

## A THREE-PHASE MODEL FOR DAMAGE INDUCED BY ASR IN CONCRETE STRUCTURES

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**Abstract.** In the present work we propose a three-phase chemo-elastic damage model for the description of the mechanical degradation of concrete induced by the Alkali Silica Reaction (ASR). The model accounts for ASR kinetics, depending on temperature and humidity conditions, and the swelling pressure exerted by the ASR reaction products on the skeleton. The model is validated on the basis of experimental tests on concrete reactive beams performed at varying ambient humidity conditions.

### 1 INTRODUCTION

The alkali-silica reaction occurring in concrete composed by certain type of aggregates and cement paste forms a hydrous alkali-calcium-silica gel. In the presence of water this gel swells, causing expansion and micro-cracking. Water in concrete is always present in little quantities as the rest of the process of hydration, but environmental conditions can change the water content.

The structural consequences of this phenomenon can be very serious, especially in massive concrete structure such as dams or bridges built some decades ago. For this reason in recent years many experimental campaigns has been performed to assess the influence of environmental factors, such as temperature and humidity conditions, on the ASR development [1, 2] and several mathematical models have been proposed to simulate the mechanical effects of ASR. In early proposed models [3, 4] the gel expansion was taken into account as an incompatible strain. Subsequently, more refined models have been proposed (see e.g. [5, 6]), with focus on the kinetics of the reaction. In [6] the kinetics law, based on Larive's proposal, also includes, in a heuristic way, the effect of the stress state. Other models address the mechanical modeling of ASR by considering a two-phase material with the concrete skeleton and the expansive gel acting in parallel [7, 8, 9, 10]. Very recently a model was developed taking into account concrete creep, the stress induced by the formation of ASR gel and the mechanical damage [11]. More in general multi-phase models have been developed to model other chemo-physical phenomena in concrete, like e.g. calcium leaching [12].

In the present work the phenomenological bi-phase isotropic damage model proposed in [13] is extended in order to catch the structural effects induced by the alkali-silica reaction (ASR) in concrete structures when ambient humidity conditions change and moisture gradients occur. The quantity of water present in the structure is taken into account through the degree of saturation of concrete and, in the framework of Biot's theory of multi-phase

porous materials, concrete is here conceived as a three-phase material constituted by the superposition of three homogenized phases: the concrete skeleton, the gel produced by the chemical reaction and the water. The assumption of three different phases allows to take into account also the effects of shrinkage at difference from the two-phase model previously proposed by the Authors [14].

The ASR kinetic is described by using an intrinsic time (as proposed in [1]), expressed as a function of the latency time and the characteristic time depending on both temperature and degree of saturation. When external humidity boundary conditions are not homogeneous, the moisture gradients are taken into account through a diffusion analysis of moisture in its liquid form. The mechanical part of the model is based on the damage theory in order to assess the decrease of stiffness of the concrete due to cracking caused by ASR.

The proposed model has been validated through the comparison with experimental results in [15] for free expansion tests on unconstrained cylindrical specimens and plain concrete beams.

## 2 CHEMO-DAMAGE MODEL

### 2.1 Field equations

The present model for concrete affected by ASR is formulated within the theory of multi-phase porous materials [16]. At the meso-scale three different phases are considered: the concrete skeleton, (*s*) including cement paste and aggregates, the gel (*g*) and the water (*w*) present in the connected porosity.

In the proposed model gel and water are considered as distinct phases, occupying two distinct and not interacting porosities. This hypothesis is in accordance to the theoretical models based on surface-chemistry principles, that attribute the expansion of the product gel in the presence of water to an electrical double-layer repulsion (see e.g. [17]), similarly to what happens for clays (see e.g. [18]).

Let us consider a representative volume element RVE of total volume  $V$  and denote by  $V_s, V_g, V_w$  the volume occupied by solid, gel and water phases, and by  $V_v = V_{vg} + V_{vw}$  the void volume, formally divided into the volume of voids  $V_{vg}$  and  $V_{vw}$ , which can be filled by the gel and by the water respectively. The total porosity  $\varphi = V_v/V$  is thus divided into two non interacting porosities  $\varphi_g = V_{vg}/V$  and  $\varphi_w = V_{vw}/V$ , and the degrees of saturation for the gel and the water are defined as  $S_g = V_g/V_{vg}$  and  $S_w = V_w/V_{vw}$ . With the above definitions, the volumetric fractions of gel and water are expressed as

$$\zeta_g = V_g/V = \varphi_g S_g, \quad \zeta_w = V_w/V = \varphi_w S_w \quad (1)$$

Petrographic investigations show that ASR gel forms between grains and cement past consuming the silica particles necessary for the reaction, so we assume that the gel porosity evolves with the gel formation and be always filled by the gel, so that  $S_g=1$ . The very low permeability of concrete with respect to gel than to water justifies the assumption that the forming gel does not fill the connected porosity of concrete, initially available for water.

Assuming small strains and quasi-static conditions the compatibility, equilibrium and mass balance equations for the multi-phase solid read

$$\boldsymbol{\varepsilon} = \frac{1}{2} (\text{grad} \mathbf{u}_s + \text{grad}^T \mathbf{u}_s) \quad (2)$$

$$\text{div} \boldsymbol{\sigma} + \rho \mathbf{b} = 0 \quad (3)$$

$$\text{div} (m_\beta \dot{\mathbf{w}}_\beta) + \frac{\partial m_\beta}{\partial t} = \pm \gamma_{\rightarrow\beta} \quad \beta = s, g, w \quad (4)$$

where:  $\boldsymbol{\varepsilon}$  is the total strain in the skeleton,  $\mathbf{u}_s$  is the skeleton displacement,  $\boldsymbol{\sigma}$  is the Cauchy stress in the combined solid and fluid mix,  $\rho = (1-\phi)\rho_s + \sum \phi_\alpha \rho_\alpha S_\alpha$  is the density of the assembly,  $\rho_\beta$  is the mass density of phase  $\beta$ ,  $\rho \mathbf{b}$  is the body force of the solid and fluid mix,  $m_\beta$  is the mass variation of phase  $\beta$ ,  $\mathbf{w}_\alpha = \phi_\alpha S_\alpha (\mathbf{u}_\alpha - \mathbf{u}_s)$ ,  $\alpha = g, w$ , is pseudo-displacement of the fluid phase  $\alpha$  relative to the skeleton,  $\mathbf{w}_s = \mathbf{0}$  and  $\gamma_{\rightarrow\beta}$  is the rate of mass formation  $+\gamma_{\rightarrow\beta}$  or consumption  $-\gamma_{\rightarrow\beta}$  for the phase  $\beta$  which coincides with the rate of mass exchanged among phase  $\beta$  and other two phases.

The mass variation  $m_\alpha$  of the fluid phase  $\alpha$  can be expressed as the product of the mass density  $\rho_\alpha$  times the volumetric fractions of gel and water  $\zeta_\alpha$  ( $\alpha = g, w$ ) which can be interpreted as the variation of fluid phase content with respect to initial conditions, i.e. the volume change of fluid phase per unit total volume:

$$m_\alpha = \rho_\alpha \zeta_\alpha = \rho_\alpha \phi_\alpha S_\alpha, \quad \alpha = g, w \quad (5)$$

In order to adapt the general equations of mixture theory to our problem, some simplifying assumptions can be introduced at the meso-scale. First of all the rate of solid mass dissolved by water and intervening in the ASR reaction can be neglected in the solid mass balance equation, while its contribution is relevant for the evolution of gel mass. Moreover the rate of mass of water absorbed by the gel is negligible with respect to the concentration of water in the porosity and the relative velocity of gel with respect to solid skeleton  $\dot{\mathbf{w}}_g$  is negligible with respect to the relative velocity of water  $\dot{\mathbf{w}}_w$ . Finally, the rate of mass of water transformed into vapor can be neglected because of the small porosity of concrete. With these approximations, using eq.(5), the conservation laws (4) for the gel the liquid and water can be written as

$$\frac{\partial (\rho_g \phi_g S_g)}{\partial t} = \gamma_{s \rightarrow g} \quad (6)$$

$$\text{div} (\rho_w \phi_w S_w \dot{\mathbf{w}}_w) + \frac{\partial (\rho_w \phi_w S_w)}{\partial t} = 0 \quad (7)$$

Proper transport law should be specified for the fluid phases. As already remarked the low permeability of concrete with respect to gel allows to neglect the transport of gel. For the water phase, we adopt the simplified equation of moisture transport proposed in [19], valid for slightly porous material and obtained by the combination of the Darcy's law for fluid flow in porous media with conservation law (7). The Darcy's law reads:

$$\varphi_w S_w \dot{\mathbf{w}}_w = -\frac{K}{\eta_w} k_{rw}(S_w) \text{grad}(p_w) \quad (8)$$

where  $K$ ,  $\eta_w$ ,  $k_{rw}$  and  $p_w$  denote respectively intrinsic permeability of concrete (independent from the saturating fluid), dynamic viscosity, relative permeability and pressure of water. Substitution of (8) into the conservation law (7) leads to the following non-linear transport law for moisture in its liquid form

$$\varphi_w \frac{\partial S_w}{\partial t} + \text{div}(D_w(S_w) \text{grad} S_w) = 0 \quad (9)$$

where  $D_w(S_w)$  is the permeability of concrete, dependent on the degree of saturation. By taking into account the approximations introduced in the conservation laws (6) and (7), the expression introduced in [21] for the relative permeability  $k_{rw}(S_w)$  and the procedure proposed in [20] the following expression for the permeability of concrete can be obtained:

$$D_w(S_w) = m_1 m_2 \left(1 - \frac{1}{m_2}\right) \frac{K}{\eta_w} \frac{\left[1 - (1 - S_w^{1/m_3})^{m_3}\right]^2}{S_w^{1/2+m_2} (S_w^{-m_2} - 1)^{1/m_2}} \quad (10)$$

where  $m_1$ ,  $m_2$  and  $m_3$  are assumed as in [20].

## 2.2 Constitutive equations

The state equations relating the static variables (total Cauchy stress  $\boldsymbol{\sigma}$ , chemical potential of each fluid phase  $\mu_\alpha$  and entropy  $\mathcal{S}$ ) to the conjugate kinematic variables (strain of the skeleton  $\boldsymbol{\varepsilon}$ , variation of fluid phase contents  $\zeta_\alpha$  and temperature variation  $\theta = T - T_0$ , being  $T_0$  the local reference temperature) are derived from the free energy potential  $\Psi$ . In order to model the concrete skeleton degradation induced by the ASR, we also introduce in the free energy an internal damage variable  $D$ . This damage variable affects the solid skeleton elastic behavior and depends on two scalar variables,  $D_t$  and  $D_c$ , which describe damage mechanisms under prevailing tension and compression conditions:  $D = 1 - (1 - D_t)(1 - D_c)$ .

The following explicit expression for the free energy is proposed:

$$\begin{aligned} \Psi &= \Psi(\boldsymbol{\varepsilon}, \theta, \zeta_w, \zeta_g, D) \\ &= \frac{1}{2}(1-D) \left[ 2G\boldsymbol{\varepsilon} : \mathbf{e} + K\text{tr}^2\boldsymbol{\varepsilon} + M_g b_g^2 \left( \text{tr}\boldsymbol{\varepsilon} - \frac{\zeta_g}{b_g} \right)^2 + M_w b_w^2 \left( \text{tr}\boldsymbol{\varepsilon} - \frac{\zeta_w}{b_w} \right)^2 \right] + \\ &+ (1-D) \left[ \frac{1}{2} \left( -\frac{C}{T_0} + M_g \alpha_g^2 + M_w \alpha_w^2 \right) \theta^2 - K\text{tr}\boldsymbol{\varepsilon} \alpha \theta \right] + \\ &+ (1-D) \left[ -M_g b_g \left( \text{tr}\boldsymbol{\varepsilon} - \frac{\zeta_g}{b_g} \right) \alpha_g \theta - M_w b_w \left( \text{tr}\boldsymbol{\varepsilon} - \frac{\zeta_w}{b_w} \right) \alpha_w \theta \right] + \psi_g \rho_g \zeta_g + \psi_w \rho_w \zeta_w \end{aligned} \quad (11)$$

In the above equation  $\mathbf{e}$  is the deviatoric strain tensor,  $G$  and  $K$  are respectively the shear and bulk moduli of the homogenized concrete skeleton,  $M_g$  and  $M_w$  are the Biot moduli referred to gel and water,  $b_g$  and  $b_w$  are the Biot coefficients,  $\alpha$ ,  $\alpha_g$ ,  $\alpha_w$  are respectively the volumetric coefficients of thermal expansion for concrete skeleton, gel and water. The terms  $\psi_\alpha \rho_\alpha \zeta_\alpha$  account for free energy supply associated with the mass variations  $\rho_\alpha \zeta_\alpha$ .

The state equations are obtained by partial derivation and read:

$$\boldsymbol{\sigma} = \frac{\partial \Psi}{\partial \boldsymbol{\varepsilon}} = (1-D) \left\{ 2G\mathbf{e} + \left[ K\text{tr}\boldsymbol{\varepsilon} - K\alpha\theta + M_g b_g (b_g \text{tr}\boldsymbol{\varepsilon} - \zeta_g - \alpha_g \theta) + M_w b_w (b_w \text{tr}\boldsymbol{\varepsilon} - \zeta_w - \alpha_w \theta) \right] \mathbf{1} \right\} \quad (12)$$

$$\rho_g \mu_g = \frac{\partial \Psi}{\partial \zeta_g} = -(1-D) M_g (b_g \text{tr}\boldsymbol{\varepsilon} - \zeta_g - \alpha_g \theta) + \rho_g \psi_g \quad (13)$$

$$\rho_w \mu_w = \frac{\partial \Psi}{\partial \zeta_w} = -(1-D) M_w (b_w \text{tr}\boldsymbol{\varepsilon} - \zeta_w - \alpha_w \theta) + \rho_w \psi_w \quad (14)$$

$$S = -\frac{\partial \Psi}{\partial \theta}; Y = -\frac{\partial \Psi}{\partial D} \quad (15)$$

The chemical potential of the fluid phases can be expressed in terms of the pressure  $p_g$  and  $p_w$  that gel and water exert on the solid skeleton and of the specific free energies of gel and water  $\psi_g$  and  $\psi_w$  as

$$\mu_\alpha = p_\alpha / \rho_\alpha + \psi_\alpha \quad (16)$$

Substituting (16) into equations (13) and (14) one obtains the following forms for the gel and water pressure

$$p_g = -(1-D) M_g (b_g \text{tr}\boldsymbol{\varepsilon} - \zeta_g - \alpha_g \theta) \quad (17)$$

$$p_w = -(1-D) M_w (b_w \text{tr}\boldsymbol{\varepsilon} - \zeta_w - \alpha_w \theta) \quad (18)$$

Consequently the macroscopic stress, equation (12), can be written as

$$\boldsymbol{\sigma} = \frac{\partial \Psi}{\partial \boldsymbol{\varepsilon}} = (1-D) \left[ 2G\mathbf{e} + K(\text{tr}\boldsymbol{\varepsilon} - \alpha\theta) \mathbf{1} \right] - b_g p_g \mathbf{1} - b_w p_w \mathbf{1} \quad (19)$$

The constitutive model should be completed by evolution equations for the variation of fluid volume contents  $\zeta_\alpha$  and damage  $D$ . For the evolution of the water content we assume the simple form

$$\dot{\zeta}_w = A \varphi_w \dot{S}_w \quad (20)$$

where the constant  $A$  is calibrated on the basis of experimental data [20] as explained in Section 3. The evolution of the gel content is assumed to be proportional to the rate of the reaction extent  $\xi$

$$\dot{\xi}_g = c \dot{\xi} \quad (21)$$

The constant  $c$  is proportional to the free asymptotic volumetric expansion due to the ASR in the fully saturated case ( $t \rightarrow \infty, S_w = 1$ )  $\mathcal{E}_{ASR, S_w=1}^\infty$

$$c = \frac{K_u}{M_g b_g} \mathcal{E}_{ASR, S_w=1}^\infty \quad (22)$$

with  $K_u = K + M_g b_g^2 + M_w b_w^2$  denoting the undrained bulk modulus.

Considering a first order reaction kinetics [9], the following form for the rate of reaction is proposed

$$\dot{\xi} = \frac{\langle f(S_w) - \xi \rangle^+}{\tilde{t}(\xi, S_w, T)} \quad (23)$$

where  $\tilde{t}$  is the intrinsic time of the reaction which depends on the local temperature and degree of saturation histories  $T(t)$  and  $S_w(t)$ , and on the reaction extent  $\xi(t)$ . As suggested by experiments [22] and assumed by various authors, we consider the simultaneity of gel formation and swelling. The intrinsic time is expressed in terms of latency time  $\tau_{lat}$  and characteristic time  $\tau_{ch}$  registered for the swelling of reactive specimens. In this work both the dependence on temperature  $T$  and degree of saturation  $S_w$  is introduced

$$\frac{1}{\tilde{t}} = \frac{\xi + \exp[-\tau_{lat}/\tau_{ch}]}{\tau_{ch}(1 + \exp[-\tau_{lat}/\tau_{ch}])} \quad (24)$$

with

$$\tau_i(T, S_w) = \left\{ \tau_i(\bar{T}, 1) + \frac{\tau_i(\bar{T}, 0) - \tau_i(\bar{T}, 1)}{1 + c_{1i} \exp\left[-\frac{c_{2i}(1 - 2S_w)}{S_w(1 - S_w)}\right]} \right\} \exp\left[ U_i \left( \frac{1}{T} - \frac{1}{\bar{T}} \right) \right], \quad i = ch, lat \quad (25)$$

The expressions (25) here proposed combine the Arrhenius law, describing the dependence on the temperature, with a dependence on the moisture, based on experimental results at the reference temperature  $\bar{T} = 38^\circ\text{C}$  (see e.g. [9]). In the equation (25)  $U_{lat}$  and  $U_{ch}$  are the activation energies [13], while the parameters  $c_{1i}$ ,  $c_{2i}$ ,  $\tau_i(\bar{T}, 0)$  and  $\tau_i(\bar{T}, 1)$  are calibrated with experimental data [14]. The function  $f(S_w)$  takes into account the influence of water content on the final extent of reaction and assumes the following form

$$f(S_w) = [1 + b_1 \exp(-b_2)] / [1 + b_1 \exp(-b_2 S_w)] \quad (26)$$

where parameters  $b_1$  and  $b_2$  are calibrated with experimental data and reported in Section 3.

The evolution of the damage variables  $D_t$  and  $D_c$  is governed by loading-unloading conditions defined in terms of the macroscopic stress  $\sigma$ , the gel pressure  $p_g$  and the water pressure  $p_w$  through the ‘inelastic effective stress’  $\sigma''$  defined as

$$\sigma'' \equiv \sigma + \beta_g p_g \mathbf{1} + \beta_w p_w \mathbf{1}$$

The non-dimensional coefficients  $\beta_g \leq b_g$  and  $\beta_w \leq b_w$  govern the damage level achievable in a concrete specimen under free expansion due ASR and the pressure exerted on skeleton by gel and water.

The loading-unloading conditions read

$$f_t \leq 0 \quad \dot{D}_t \geq 0 \quad f_t \dot{D}_t = 0; \quad f_c \leq 0 \quad \dot{D}_c \geq 0 \quad f_c \dot{D}_c = 0$$

where  $f_t$  and  $f_c$  are the damage activation function in tension and compression, defined as

$$f_t(\sigma'', D_t) = \frac{1}{2} \mathbf{s} : \mathbf{s} - a_{t0} (\text{tr} \sigma'')^2 + a_{t1} \text{tr} \sigma'' h_t - a_{t2} h_t^2$$

$$f_c(\sigma'', D_c) = \frac{1}{2} \mathbf{s} : \mathbf{s} - a_{c0} (\text{tr} \sigma'')^2 + a_{c1} \text{tr} \sigma'' h_c - a_{c2} h_c^2$$

where  $\mathbf{s}$  is the deviatoric stress,  $h_t$  and  $h_c$  are the hardening-softening functions and  $a_{i0}$ ,  $a_{i1}$ ,  $a_{i2}$  ( $i=t,c$ ) are non-negative material parameters governing the shape and the size of the elastic domain (see [23] for details).

### 3 MODEL CALIBRATION

The proposed model requires the identification of four distinct sets of material parameters, namely: (i) elastic parameters for concrete skeleton, gel and water ( $K$ ,  $G$ ,  $M_g$ ,  $b_g$ ,  $M_w$ ,  $b_w$ ) (ii) parameters governing the damage response ( $a_{i0}$ ,  $a_{i1}$ ,  $a_{i2}$ ,  $i=t,c$ ), (iii) parameters defining the expansion due to ASR ( $U_i$ ,  $\tau_i(\bar{T}, 1)$ ,  $\tau_i(\bar{T}, 0)$ ,  $c_{i1}$ ,  $c_{i2}$ , with  $i=lat, ch$ ,  $b_1$ ,  $b_2$ ,  $\beta_g$ ,  $\beta_w$ ) and (iv) parameters governing the hygroscopic behavior of concrete ( $m_1$ ,  $m_2$ ,  $m_3$  and  $A$ ). The procedure for the choice of these parameters is briefly illustrated in the following. Table 1 collects the values identified for the concrete used in the experimental test performed by [15], simulated in Section 4.

**Table 1:** Parameters of the model

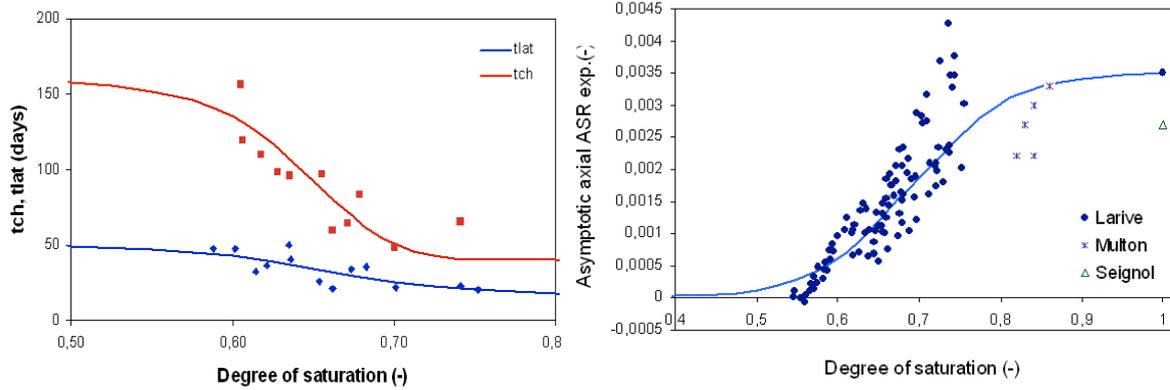
$a_{t0}$	0.3	-	$a_{t1}$	5.01	MPa	$a_{t2}$	21.33	MPa <sup>2</sup>	$K$	1e-21	m <sup>2</sup>
$a_{c0}$	0.0025	-	$a_{c1}$	3.8	MPa	$a_{c2}$	349.29	MPa <sup>2</sup>	$\eta_1$	0.001	Kg/ms
$\tau_{lat}(S_w=1)$	80	days	$\tau_{ch}(S_w=1)$	70	days	$b_1$	130000	-	$m_1$	37.55	MPa
$\tau_{lat}(S_w=0)$	200	days	$\tau_{ch}(S_w=0)$	150	days	$b_2$	17	-	$m_2$	2.17	-
$\phi_w$	0.16	-	$\phi_g$	0.003	-	$\epsilon_{ASR, Sw=1}$	0.0035	-	$m_3$	0.46	-
$b_w$	0.41	-	$b_g$	0.0089	-	$\beta_w$	0.025	-	$A$	0.025	-
$M_w$	11923.85	MPa	$M_g$	612496.35	MPa	$\beta_g$	0.0008	-			
$c_{1lat}$	0.0145	-	$c_{2lat}$	3.43	-	$c_{1ch}$	0.049	-	$c_{2ch}$	2.2984	-

(i) The elastic parameters for concrete can be computed from the experimental values of Young’s modulus and Poisson ratio; Biot’s parameters  $M_w$  and  $b_w$  can be obtained from the

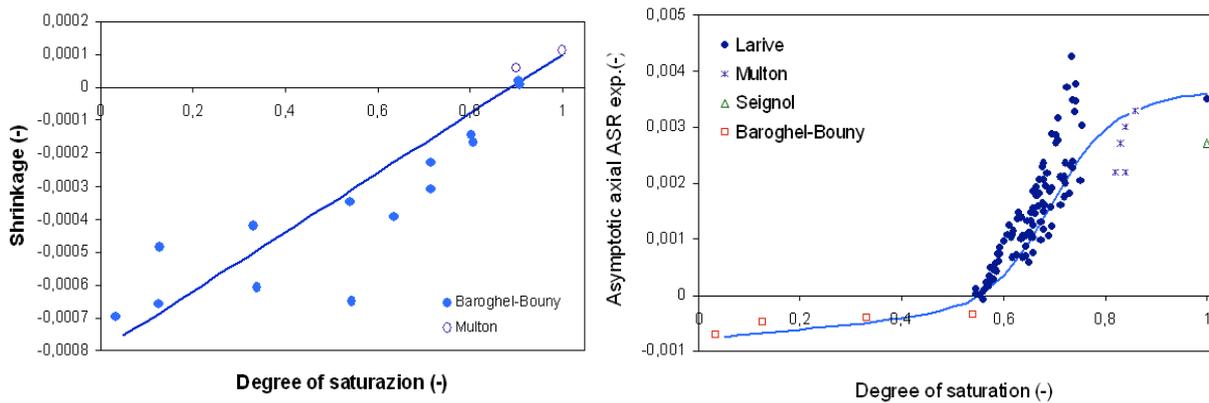
initial porosity of concrete through the homogenized relations proposed in [25];  $M_g$  and  $b_g$  can be estimated from the experimental data on the volume fraction occupied by the gel (data in [11] are used).

(ii) The second set of parameters can be identified on the basis of standard uniaxial and biaxial mechanical tests on concrete. Feasible intervals for the model parameters, together with their units and experimental data required for their identification can be found in [23].

(iii) The activation energies for ASR kinetics in (25) have been estimated by Larive on the basis of experiments on reactive concrete specimens, at different temperatures. The parameters  $c_{1i}$ ,  $c_{2i}$ ,  $\tau_i(\bar{T}, 0)$  and  $\tau_i(\bar{T}, 1)$  in (25) and  $b_1$  and  $b_2$  in (26) have been calibrated by using isothermal free expansion tests on reactive concrete cylindrical specimens at different moisture conditions as shown in [14]. Figure 1 shows the model variation of characteristic time, latency time and asymptotic axial ASR expansion with degree of saturation and experimental data. The values of coefficients  $\beta_g$  and  $\beta_w$  can be obtained by calibration on the basis of experimental data or by a proper micromechanical model (e.g. [24] for a bi-phase model).



**Figure 1:** (a) Variation of characteristic time and latency time and (b) asymptotic axial ASR expansion with degree of saturation  $S_w$ : experimental points from [1, 2] and proposed model

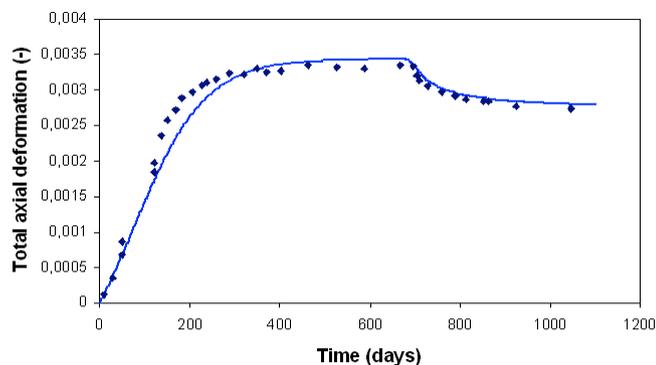


**Figure 2:** (a) Drying shrinkage of hardened concrete: experimental points from [2, 20] and proposed model (b) Total axial deformation of cylindrical specimens of reactive concrete with degree of saturation: experimental points from [1, 2] and proposed model

(iv) For the calibration the data reported in [20] for ordinary and high performance hardened concrete can be used since experimental studies [2, 15] show the same drying behavior for reactive and non-reactive concrete. The constant  $A$  governing the evolution of the water content in (20) is calibrated on the basis of experimental drying and adsorption tests reported in Figure 2a. Parameters  $m_1$ ,  $m_2$  and  $m_3$ , governing the dependence of concrete permeability on the degree of saturation in (10), has been taken from [20] for an ordinary concrete.

Figure 2b shows the total axial deformation for a reactive-concrete obtained with the model together with the experimental data used for calibration.

The model has been validated by simulating the experimental tests reported in [2] for a reactive cylindrical specimen kept in water for two years and then exposed to drying air at 30% RH, as shown in Figure 3.



**Figure 3:** Free expansion tests at  $T=38^{\circ}\text{C}$  for a reactive specimen kept in water for two years and then exposed to drying air at 30% RH: experimental points from [15] and model prediction

#### 4 REACTIVE CONCRETE BEAMS SUBMITTED TO MOISTURE GRADIENTS

The proposed model has been implemented in a finite element code and has been employed to simulate the experimental test reported in [15] performed on reactive and non-reactive plain concrete beams subjected to controlled imposed histories of humidity and temperature.

To compute the degradation effects of ASR on concrete structures when temperature and moisture gradients vary in time, the mechanical analysis is preceded by two uncoupled diffusion analyses: the heat diffusion analysis, governed by Fourier law, and the liquid moisture diffusion analysis, governed by equation (9). A 2D (plane strain) finite element discretization of the beam section has been used. The experimental conditions adopted in [15] are reproduced in the analysis by proper initial and boundary conditions: after curing under aluminium sealing, the lower face of the beam was immersed in water, while the upper face was in contact with air at 30% RH for 14 months and then submitted to permanent water supply for 9 months. Figure 4 shows the computed patterns of degree of saturation and reaction extent after 14 and 23 months respectively. The comparison between experimental results and numerical analysis is shown in Figure 5 in terms of strain evolution at different depths of the beam. At 0.08 m from the top the model predicts shrinkage due to the decrease of external humidity conditions. With the model parameters calibrated in Section 3, on the

basis of the experimental results in [1, 2], no ASR expansion occurs at that level of humidity. On the contrary the experimental data show a limited expansion in the upper part of the beam which seems to indicate that ASR is not completely stopped despite the low humidity conditions.

Figure 6 shows the corresponding damage evolution at four depths of the reactive beam. In the upper part (0.08m depth) a limited damage develops during the first months due to drying, while damage induced by ASR develops after the water supply. The damage evolution is faster in the lower part of the beam, where the degree of saturation is high and ASR evolves rapidly.

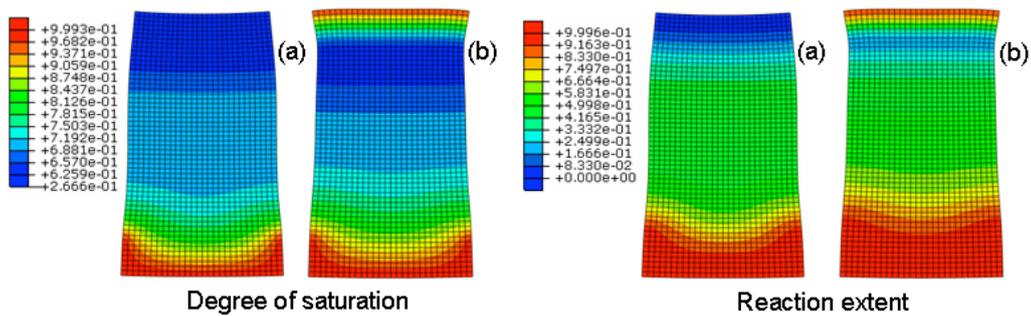


Figure 4: Patterns of degree of saturation and reaction after (a) 14 and (b) 23 months

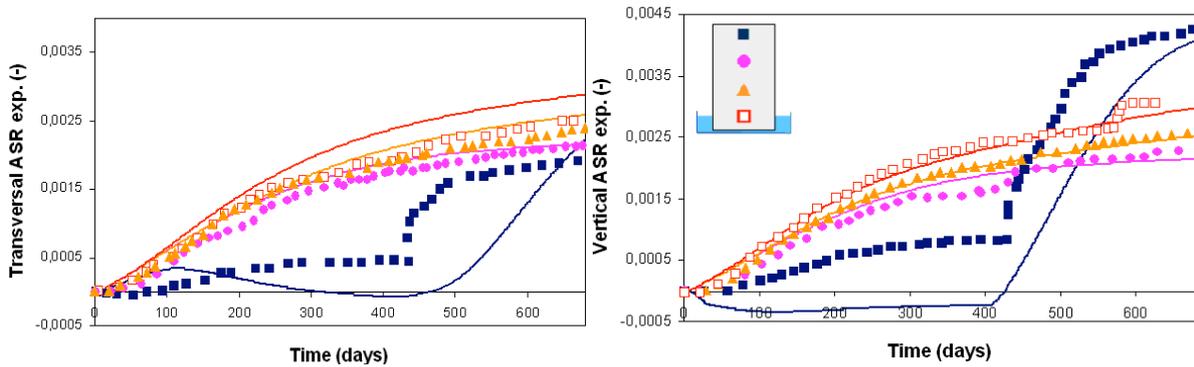


Figure 5: Vertical and transverse strains of reactive beams measured at four depths (0,08, 0,17, 0,27, 0,37 m from the upper face): experimental points from [15] and model prediction

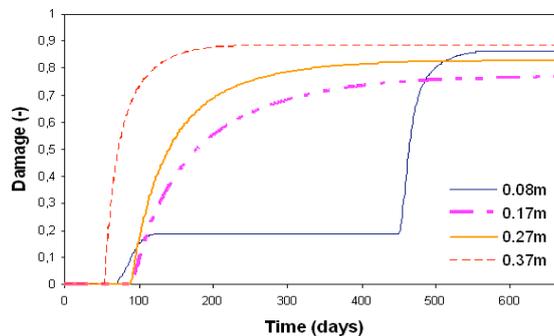
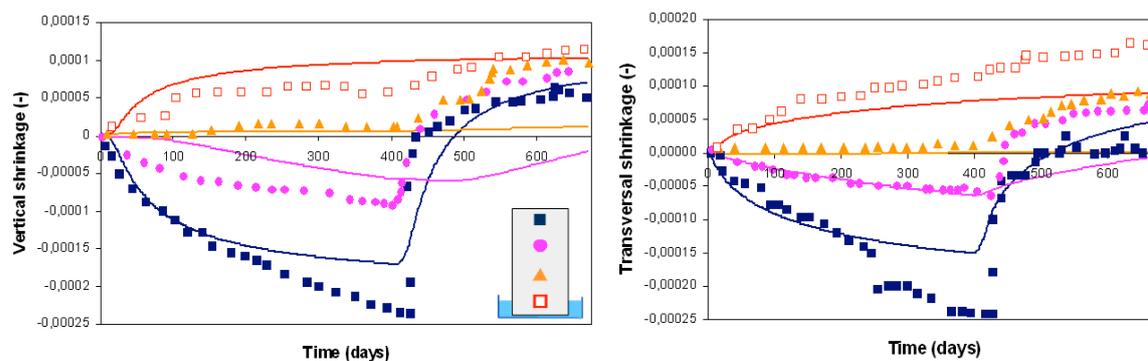


Figure 6: Damage of reactive beams measured at four depths (0,08, 0,17, 0,27, 0,37 m): model prediction

Figure 7 shows vertical and transversal deformations for a non-reactive beam submitted to the same moisture gradient history. A good qualitative agreement is observed. The quantitative discrepancy between experimental data and model results could be caused by a disagreement between the moisture pattern obtained with the theoretical expression in (6) and the real adsorption-desorption behavior of the beam. This aspect needs a closed examination.



**Figure 7:** Vertical and transverse strains of non-reactive beams measured at four depths (0,08, 0,17, 0,27, 0,37 m from the upper face): experimental points from [15] and model prediction

## 5 CONCLUSIONS

In this study a three-phase chemo-elastic damage model for the description of the mechanical degradation of concrete induced by the ASR has been presented. The model takes into account the influence of temperature and humidity histories on the kinetic of the reaction and on the final expansion of the gel produced by the ASR. The consequent mechanical degradation of concrete is described by introducing an isotropic damage variable. The application of the model to beams subjected to moisture gradients shows the capability of the isotropic model to predict the structural degradation due to ASR.

Although the attention has been focused on the rigorous thermodynamic formulation, the model is simple enough to be used in structural analysis.

With respect to the two-phase model previously proposed by the Authors [14], the present model permits to take into account shrinkage effects in a straightforward way.

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