,[8,9] and terrestrial terminals working in saturated scintillation scenarios.

2 Received Signal Fluctuations

We assume that the basic configuration of the optical receiving system consists of N identical apertures which we characterize from the point of view of the collected power, by an intensity pupil[10] $W(\mathbf{r})$. By terming $\mathbf{r}_i$ the position vector of the i'th aperture center (Figure 1), the instantaneous collected power by the aperture system can be written as

$$P=\sum_{i=1}^{N}\int_{S_i} W(\boldsymbol{\rho}-\mathbf{r}_i)I(\boldsymbol{\rho})\mathrm{d}\boldsymbol{\rho}, \tag{1}$$

where $I(\boldsymbol{\rho})$ is the wave intensity. Through the variables change $\mathbf{R}_i=\boldsymbol{\rho}-\mathbf{r}_i$, Eq. (1) can be rewritten as

$$P=\sum_{i=1}^{N}\int_{S_i} W(\mathbf{R}_i)I(\mathbf{R}_i+\mathbf{r}_i)\ \mathrm{d}\mathbf{R}_i. \tag{2}$$

Under the assumption of identical subapertures, Eq. (3) simplifies to

$$P=\int_{S} W(\mathbf{R})\sum_{i=1}^{N} I(\mathbf{R}+\mathbf{r}_i)\ \mathrm{d}\mathbf{R}, \tag{3}$$

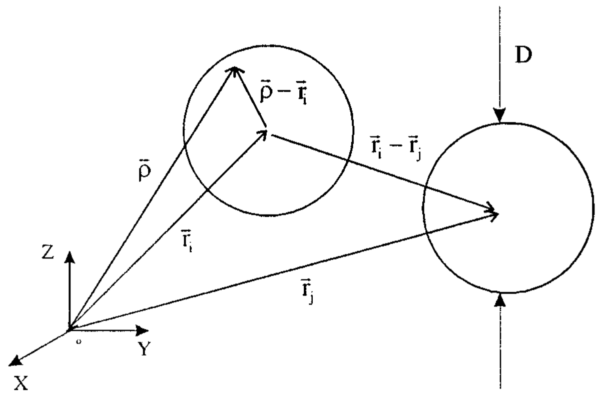


Fig. 1 Geometry of circular apertures with center to center separation $|\mathbf{r}_i - \mathbf{r}_j|$ and individual aperture diameter D.

from which the average power $\langle P \rangle$ can be written as

$$\langle P \rangle = \int_S W(\mathbf{R}) \sum_{i=1}^{N} \langle I(\mathbf{R}+\mathbf{r}_i) \rangle \, \mathrm{d}\mathbf{R}. \tag{4}$$

The variance of the collected power is defined as $\sigma_P^2 = \langle (P - \langle P \rangle)^2 \rangle = \langle P^2 \rangle - \langle P \rangle^2$, which can be written, using Eqs. (3) and (4), and writing the products of integrals as double integrals, as

$$\begin{aligned} \sigma_P^2 = & \int_s \int_{s'} W(\mathbf{R}) W(\mathbf{R}') \sum_{i=1}^{N} \sum_{j=1}^{N} \langle I(\mathbf{R}+\mathbf{r}_i) \\ & \times I(\mathbf{R}'+\mathbf{r}_j) \rangle \mathrm{d}\mathbf{R} \, \mathrm{d}\mathbf{R}' \\ & - \int_s \int_{s'} W(\mathbf{R}) W(\mathbf{R}') \sum_{i=1}^{N} \sum_{j=1}^{N} [\langle I(\mathbf{R}+\mathbf{r}_i) \rangle \\ & \times \langle I(\mathbf{R}'+\mathbf{r}_j) \rangle] \, \mathrm{d}\mathbf{R} \, \mathrm{d}\mathbf{R}', \end{aligned} \tag{5}$$

where it can be recognized in the integrand the definition of the spatial covariance function of intensity $C_I(\mathbf{R}+\mathbf{r}_i, \mathbf{R}+\mathbf{r}_j) = \langle I(\mathbf{R}+\mathbf{r}_i) I(\mathbf{R}+\mathbf{r}_j) \rangle - \langle I(\mathbf{R}+\mathbf{r}_i) \rangle \langle I(\mathbf{R}'+\mathbf{r}_j) \rangle$. So, by making the change of variables $\boldsymbol{\rho} = \mathbf{R} - \mathbf{R}'$, Eq. (5) is rewritten as

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \int_s \int_{s'} W(\mathbf{R}) W(\mathbf{R}-\boldsymbol{\rho}) C_I(\boldsymbol{\rho}+\mathbf{r}_i-\mathbf{r}_j) \, \mathrm{d}\mathbf{R} \, \mathrm{d}\boldsymbol{\rho}, \tag{6}$$

where spatial invariance of the function $C_I(\boldsymbol{\rho}, \boldsymbol{\rho}')$ has been assumed. By setting

$$K_W(\boldsymbol{\rho}) = \int_S W(\mathbf{R}) W(\mathbf{R}-\boldsymbol{\rho}) \, \mathrm{d}\mathbf{R}, \tag{7}$$

and carrying out the integration with respect to $\mathbf{R}$ in Eq. (6), the variance of power fluctuations is written as

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} C_P(\mathbf{r}_i - \mathbf{r}_j), \tag{8}$$

where

$$C_P(\boldsymbol{\rho}) = \int_S K_W(\boldsymbol{\rho}') C_I(\boldsymbol{\rho}'+\boldsymbol{\rho}) \, \mathrm{d}\boldsymbol{\rho}' \tag{9}$$

is the spatial covariance function of the power collected by an aperture. Now, using the definition for the aperture-averaging factor G, that considers the reduction of collected-power fluctuations with respect to intensity fluctuations due to the partial decorrelation of the latter over the collecting aperture surface,

$$\frac{\sigma_P^2}{P_0^2} = G \, \frac{\sigma_I^2}{I_0^2}, \tag{10}$$

and taking into account that, for a system of N identical apertures, the total average collected power is

$$P_0 = N I_0 \int_S W(\boldsymbol{\rho}) \, \mathrm{d}\boldsymbol{\rho}, \tag{11}$$

the averaging factor can be expressed in a very compact form as

$$G = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} C_P(\mathbf{r}_i - \mathbf{r}_j)}{\sigma_I^2 N^2 (\int_S W(\boldsymbol{\rho}) \, \mathrm{d}\boldsymbol{\rho})^2}. \tag{12}$$

Equation (12) can be used to compute the averaging factor of a multiaperture system for any number N of identical subapertures with given relative positions once the spatial covariance function of intensity fluctuations is known. In the following section, we consider a set of calculations as application examples of all previous considerations.

3 Calculations

As an example of application of the previous result [Eq. (12)], let us consider several simple configurations for the collector system enabling us to analyze the evolution of the averaging effect versus some significant parameters. We use the intensity normalized covariance function computed under the hypothesis $\sigma_I^2 \ll 1$, which makes the intensity covariance proportional to the log- amplitude covariance. The latter can, in turn, be computed using the Rytov approximation. Figure 2 shows the intensity normalized covariance function versus normalized spatial separation and the corresponding collected power covariance [Eq. (9)] for different aperture diameters. Note, however, that the expression giving the averaging factor for a multiaperture collecting system can be used for any turbulence regime as far as the spatial covariance function of the intensity fluctuations is known.

Figure 3 shows the averaging factor versus the number of subapertures, assuming the subapertures are arranged in a row, for several separations l between consecutive subapertures, along with the limiting case of infinite separation ($1/N$ dependence). In this way, it is possible to study the total number of subaperture effects. Figure 4 gives the averaging factor versus distance between two subapertures.

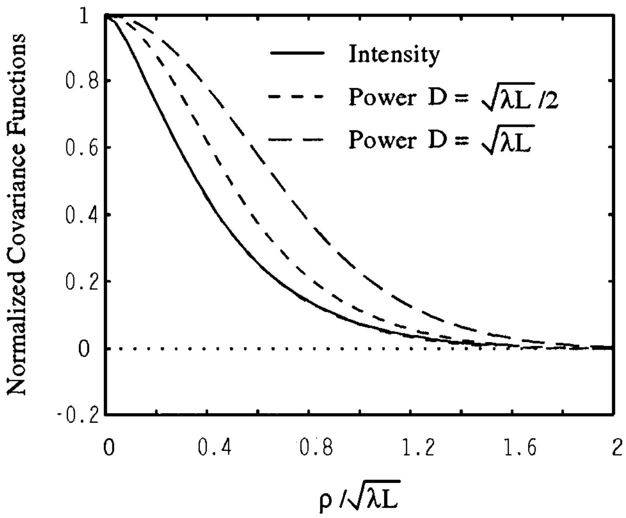


Fig. 2 Intensity normalized covariance function versus normalized spatial separation for spherical wave propagation geometry. The corresponding power-collected covariance function is shown for two aperture diameters. The curves are computed from the Rytov approximation[1] with the added assumption that the turbulence is uniform along the path.

As a final example, Figure 5 shows the averaging factor for square geometry arrays of 2×2 subapertures versus distance between subapertures.

The results of our calculations enable us to draw some preliminary conclusions. First, an increase of the number of subapertures N makes the averaging of collected power fluctuations more effective (see Figure 3). Second, we can observe that an increase of the elementary lattice side cause an increase of the averaging effect as a consequence of the greater decorrelation between the collected power for each

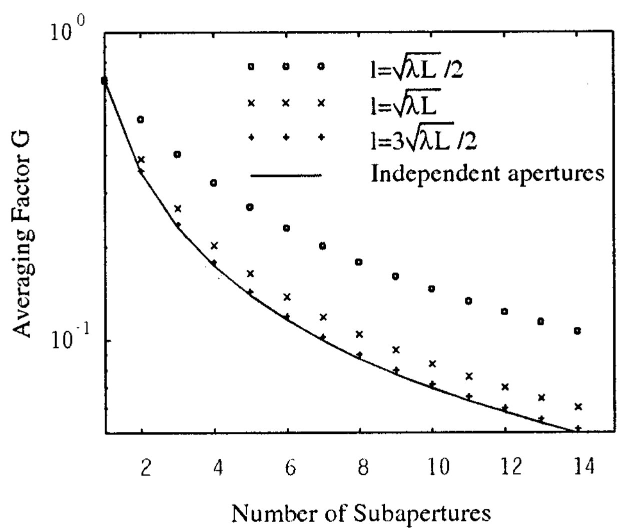


Fig. 3 Averaging factor versus number of subapertures for an atmospheric path characterized by the covariance functions in Figure 2. It is assumed that $D=\sqrt{\lambda L}/2$ and $l=\sqrt{\lambda L}$, where D is the individual aperture diameters and l is the center to center distance between subapertures; $\sqrt{\lambda L}$ is the intensity correlation length when the covariance functions are computed from the Rytov approximation.

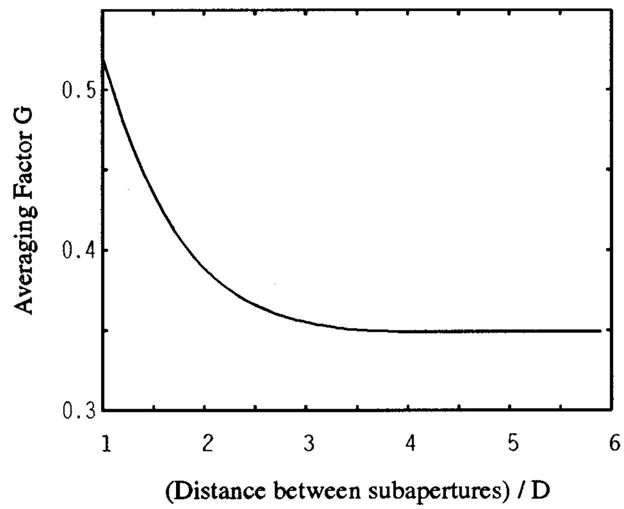


Fig. 4 Averaging factor versus distance between two subapertures for atmospheric path characterized by the covariance functions in Figure 2; $D=\sqrt{\lambda L}/2$.

aperture. Moreover, when the elementary lattice side is larger than the intensity correlation length (in our examples, $\sqrt{\lambda L}$), a further increase of its size does not cause any decrease of the averaging factor, which tends to a constant equal to the averaging factor for a single subaperture divided by the number of subapertures in the array (compare Figure 3 with $N = 1$ with the tails in Figures 4 and 5).

4 Conclusions

We have developed a generalized expression for the averaging factor of a multiaperture receiver system that enables us to predict the impact of turbulence in the total collected power fluctuations. This expression was used to assess the decrease in the total collected power fluctuations for several multiaperture receiver configurations in a weak-turbulence

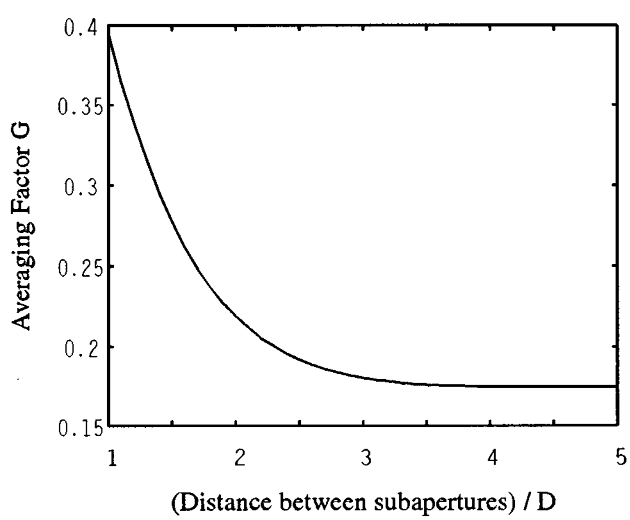


Fig. 5 Averaging factor for a square array of 2×2 subapertures versus distance between subapertures for atmospheric path characterized by the covariance function in Figure 2; $D=\sqrt{\lambda L}/2$.

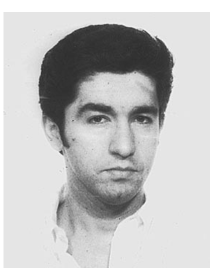
atmospheric optical propagation. However, the expression giving the averaging factor for a multiaperture collecting system can be used for any turbulence regime if the spatial covariance function of the intensity fluctuations is known. With this generalized formalism, we considered several simple configurations for the collector system that help us to understand the performance of the receiver system when it consists of more than one aperture.

Acknowledgments

This work was carried out under the European Space Agency Contract No. 8973/90/NL/SG and Purchase Order No. 114918 and Spain's CICYT grant No. ESP 978/90.

References

1. V. I. Tatarskii, Wave Propagation in a Turbulent Medium, McGraw-Hill, New York (1961).
2. D. L. Fried, ''Aperture averaging of scintillation,'' *J. Opt. Soc. Am.* **57**, 169–175 (1967).
3. J. H. Churnside, ''Aperture averaging of optical scintillation in the turbulent atmosphere,'' *Appl. Opt.* **30**, 1982–1994 (1991).
4. E. V. Hoversten, R. O. Harger, and S. J. Halme, ''Communication theory for the turbulent atmosphere,'' *Proc. IEEE* **58**, 1626–1650 (1970).
5. M. C. Teich and S. Rosenberg, ''Photocounting array receivers for optical communication through the lognormal atmospheric channel. 1: Optimum and suboptimum receiver structures,'' *Appl. Opt.* **12**, 2616–2624 (1973).
6. S. Rosenberg and M. C. Teich, ''Photocounting array receivers for optical communication through the lognormal atmospheric channel. 2: Optimum and suboptimum receiver structures,'' *Appl. Opt.* **12**, 2625–2635 (1973).
7. J. H. Churnside and C. M. McIntyre, ''Averaged threshold receiver for direct detection of optical communications through the lognormal atmospheric channel,'' *Appl. Opt.* **16**, 2669–2676 (1977).
8. A. F. Popescu and B. Furch, ''Status of the European developments for laser intersatellite communications,'' in Free Space Laser Communication Technologies V, G. Stephen Mecherle, Ed., *Proc. SPIE* **1866**, 10–20 (1993).
9. M. Shikatani, M. Toyoda, H. Takami, K. Araki, M. Isogai, Y. Suzuki, and T. Aruga, ''Ground system development for the ETS-VI/LCE laser communication experiment,'' in Free Space Laser Communication Technologies V, G. Stephen Mecherle, Ed., *Proc. SPIE* **1866**, 21–29 (1993).
10. J. W. Goodman, Statistical Optics, p. 222, John Wiley & Sons, New York (1985).



Aniceto M. Belmonte received his BS degree in physics from Barcelona University in 1988 and his PhD degree in telecommunications engineering from the Polytechnic University of Catalonia in 1995. His current research interests include statistical optics and all aspects of wave propagation in random media. He has been involved in the definition and achievement of experiments that showed the suitability of the Canary Islands scenario to host an optical ground station.

Adolfo Comerón received a telecommunication engineer degree from the Telecommunication Engineering School of Barcelona in 1976 and DEA and DrIng degrees from the Paris-XI University, Orsay, in 1977 and 1980, respectively. He is currently a professor at the Polytechnic University of Catalonia, Barcelona, Spain. His research activities have included the study of nonlinear devices at IR wavelengths and the development of microwave and milimeter-wave receivers for satelllite communication systems and currently focuses on free-space optical communications and remote detection and sensing at optical wavelengths.

Javier Bará received a senior telecommunication engineer degree from the School of Telecommunication Engineering, Madrid in 1968 and ScM and PhD degrees, both in electrical engineering, from Brown University, Providence, Rhode Island in 1969 and 1972, respectively. Since 1972 he has been a professor at the Polytechnic University of Catalonia (UPC), Barcelona, Spain, where he has been school dean, dean and department director. Presently he is dean of the College of Telecommunication Engineering "Baix Llobregat." His research interests have been microwaves (ferrites, integrated circuits, satellite communications, industrial heating and drying processes) and he is presently involved in projects in nonguided optical communications in the near IR and interferometric radiometry for remote sensing of the earth.

J. A. Rubio received his senior telecommunication engineer degree from the School of Telecommunication Engineering of Barcelona in 1992. From 1992 to 1995 he was a research assistant in the Department of Signal Theory and Communications of the Polytechnic University of Catalonia. He is currently working toward a PhD in engineering. His current research is in unguided optical communications systems and lidar.