Flexural stiffness reduction for stainless steel SHS and RHS members prone to local buckling

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Abstract:

In this paper, flexural stiffness reduction factor formulations, applicable to stainless steel members with compact cold-formed square and rectangular hollow section (SHS and RHS), are extended to account for local buckling effects and initial localized imperfection (ω). Local buckling effects and the influence of ω are accounted for by means of reducing the gross section resistance using a strength reduction factor ρ. ρ, determined by the Direct Analysis Method, depending on cross-section slenderness, is adopted. For in-plane stainless steel elements with non-compact and slender sections, results determined by the extended flexural stiffness reduction factor coupled with Geometrically Nonlinear Analysis (GNA) are verified against those determined by Geometrically and Materially Nonlinear Analysis with Imperfections (GMNIA). It is found that GNA with the extended flexural stiffness reduction factor (using beam element) generally achieves the accuracy of GMNIA (using shell element). Probabilistic studies based on 3D models with random ω are carried out to evaluate the effect of uncertainty in ω on the accuracy of GNA with the extended beam-column flexural stiffness reduction factor.

Keywords: Local buckling; stiffness reduction; stability; probabilistic studies; stainless steel

Notation

λ_c :	Column slenderness
λ_l :	Cross-sectional slenderness
λ_r and λ_p :	Limiting width-to-thickness ratio
ρ:	Strength reduction factor accounting for local buckling effects
τ_M:	Flexural stiffness reduction factor for beam with compact cross-sections
τ_N:	Flexural stiffness reduction factor for column with compact cross-sections
τ_{MN}:	Flexural stiffness reduction factor for beam-column with compact cross-sections
$\tau_{M-\rho}$:	Flexural stiffness reduction factor for beam with non-compact and slender cross-sections considering local buckling effects
$\tau_{M-shell}$	τ_M derived from the M-k curve determined by GMNIA-shell
τ_{M-beam}	τ_M derived from the M-k curve determined by GMNIA-beam
$\tau_{N-\rho}$:	Flexural stiffness reduction factor for column with non-compact and slender cross-sections considering local buckling effects

$\tau_{MN-\rho}$: Flexural stiffness reduction factor for beam-columns with non-compact and slender cross-sections considering local buckling effects (determined based on minimum strength reduction factors)

$\tau_{MN-\rho mem}$: Flexural stiffness reduction factor for beam-columns with non-compact and slender cross-sections considering local buckling effects (determined based on corresponding strength reduction factors)

ω: Initial localized imperfection

ω_{max}: The maximum initial localized imperfection

γ: A parameter used to facilitate the comparison of results provided by different methods

γ_S: γ determined by GMNIA-shell

γ_B: γ determined by GMNIA-beam

γ_ρ: γ determined by GNA-$\tau_{MN-\rho}$

γ_m: γ determined by GNA-$\tau_{MN-\rho mem}$

μ: Mean value

B_{2-E} : Amplification factor evaluates P-Δ effects (including P-δ effects) on elastic beam-columns

C_m : Equivalent uniform moment factor

COV: Coefficients of Variation

DM: Direct Analysis Method provided in AISC 360-16

DSM: Direct Strength Method

GMNIA: Geometrically and Materially Non-linear Analysis with Imperfections

GMNIA-shell: GMNIA using shell element

GMNIA-beam: GMNIA using beam element

GNA: Geometrically Non-linear Analysis

GNA-$\tau_{N-\rho}$: GNA coupled with $\tau_{N-\rho}$

GNA-$\tau_{MN-\rho}$: GNA coupled with $\tau_{MN-\rho}$

GNA-$\tau_{MN-\rho mem}$: GNA coupled with $\tau_{MN-\rho mem}$

LA: Linear Elastic Analysis

M_1 and M_2 :Applied external end moments, $|M_1| \leq |M_2|$.

M_n: Nominal flexural strength of a beam

M_{ne}: Nominal global (lateral-torsional) buckling moment

M_{nl}: Nominal local buckling moment

M_{crl}: Elastic critical local buckling moment.

M_P : Moment at full cross-section yielding

M_y : Moment at yielding of the extreme fiber

M_{r1}: Maximum internal first order moment within the member

M_{r2}: Maximum internal second order moment within the member

$M_{r2\text{-GMNIA-S}}$: M_{r2} determined by GMNIA-shell

$M_{r2\text{-}\tau MN\text{-}\rho}$: M_{r2} determined by GNA-$\tau_{MN\text{-}\rho}$

$M_{r2\text{-}\tau MN\text{-}\rho mem}$: M_{r2} determined by GNA-$\tau_{MN\text{-}\rho mem}$

M_u: Ultimate external moment

$M_{u\text{-GMNIA-B}}$: M_u determined by GMNIA-beam

$M_{u\text{-GMNIA-S}}$: M_u determined by GMNIA-shell

$M_{u\text{-}\tau MN\text{-}\rho}$ M_u determined by GNA-$\tau_{MN\text{-}\rho}$

$M_{u\text{-}\tau MN\text{-}\rho mem}$ M_u determined by GNA-$\tau_{MN\text{-}\rho mem}$

$M_{u\text{-rand}}$ M_u for each model with random ω

P_{crl}: Elastic critical local buckling strength of the column

P_e : Elastic critical global buckling strength of the column

$P_{e\text{-}\tau N\text{-}\rho}$: P_e determined by the reduced flexural stiffness ($\tau_{N\text{-}\rho}$times initial flexural stiffness)

P_n: Nominal compressive strength of a column

P_{ne}: Nominal global buckling strength in compression

P_{nl}: Nominal local buckling strength in compression

P_{r1}: Maximum internal first order axial force within the member

P_{r2}: Maximum internal second order axial force within the member

P_u: Ultimate axial load of the member

$P_{u\text{-GMNIA-B}}$: P_u determined by GMNIA-beam

$P_{u\text{-GMNIA-S}}$: P_u determined by GMNIA-shell

$P_{u\text{-}\tau MN\text{-}\rho}$: P_u determined by GNA-$\tau_{MN\text{-}\rho}$

$P_{u\text{-}\tau MN\text{-}\rho mem}$: P_u determined by GNA-$\tau_{MN\text{-}\rho mem}$

$P_{u\text{-}\tau N\text{-}\rho}$: P_u determined by GNA-$\tau_{N\text{-}\rho}$

$P_{u\text{-rand}}$: P_u for each model with random ω

P_y : Cross-section yield strength

R_M: Factor accounts for P-δ effects on the global behavior of the structure

1. Introduction

Stainless steel is a high-performance material for the construction industry and has attracted much attention [1-2]. It has been studied for structural applications at material, member, and system level [3-9]. A stiffness reduction-based design approach: Geometrically Nonlinear Analysis (GNA) coupled with flexural stiffness reduction factor, for the in-plane stability design of stainless steel elements and frames has been established by Shen and Chacón [10]. In this approach, the flexural stiffness reduction factor accounts for the deleterious influence of spread of plasticity, residual stresses, and member out-of-straightness, while GNA accounts for second order effects. The ultimate limit state for this approach is the formation of first plastic hinge.

In [10], flexural stiffness reduction functions were developed and verified for compact cold-formed rectangular hollow section (RHS) and square hollow section (SHS). In practical situations, many economical cold formed stainless steel hollow box sections contain slender thin-walled elements that are susceptible to local buckling. For these sections, adequate flexural stiffness reduction functions that are capable of taking into consideration local buckling effects should be developed. Although great efforts have been made to stiffness reduction-based design approaches [11-18], when it comes to local buckling effects on the flexural stiffness reduction of slender elements, less information is available. For Direct Analysis Method (DM) provided in AISC 360-16[19], which is a representative example of GNA with stiffness reduction approach, the influence of local buckling on the flexural stiffness reduction of slender elements subjected to compression are accounted for by means of reducing the resistance of the gross section. A similar approach to account for local buckling effects on column flexural stiffness was adopted by White et al. [15]. This reduction in gross section resistance can be considered through either using the effective cross-sectional area (effective widths of elements) determined by Effective Width Method (EWM)[20-23], or adopting a strength reduction factor (ρ) that accounts for local buckling effects for compression elements [24-26].

In the current paper, for stainless steel elements with non-compact and slender sections, the flexural stiffness reduction formulations provided in [10] are extended by reducing gross section resistance. Non-compact section here refers to cross-section that is able to reach the yield stress (0.2% proof stress) in its compression elements before inelastic local buckling occurs, but is unable to develop fully plastic stress distribution due to local buckling. Slender section here refers to cross-section in which inelastic local buckling will occur in the range between proportional limit and yield stress (0.2% proof stress). The proportional limit of stainless steels ranges from 40% to 70% of the 0.2% proof strength [27]. Cross-sections in which elastic local buckling occurs below 40% of the 0.2% proof strength are not considered in this paper. According to [28], stainless steel hollow sections under compression containing elements with width-to-thickness ratios greater than λ_r from Table 1, are defined as slender. For stainless steel hollow box sections subjected to bending, if one or more compression element with width-to-thickness ratios great than λ_p but less than λ_r provided in Table 1, these sections are defined as non-compact, while if the width-to-thickness ratio of any compression element exceeds λ_r, these sections are designated as slender.

Table 1. Limiting width-to-thickness ratios for stainless steel box sections (E and f_y are Young's Modulus and 0.2% proof stress, respectively)

Limiting width-to-thickness ratios	Compression elements subject to axial compression	Compression elements subject to flexure	
	λ_r	λ_p	λ_r
Flange	$1.24\ (E/f_y)^{0.5}$	$1.12\ (E/f_y)^{0.5}$	$1.24\ (E/f_y)^{0.5}$
Web	$1.24\ (E/f_y)^{0.5}$	$2.42\ (E/f_y)^{0.5}$	$3.01\ (E/f_y)^{0.5}$

Besides local buckling effects, cross-sections that contain slender elements are susceptible to initial localized imperfection **ω** (as illustrated in Fig.1.). Localized imperfections, which are induced from rolling and fabrication process, have sufficient variability and have no definitive characterization [29]. Therefore, it is necessary to evaluate the sensitivity of the extended stiffness reduction factor (in conjunction with GNA) to random **ω** (both the shape and magnitude of ω varied randomly).

In the following sections of the paper, strength reduction factors used to reduce the resistance of the gross section are first presented, followed by a brief description of the adopted finite element modelling approach. The extension of the flexural stiffness reduction formulations by incorporating strength reduction factors, and subsequent verification are presented in Section 4. Probabilistic studies to evaluate the effect of uncertainty in localized imperfection on the accuracy of GNA coupled with the extended flexural stiffness reduction factor are then presented.

2 Strength reduction factors for considering local buckling effects

A strength reduction factor (ρ), which is a function of cross-sectional slenderness (λ_l), accounting for local buckling effects for compression elements, is used to reduce the resistance of a cross-section due to local buckling reduction. In general, ρ can be determined by two methods: the Effective Width Method (EWM) and Direct Strength Method (DSM). For DSM, ρ, depending on λ_l, is given directly. And the same strength reduction curve is adopted for both columns and beams to take into consideration the interaction of global buckling with local buckling [26]. For EWM, since formulations that determine the resistance of members subject to local buckling reduction are based on effective cross-sectional area (effective widths of elements) [30-33], they have to be rewritten in terms of cross-sectional slenderness, to obtain the expression of ρ. The strength reduction factors derived from different design codes and specifications that adopt EWM may vary slightly.

The accuracy of both DSM and EWM highly depends on the accuracy of the adopted flexural buckling strength curves that determine the global buckling strength. The study of Arrayago et al. [34-35] showed that, compared to EWM, DSM-based approach gave more accurate predictions for most cold-formed stainless steel RHS and SHS members, when the same flexural buckling strength curve was adopted in DSM and EWM to determine the global buckling strength. Therefore, the strength reduction factors determined by DSM rather than EWM are adopted in this paper. They are shown in the following sections:

2.1 ρ for members subjected to axial compression

For members in compression, ρ is given by Eq.(1) and (2), shown in Fig.2. The strength reduction factor ρ considers interaction between global and local buckling.

when $\lambda_l \leq 0.776$ $\quad \frac{P_{nl}}{P_{ne}} = \rho = 1$ (1)

when $\lambda_l > 0.776$ $\quad \frac{P_{nl}}{P_{ne}} = \rho = {\lambda_l}^{-0.8} - 0.15{\lambda_l}^{-1.6}$ (2)

where λ_l=(P_{ne}/P_{crl})^0.5; P_{nl} is the nominal local buckling strength in compression; P_{nl} is equal to the nominal compressive strength (P_n) of a column (without distortional buckling); P_{ne} is the nominal global buckling strength in compression; P_{crl} is the elastic critical local buckling strength.

2.2 ρ for members subjected to bending

The strength reduction curve shown in Fig.2 is applicable to members subjected to bending, provided that inelastic reserve strength (corresponding to $\lambda_l \leq$0.776) resulted from partial yielding of the cross-section under bending is not considered [26].The strength reduction factor for members in bending is given by

when $\lambda_l \leq 0.776$ $\quad \frac{M_{nl}}{M_{ne}} = \rho = 1$ (3)

when $\lambda_l > 0.776$ $\quad \frac{M_{nl}}{M_{ne}} = \rho = {\lambda_l}^{-0.8} - 0.15{\lambda_l}^{-1.6}$ (4)

where λ_l=(M_{ne}/M_{crl})^0.5; M_{nl} is the nominal local buckling moment; For a beam without distortional buckling, M_{nl} is equal to the nominal flexural strength (M_n); M_{ne} is the nominal global (lateral-torsional) buckling moment; M_{crl} is elastic critical local buckling moment.

2.3 Calculation of nominal buckling strength and moment

For Eq. (1), (2), (3) and (4), the corresponding nominal buckling strength and moment (P_{ne}, P_{crl}, M_{ne}, and M_{crl}) are determined in accordance with rules that are applicable to stainless steels, as follows:

(1) The nominal global buckling strength P_{ne}, given by Eq. (5) and (6), is determined in accordance with [28].

When $\lambda_c \leq 1.2$ $\quad P_{ne} = 0.5^{{\lambda_c}^2} P_y$ (5)

When $\lambda_c > 1.2$ $\quad P_{ne} = 0.531 P_e = \frac{0.531}{{\lambda_c}^2} P_y$ (6)

where λ_c is member slenderness; λ_c=(P_y/P_e)^0.5; P_y is full cross-section yield strength; P_y=Af_y ; f_y is 0.2% proof stress; A is gross section area; $P_e=(\pi^2 EI)/(Kl)^2$; E is Young's Modulus, I moment of inertia, K effective length factor, *l* length of the member.

(2) The elastic critical local buckling strength P_{crl} is given by

$$P_{crl} = f_{crl}A \quad (7)$$

where f_{crl} is the elastic critical local buckling stress. f_{crl} can be determined by Eq. (8) or determined by appropriate Software (such as CUFSM recommended by AISI S100-16 [26]).

$$f_{crl} = \frac{k_b \pi^2 E t^2}{12(1-\nu)^2 b^2} \quad (8)$$

where t is plate thickness, ν Poisson's ratio, b width of the slender element, K_b the buckling factor.

(3) The nominal global (lateral-torsional) buckling moment M_{ne} is determined based on [28]. According to [28], hollow box sections with height to width ratio less than 3 are not susceptible to lateral-torsional buckling (LTB). Since all the studied hollow sections in the present paper are within this limit, M_{ne} is taken as M_p (moment at full cross-section yielding) for beams with non-compact sections while M_{ne} is taken as M_y (moment at yielding of the extreme fiber) for beams with slender sections.

(4) The elastic critical local buckling moment M_{crl} is given by

$$M_{crl} = W_{el} f_{crl} \quad (9)$$

where W_{el} is elastic gross section modulus; the elastic critical local buckling stress f_{crl} can be determined by the software CUFSM, or determined by Eq.(8).

It should be pointed out that, for stainless steel members with RHS and SHS, initial localized imperfection (ω) considered in the ρ factor is a random variable. In the current paper, the value of ω considered in the ρ factor is conservatively taken as the mean value of the maximum localized imperfection (ω_{max}) collected from reported tests. Statistical analysis of ω_{max} of a total of 161 cold-formed stainless steel RHS and SHS members has already been provided in [29, 36]. The study of Shen [36] showed that the mean value of ω_{max} of the collected samples is 0.185. A brief summary of the samples is shown in Table A.1 of Appendix.

3. Numerical modelling

The in-plane structural behavior of stainless steel elements susceptible to local buckling is studied using finite element (FE) software Abaqus 6.13 [37].

3.1 Elements, material properties and residual stresses

Two types of elements are employed: one-dimensional beam element with in-plane behavior (B21) and three-dimensional shell element (S4R). When conducting GNA coupled with flexural stiffness reduction, beam elements are employed, while both beam and shell elements are employed when implementing Geometrically and Materially Non-linear Analysis with

Imperfections (GMNIA). The cross-section (without rounder corner) is defined as box section for beam element. To make the results determined by beam element and those determined by shell element comparable, the same box section is used for shell element.

The nonlinear two-stage stress-strain curve, provide in [31], is adopted for material modelling. It is given by Eq. (10) and (11), and shown in Fig. 3. The expression of the curve involves three basic parameters for $\sigma \leq f_y$: Young's Modulus (E), 0.2% proof stress (f_y), and the first stage strain hardening exponent (n), and three additional parameters for $\sigma > f_y$: the ultimate strain(ε_u), the ultimate stress (f_u), and the second stage strain hardening exponent (m). The additional parameters can be determined in terms with the basic parameters [38].

$$\varepsilon = \frac{\sigma}{E} + 0.002\left(\frac{\sigma}{f_y}\right)^n \qquad \text{for } \sigma \leq f_y \qquad (10)$$

$$\varepsilon = 0.002 + \frac{f_y}{E} + \frac{\sigma - f_y}{E_y} + \varepsilon_u\left(\frac{\sigma - f_y}{f_u - f_y}\right)^m \qquad \text{for } f_y < \sigma \leq f_u \qquad (11)$$

where E_y is the tangent modulus at the 0.2% proof stress; $E_y = E/(1+0.002nE/f_y)$.

To take the enhanced material properties of the corner regions (including the extended area) of the cold-formed cross sections into consideration, the weighted material property method proposed by Hradil and Talja [39] is adopted. The accuracy of the weighted average material property method for cold-formed stainless steel RHS and SHS members has been extensively verified against experimental results by Arrayago [40], in which results demonstrated that FE models with weighted average material properties provided excellent results for both compression and combined loading conditions.

Longitudinal bending residual stresses are considered in this paper. They are accounted for by modifying the stress-strain curve used for FE models, in which the assumption that the material properties of stainless steel satisfy von Mises yield criterion and Prandtl-Reuss flow rules is made. The adopted amplitude of longitudinal residual stresses is based on the pattern suggested by Gardner and Cruise [41]. Detailed procedures of modifying the stress-strain curve for cold-formed stainless steel RHS and SHS are provided in [36].

3.2 Initial geometric imperfections

For sway-restrained members, out-of-straightness (δ/L) and localized imperfection (ω) are considered and modelled directly. Out-of-straightness and localized imperfection are combined together by means of linear superposition of relevant modes (local and global buckling modes). These modes are obtained from preliminary Buckle Analysis through Abaqus. The deterministic value for out-of-straightness is taken as 0.001. The deterministic value for localized imperfection, is taken as the mean value of the maximum ω (ω_{max}) collected from the reported tests results. According to [36], the mean value of ω_{max} for a total of 161 cold-formed stainless steel RHS and SHS members is 0.185. For linear superposition, the global buckling mode is multiplied by 0.001, while local buckling mode is multiplied by the 0.185. For sway-permitted members, out-of-plumbness (Δ/h), out-of-straightness (δ/L) and localized imperfection (ω) are considered. Localized imperfection is directly modelled through local buckling mode times the mean value of ω_{max}. Out-of-plumbness is taken as 0.002 and out-of-straightness is 0.001. Out-of-straightness is taken into consideration by applying a concentrated load at the mid-height of the member, while out-of-plumbness is modelled by applying a concentrated notional load at cantilever end in the direction of sway deformation. All notional loads are applied to the directions that produce most destabilizing effects. To avoid additional shear force at the member base due to notional loads, corresponding horizontal reaction forces, equal and opposite in direction to the sum of all notional loads, are applied. It should be noted that modelling of initial geometric imperfections presented here is not applicable to probabilistic studies.

3.3 FE model validation

A validation of the developed FE models against experimental results reported in [5] is shown in Fig.4. For the validation study, the full stage Ramberg-Osgood curve was adopted. The longitudinal bending residual stresses were not modelled, since they are implicitly included in the tested material parameters. In Fig. 4, the FE model using beam element was validated against the beam-column with compact cross-section (S1-EC1), while the FE model using shell element was validated against the beam-column prone to local buckling reduction (S4-EC1). It is seen that the numerical results are in very close agreement with experimental results.

4. Flexural stiffness reduction accounting for local buckling effects and initial localized imperfection

In this section, flexural stiffness reduction formulations, presented in [10], are extended to account for local buckling effects and initial localized imperfection (ω) by means of incorporating the strength reduction factor (ρ) to reduce the resistance of the gross section. Verification studies for GNA with extended flexural stiffness reduction factor are then carried out numerically. Predicted results by GNA with extended flexural stiffness reduction (using beam element) factor are compared against those determined by GMNIA using shell element. To evaluate local buckling effects and influence of initial localized imperfection (ω), predicted results by GMNIA using shell element are compared against those obtained from GMNIA using beam element.

4.1 Extension of column flexural stiffness reduction factor

Column flexural stiffness reduction factor (τ_N) developed in [10] was derived from AISC-based stainless column strength curve provide in [28]. To account for stiffness reduction caused by local buckling and initial localized imperfection (ω), the resistance of the gross section is reduced through incorporating the strength reduction factor ρ (ρ≤1) determined by DSM. The extended column flexural stiffness reduction factor ($\tau_{N\text{-}\rho}$) formulation is given by

$$\tau_{N-\rho} = 1 \qquad \text{for } \frac{P_{r1}}{\rho P_y} \leq 0.37 \qquad (12)$$

$$\tau_{N-\rho} = -2.717\frac{P_{r1}}{\rho P_y} ln \frac{P_{r1}}{\rho P_y} \qquad \text{for } \frac{P_{r1}}{\rho P_y} > 0.37 \qquad (13)$$

where P_{r1} is maximum internal first order axial force within the member.

A plot of the extended column flexural stiffness reduction factor ($\tau_{N-\rho}$) against $P_{r1}/\rho P_y$ is shown in Fig.5 (a). Regardless of the strength reduction factor ρ, the curve of $\tau_{N-\rho}$ versus $P_{r1}/\rho P_y$ (non-compact and slender sections) is same to the curve of τ_N versus P_{r1}/P_y (compact sections). In order to show the influence of ρ on $\tau_{N-\rho}$, cross-section slenderness (λ_l) is assumed to be varied from 0 to 2 , as shown in Fig.5 (b). In the figure, the curve with $\lambda_l \leq 0.776$ (ρ=1) represents flexural stiffness reduction for columns with compact cross-sections.

4.2 Verification of the extended column flexural stiffness reduction factor

The accuracy of the extended column flexural stiffness reduction factor ($\tau_{N-\rho}$) for stainless steel members susceptible to local buckling effects subjected to axial load is assessed in this section. Simply supported columns with cross-section 120x80x2.5 (basic material parameters: E=200GPa, f_y=350MPa, n=6) subjected to axial loads are studied. The length of the columns varies from 100mm to 7000 mm. The applied axial load is factored load. For each column, GMNIA using shell element (denoted by GMNIA-shell), GMNIA using beam element (denoted by GMNIA-beam), and GNA with $\tau_{N-\rho}$ (denoted by GNA-$\tau_{N-\rho}$) using beam element are conducted.

Verification study for the studied beam-columns are conducted through the following steps, as illustrated in Fig.6.

(1) Perform GMNIA-shell and GMNIA-beam analysis to obtain the ultimate axial load (P_u) of the columns.

The introduced out-of-straightness is 0.001. The adopted amplitude of maximum localized imperfection (ω_{max}) is 0.185 when conducting GMNIA-shell. P_u predicted by GMNIA-beam is denoted by $P_{u\text{-}GMNIA\text{-}B}$, while P_u predicted by GMNIA-shell is denoted by $P_{u\text{-}GMNIA\text{-}S}$.

(2) Perform Linear Elastic Analysis (using beam element) to obtain maximum first order axial force.

The applied load is $P_{u\text{-}GMNIA\text{-}S}$. Maximum first order axial force obtained from Linear Elastic Analysis (LA) is referred to as P_{r1}. For all the studied simply supported columns, P_{r1} is equal to $P_{u\text{-}GMNIA\text{-}S}$.

(3) Calculate the ρ factor and the extended column flexural stiffness reduction factor $\tau_{N-\rho}$.

The ρ factor is calculated according to Eq. (1) and (2). $\tau_{N-\rho}$ is determined according to Eq.(12) and (13).

(4) Perform GNA-$\tau_{N-\rho}$ (using beam element) analysis to predict the ultimate axial load of the columns.

Ultimate axial load predicted by GNA-$\tau_{N-\rho}$ is denoted by $P_{u\text{-}\tau N\text{-}\rho}$.

Note that even though out-of-straightness of 0.001 is included in $\tau_{N-\rho}$, an imperfection value much smaller than 0.001is introduced into the columns to ensure that these columns can buckle in GNA (columns without any imperfection would not buckle in GNA). If the extended $\tau_{N-\rho}$ expression is "perfect", the ultimate axial load determined by GNA-$\tau_{N-\rho}$ should be equal to the ultimate axial load determined by GMNIA-shell. The discrepancy between them shows the quality of $\tau_{N-\rho}$.

As expected, the ultimate load (P_u) of the simply supported columns predicted by GNA-$\tau_{N-\rho}$ matches the bifurcation load (or elastic critical buckling load) $P_{e-\tau N\rho}$ determined by the reduced flexural stiffness ($\tau_{N-\rho}$ times EI), as shown in Fig.7. $P_{e-\tau N\rho}$ is given by

$$P_{e-\tau N\rho} = \frac{\pi^2(\tau_{N-\rho}EI)}{(L)^2} \quad (14)$$

where EI is initial flexural stiffness; L is unbraced length of the column.

Comparison of the results determined by GNA-$\tau_{N-\rho}$, GMNIA-shell and GMNIA-beam is shown in Fig.8, where the ultimate axial load (P_u) predicted by different method is normalized by full cross-section yield strength (P_y). The difference between the curve of GMNIA-beam and the curve of GMNIA-shell is mainly resulted from local buckling effects. It is observed that the smaller the column slenderness (λ_c) is, the more significant the difference is. This can be explained as follows: For a given cross-section, since elastic critical local buckling strength (P_{crl}) is constant, the cross-sectional slenderness λ_l (λ_l=(P_{ne}/P_{crl})^0.5) is governed by P_{ne}. According to Eq. (5) and (6), P_{ne} increases with λ_c decreasing. It means that for a given cross-section, the smaller λ_c is, the larger λ_l . As a consequence, the difference between the two curves due to the influence of local buckling becomes more considerable.

It is observed that the ultimate axial loads predicted by GNA-$\tau_{N-\rho}$ using beam element are in very close agreement with those predicted by GMNIA-shell. For columns with low λ_c, GNA-$\tau_{N-\rho}$ slightly overestimates the ultimate axial load. One possible explanation is that the incorporated strength reduction factor ρ somewhat underestimates local buckling effects, which results in $\tau_{N-\rho}$ higher than the actual flexural stiffness reduction factor. Since the ultimate load predicted by GNA-$\tau_{N-\rho}$ is equal to the bifurcation load determined by Eq.(14), in which the bifurcation load is directly proportional to $\tau_{N-\rho}$, a higher $\tau_{N-\rho}$ leads to overestimate the ultimate axial load. Note that the discrepancy between the predicted results of GNA-$\tau_{N-\rho}$ and those determined by GMNIA may also be caused by the deterministically introduced initial localized imperfection.

4.3 Extension of beam flexural stiffness reduction factor

In [10], beam flexural stiffness reduction factor (τ_M) formulation for members with compact sections under bending was developed from moment-curvature relationship for stainless steel RHS and SHS members. In the current paper, local buckling effects and the influence of initial localized imperfection are accounted for by means of incorporating the strength reduction factor (ρ) to reduce the resistance of the gross section (M_y for slender section, M_p for non-compact section).

The extended beam flexural stiffness reduction factor ($\tau_{M-\rho}$) formulation for slender section is given by:

When $0 < M_{rl} \leq \rho M_y$

$$\tau_{M-\rho} = \left[1 + (n-1)\frac{0.001E}{f_y}\left(\frac{M_{r1}}{\rho M_y}\right)^{n-2}\right]^{-1} \quad (15)$$

The extended beam flexural stiffness reduction factor ($\tau_{M-\rho}$) formulation for non-compact section is given by:

When $0 < M_{r1} \leq \rho M_y$
$$\tau_{M-\rho} = \left[1 + (n-1)\frac{0.001E}{f_y}\left(\frac{M_{r1}}{\rho M_p}\frac{W_{pl}}{W_{el}}\right)^{n-2}\right]^{-1} \quad (16)$$

When $\rho M_y < M_{r1} \leq \rho M_p$
$$\tau_{M-\rho} = \left[\left(1 - \frac{M_{r1}}{\rho M_p}\right)\frac{1}{1-\frac{W_{el}}{W_{pl}}}\right]^{0.9}\left[1 + (n-1)\frac{0.001E}{f_y}\right]^{-1} \quad (17)$$

where M_{r1} is the maximum internal first order moment within a member; M_y is moment at yielding of the extreme fiber; $M_y = W_{el} f_y$; W_{el} is elastic gross section modulus; M_p is full plastic bending moment; $M_p = W_{pl} f_y$; W_{pl} is plastic gross section modulus.

As an example for a non-compact cross-section (basic parameters: f_y = 430MPa, E=200GPa, and n=6, W_{el}/W_{pl}=0.82), a plot of the extended beam flexural stiffness reduction determined by Eq.(16) and (17) against $M_{r1}/\rho M_y$ is shown in Fig.9 (a). In order to show the influence of ρ on beam flexural stiffness reduction, cross-section slenderness (λ_l) for this cross-section is assumed to be varied from 0 to 1.1. A plot of $\tau_{M-\rho}$ with different λ_l or ρ is shown in Fig.9 (b), in which $\tau_{M-\rho}$ decreases as the assumed λ_l increases.

4.4 Verification of the extended beam flexural stiffness reduction factor

The ability of $\tau_{M-\rho}$ capturing the effects of local buckling and spread of plasticity through cross-section and along member length is verified in this section. It should be noted that, due to the non-linear stress-strain behavior (beyond proportional limit) of stainless steel, the cross-section already undergoes plastic straining before internal moment reaches M_y.

Simply supported beams with slender cross-section 120x80x2 (E=200GPa, f_y=350MPa, n=7, M_y=9.53kN*m, ρ=0.97, M_u =9.24kN*m, M_u is the maximum bending moment predicted by GMNIA-shell) and non-compact cross-section 250x150x5 (E=190GPa, f_y=450MPa, n=7,W_{pl}/W_{el}=1.204, M_p=147.49 kN*m, ρ=0.95, M_u =9.24kN*m) are studied. The beam with slender cross-section is subjected to a pair of identical end moments, while the beam with non-compact cross-section is subjected to uniformly distributed loads. GMNIA-shell and GMNIA-beam are conducted to obtain M-k curves, where the introduced out-of-straightness is 0.001 and the amplitude of the maximum localized imperfection (ω_{max}) is 0.185 in implementing GMNIA-shell.

$\tau_{M-\rho}$ determined by Eq. (15), (16) and (17) are compared against flexural stiffness reduction factors derived from M-k curves provided by GMNIA-shell. Flexural stiffness reduction factor derived from M-k curve of GMNIA-shell is denoted by $\tau_{M-shell}$, and that derived from M-k curve of GMNIA-beam is denoted by τ_{M-beam}.

The derivation of flexural stiffness reduction factor is based on

$$\tau_{M-shell}(or \;\; \tau_{M-beam}) = \frac{(EI)_t}{EI} = \frac{\frac{dM_{r1}}{d\kappa}}{EI} \quad (18)$$

where $(dM_{r1})/d\kappa$ is the slope of the tangent at a given point on the M-k curve. The procedure of calculating tangent slope is conducted through MATLAB 2017b [42].

Comparison of $\tau_{M\text{-}\rho}$ against $\tau_{M\text{-shell}}$ and $\tau_{M\text{-beam}}$ is shown in Fig.10. In the figure, the difference between $\tau_{M\text{-beam}}$ and $\tau_{M\text{-shell}}$ is mainly attributed to the influence of local buckling. Compared to the curve of $\tau_{M\text{-beam}}$, the curve of $\tau_{M\text{-shell}}$ decreases at a higher rate after local buckling occurs in the inelastic range. It is observed that the $\tau_{M\text{-}\rho}$ curves generally agree well with $\tau_{M\text{-shell}}$ curves. The difference between $\tau_{M\text{-}\rho}$ and $\tau_{M\text{-shell}}$ may be attributed to the incorporated strength reduction factor ρ or the introduced initial localized imperfection. It should be pointed out that, besides the influence of the factor ρ and initial localized imperfection, the difference between $\tau_{M\text{-}\rho}$ and $\tau_{M\text{-shell}}$ also relies on the accuracy of the beam flexural stiffness reduction factor formulations applicable to compact sections to capture the spread of plasticity of the beams.

4.5 Extension of beam-column flexural stiffness reduction factor

Similar to the above approach, local buckling effects and the influence of initial localized imperfection on beam-columns are taken into consideration by reducing the resistance of the gross section through the strength reduction factor ρ. The extended $\tau_{MN\text{-}\rho}$ formulation is given by

$$\tau_{MN-\rho} = \gamma \Omega_M \tau_{N-\rho} \tau_{M-\rho} \left[1 - \left(\frac{P_{r1}}{\rho P_y}\right)^{0.9} \left(C_m \frac{M_{r1}}{\rho M_p}\right)^{\frac{W_{el}}{W_{pl}}}\right] \quad (19)$$

$$0.8 \le \gamma = 2(B_{2-E} - 0.6) < 1 \quad \text{for } 1 \le B_{2\text{-}E} < 1.1 \quad (20)$$

$$\gamma = 1 \quad \text{for } 1.1 \le B_{2\text{-}E} \quad (21)$$

$$\Omega_M = 1 \quad \text{for } 0 \le \frac{M_{r1}}{\rho M_p} < 0.4 \quad (22)$$

$$\Omega_M = \left(0.6 + \frac{M_{r1}}{\rho M_p}\right)^{1.4} \quad \text{for } 0.4 \le \frac{M_{r1}}{\rho M_p} \le 1 \quad (23)$$

$\tau_{N\text{-}\rho}$ and $\tau_{M\text{-}\rho}$ that included in Eq.(19) are calculated based on strength reduction factor for compression (ρ-column) and strength reduction factor for bending (ρ-beam), respectively. . ρ-column is determined by Eq. (1) and (2), while ρ-beam is determined by Eq. (3) and (4). Since flexural stiffness reduction for beam-columns was expected to be the combination of flexural stiffness reduction under compression and that under bending, ρ-column and ρ-beam ought to be used to reduce axial compression resistances and bending moment, respectively. Nevertheless, preliminary finite element analysis of some beam-columns showed that, the adoption of the min {ρ-column, ρ-beam} to reduce resistance of the gross section gave more accurate results, compared to those using corresponding ρ-column and ρ-beam. One explanation is the accuracy of the adopted strength reduction factor depends heavily on the accuracy of the adopted flexural stiffness reduction formulation for composite sections. For the flexural stiffness reduction formulation, the influence of local buckling reduction and the interaction of axial compression and bending may be more accurately captured by adopting minimum of strength reduction factors to reduce resistance of the gross section. For a series of beam-columns, comparison of the predicted results determined by min {ρ-column, ρ-beam} against those determined by corresponding strength reduction factor is presented in Section 4.6 of this paper.

For Eq.(20) and (21), the factor $B_{2\text{-}E}$ evaluates P-Δ effects and together with P-δ effects on sway-permitted elastic beam-columns. For sway-restrained beam-columns, $B_{2\text{-}E}$ is equal to 1. For sway-permitted isolated beam-column, $B_{2\text{-}E}$ is given by

$$B_{2-E} = \frac{1}{1-\frac{P_{r1}}{0.85\ P_{es}}} \geq 1 \tag{24}$$

where the factor 0.85 accounts for the influence of P-δ effects on the global behavior of a sway-permitted member; $P_{es}=(F_H L)/\Delta$; F_H is first order shear force; Δ is relative drift between member ends due to F_H; L is length of the member. In addition, for beam-columns with slender sections, plastic bending moment (M_p) is replaced by extreme fiber yielding moment (M_y).

4.6 Verification of the extended beam-column flexural stiffness reduction factor

The accuracy of the extended beam-column flexural stiffness reduction factor $\tau_{MN\text{-}\rho}$ (in conjunction with GNA) for in-plane stainless steel beam-columns with non-compact and slender sections are evaluated. Simply supported beam-columns and cantilever beam-columns are studied. Simply supported beam-columns, with different cross-sections and material properties (shown in Table.2), are subjected to combined axial load (P) and varied moments (M_1, M_2) at the member ends. The applied P is factored load, M_2=e*P; e ranges from 1 to 150 (e= [0,10,30,50,80,100,150]) and the unit of e is mm; $|M_2|\geq|M_1|$. The applied end moments are varied for different cross-sections: a pair of equal but opposite end moments for cross-section 120x80x2, one end moment for cross-section 200x100x3, and a pair of identical end moments for cross-section 250x150x5. Cantilever beam-columns, with different cross-sections and material properties (shown in Table.2), are subjected to combined axial load (P) and horizontal load (iP) at the cantilever end, where the applied load P is factored load, and i=[0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3].

Table.2 Basic material parameters and cross-sections of the studied beam-columns

Beam-column		Cross-section	L(mm)	E(GPa)	f_y(MPa)	n	W_{pl}/W_{el}
Simply supported	a	120x80x2	2000	200	350	6	1.19
	b	200x100x3	2500	175	400	8	1.22
	c	250x150x5	3000	190	450	7	1.20
Cantilever	a	120x80x1.5	2000	200	350	6	1.19
	b	200x200x3	2500	175	300	7	1.14

4.6.1 Steps for verification

Verification study for the studied beam-columns are conducted through the following steps, as illustrated in Fig.11.

(1) Perform GMNIA-shell and GMNIA-beam to obtain the ultimate axial load and moment (P_u and M_u) of the beam-columns.

The introduced maximum localized imperfection (ω_{max}) is 0.185 when conducting GMNIA-shell. Out-of-straightness of 0.001 is introduced to simply supported beam-columns, while out-of-straightness of 0.001 and out-of plumbness of

0.002 are introduced to cantilever beam-columns. For simply supported beam-columns, M_u is the end moment M_2, and it is equal to e*P_u. For cantilever beam-columns, M_u is equal to horizontal load (iP_u) multiplied by member length (L). P_u and M_u determined by GMNIA-beam are denoted by $P_{u\text{-GMNIA-B}}$ and $M_{u\text{-GMNIA-B}}$, respectively, while P_u and M_u determined by GMNIA-shell are denoted by $P_{u\text{-GMNIA-S}}$ and $M_{u\text{-GMNIA-B}}$, respectively. For GMNIA-shell analysis, maximum internal second order moment (denoted by $M_{r2\text{-GMNIA-S}}$) within the beam-column, corresponding to $P_{u\text{-GMNIA-S}}$ and $M_{u\text{-GMNIA-S}}$, are obtained.

(2) Perform Linear Elastic Analysis (using beam element) to obtain maximum first order internal axial force (P_{r1}) and moment (M_{r1})

The applied axial load and end moment are $P_{u\text{-GMNIA-S}}$ and $M_{u\text{-GMNIA-S}}$, respectively. For cantilever beam-columns, the applied horizontal load multiplied by member length is equal to end moment. The factor $B_{2\text{-E}}$ is calculated according to Eq. (24).

(3) Calculate the strength reduction factor and the extended beam-column flexural stiffness reduction factor

ρ-column is calculated according to Eq. (1) and (2), while and ρ-beam is calculated according to Eq. (3) and (4). For the calculation of the ρ factor, the nominal local buckling strength (P_{nl}) is taken as $P_{u\text{-GMNIA-S}}$ for the column case, while the nominal local buckling moment (M_{nl}) is taken as $M_{u\text{-GMNIA-S}}$ for the beam case. Two types of flexural stiffness reduction factors, $\tau_{MN\text{-}\rho}$ and $\tau_{MN\text{-}\rho mem}$, are considered, in which $\tau_{MN\text{-}\rho}$ denotes that the adopted flexural stiffness reduction factor is min {ρ-column and ρ-beam}, while $\tau_{MN\text{-}\rho mem}$ represents that corresponding ρ-column and ρ-beam are used in Eq. (19).

(4) Perform GNA-$\tau_{MN\text{-}\rho}$ and GNA-$\tau_{MN\text{-}\rho mem}$ (using beam element) to predict the maximum internal second order moment (M_{r2})

M_{r2} determined by GNA-$\tau_{MN\text{-}\rho}$ and GNA-$\tau_{MN\text{-}\rho mem}$ are denoted by $M_{r2\text{-}\tau MN\text{-}\rho}$ and $M_{r2\text{-}\tau MN\text{-}\rho mem}$, respectively. For both GNA-$\tau_{MN\text{-}\rho}$ and GNA-$\tau_{MN\text{-}\rho mem}$, the ultimate axil load ($P_{u\text{-}\tau MN\text{-}\rho}$ or $P_{u\text{-}\tau MN\text{-}\rho mem}$) and end moment ($M_{u\text{-}\tau MN\text{-}\rho}$ or $M_{u\text{-}\tau MN\text{-}\rho mem}$) of the beam-columns are achieved when $M_{r2\text{-}\tau MN\text{-}\rho}$ ($M_{r2\text{-}\tau MN\text{-}\rho mem}$) is equal to $M_{r2\text{-GMNIA-S}}$.

4.6.2 Comparison of γ determined by different methods

In order to facilitate the comparison of results determined by different methods, a parameter γ determined by Eq. (25) is used.

$$\gamma = \sqrt{\left(\frac{P_u}{P_n}\right)^2 + \left(\frac{M_u}{M_n}\right)^2} \quad (25)$$

where P_u and M_u are the ultimate axial load and moment, respectively; P_n and M_n are the nominal compressive strength and nominal flexural strength, respectively. γ determined by GMNIA-shell, GMNIA-beam, GNA-$\tau_{MN\text{-}\rho}$, and GNA-$\tau_{MN\text{-}\rho mem}$ are denoted by γ_S, γ_B, γ_ρ, $\gamma_{\rho m}$, respectively.

Predicted results for simply supported beam-columns and cantilever beam-columns are shown in Table 3 and 4, respectively. μ , COV, Max and Min denote mean value, coefficient of variation, maximum value and minimum value, respectively. γ_S determined by GMNIA-shell are taken as "exact" results. γ_B, γ_ρ and $\gamma_{\rho m}$ are compared against γ_S.

Table 3. Predicted results of simply supported beam-columns

e (mm)	120x80x2				200x100x3				250x150x5			
	γ_S	γ_ρ/γ_S	$\gamma_{\rho m}/\gamma_S$	γ_B/γ_S	γ_S	γ_ρ/γ_S	$\gamma_{\rho m}/\gamma_S$	γ_B/γ_S	γ_S	γ_ρ/γ_S	$\gamma_{\rho m}/\gamma_S$	γ_B/γ_S
10	0.85	1.04	1.05	1.43	0.95	1.00	1.09	1.47	0.99	1.00	1.04	1.24
30	0.74	0.91	0.94	1.18	0.87	1.01	0.91	1.47	0.97	1.03	1.09	1.26
50	0.72	0.95	1.02	1.35	0.83	1.03	0.89	1.38	0.95	1.09	0.93	1.18
80	0.74	0.97	1.09	1.10	0.81	1.06	1.07	1.35	0.94	1.07	1.11	1.16
100	0.76	0.95	1.12	1.04	0.82	1.09	1.11	1.29	0.95	0.90	0.91	1.10
150	0.82	1.01	0.87	1.08	0.83	1.07	0.88	1.26	0.96	0.97	1.08	1.08
μ		0.97	1.02	1.20		1.04	0.99	1.37		1.01	1.03	1.17
COV		0.05	0.09	0.13		0.03	0.11	0.06		0.07	0.08	0.06
Max		1.04	1.12	1.43		1.09	1.11	1.47		1.08	1.11	1.26
Min		0.91	0.87	1.02		1.00	0.88	1.26		0.90	0.91	1.08

Table 4. Predicted results of cantilever beam-columns

i	120x80x1.5				200x200x3			
	γ_S	γ_ρ/γ_S	$\gamma_{\rho m}/\gamma_S$	γ_B/γ_S	γ_S	γ_ρ/γ_S	$\gamma_{\rho m}/\gamma_S$	γ_B/γ_S
0.05	0.70	0.99	1.01	1.19	0.68	1.02	0.97	1.08
0.10	0.75	0.92	0.79	1.23	0.71	1.09	1.13	1.12
0.15	0.81	0.81	0.94	1.24	0.79	1.03	1.15	1.08
0.20	0.85	0.88	0.74	1.25	0.83	0.96	0.87	1.11
0.25	0.90	0.91	0.82	1.21	0.86	1.00	1.07	1.07
0.30	0.97	1.03	0.93	1.16	0.86	1.01	1.09	1.09
μ		0.92	0.87	1.21		1.02	1.05	1.10
COV		0.09	0.12	0.03		0.04	0.10	0.02
Max		1.03	1.01	1.25		1.09	1.15	1.12
Min		0.81	0.74	1.16		0.96	0.87	1.07

It is observed that, the range of mean values and COVs for γ_ρ/γ_S are 0.92-1.02, and 0.03-0.09, respectively, while the range of mean values and COVs for $\gamma_{\rho m}/\gamma_S$ are 0.87-1.05 and 0.08-0.12, respectively. It indicates that predicted results of both GNA-$\tau_{MN\text{-}\rho}$ and GNA-$\tau_{MN\text{-}\rho mem}$ are in close agreement with those determined by GMNIA-shell. With the mean value of γ_B/γ_S ranging from 1.10 to 1.37, GMNIA-beam overestimates the ultimate strength of all the studied beam-columns. This is because the influence of local buckling reduction and initial localized imperfection is not considered in it.

The maximum errors of overestimation vary from 3% to 9% for GNA-$\tau_{MN-\rho}$, and 1% to 15% for GNA-$\tau_{MN-\rho mem}$, while the maximum errors of underestimation vary from 4% to 19% for GNA-$\tau_{MN-\rho}$, and 9% to 26% for GNA-$\tau_{MN-\rho mem}$. It demonstrates that, GNA-$\tau_{MN-\rho}$ whose flexural stiffness reduction factors adopting min{ρ-column, ρ-beam}provide more accurate results, compared to GNA-$\tau_{MN-\rho mem}$ in which corresponding ρ-column and ρ-beam are used in the calculation of beam-column flexural stiffness reduction factor. This is probably because the accuracy of the adopted strength reduction factor depends heavily on the accuracy of the adopted flexural stiffness reduction formulation for composite sections, where the influence of local buckling reduction and the interaction of axial compression and bending can be more accurately captured by using minimum of strength reduction factors to reduce resistance of the gross section.

4.6.3 Comparison of strength curves determined by different methods

The predicted strength curves for simply supported beam-columns and cantilever beam-columns are shown in Fig.12 and 13, respectively. The strength curves determined by GNA-$\tau_{MN-\rho}$ are compared against those determined by GMNIA-shell, to evaluate the accuracy of the adopted $\tau_{MN-\rho}$, while the strength curves determined by GMNIA-beam are compared against those determined by GMNIA-shell, to evaluate local buckling effects and the influence of initial localized imperfection (ω). In the two figures, P_n and M_n are the nominal compressive strength of the column and nominal flexural strength of the beam, respectively; P_n and M_n determined by relevant equations provided in the above Section 2 are very close to P_u (column case) and M_u (beam case) determined by GMNIA-shell, respectively; P_u and M_u predicted by different methods are normalized by P_n and M_n, respectively. It should be mentioned that for all the beam cases, $M_{u-\tau MN-\rho}$ is taken as the ultimate end moment determined by GMNIA-shell.

For the studied beam-columns, the considerable difference between the curve of GMNIA-beam and the curve of GMNIA-shell is attributed to local buckling effects and the influence of initial localized imperfection (ω). It is observed that the results predicted by GNA-$\tau_{MN-\rho}$ are in close agreement with those determined by GMNIA-shell. For most of the studied beam-columns, the difference between the predicted results of GNA-$\tau_{MN-\rho}$ and those determined by GMNIA-shell mainly occurs in the intermediate part of the interaction curves (P_u/P_n versus M_u/M_n). It may result from either the incorporated strength reduction factor or the amplitude of introduced maximum initial localized imperfection (ω_{max}) in implementing GMNIA-shell analysis. From Fig.12 and Fig.13, it is concluded that, besides capturing the influence of spread of plasticity, the extended flexural stiffness reduction factor $\tau_{MN-\rho}$ can well capture local buckling effects.

For member-based ultimate limit design checks using internal axial forces and moments determined by GNA-τ_{MN}-ρ or GMNIA-shell conducted in this paper, only cross-section strength check is needed and member buckling strength check is eliminated. This is because second order effects (P-Δ and P-δ) and all initial geometric imperfections (out-of-plumbness, out-of-straightness, and localized imperfection) are considered. In addition, for the design check of non-compact and slender cross-sections, full cross-section resistance has to be reduced to account for local buckling effects.

5. Probabilistic studies

Since the capacity of members with non-compact and slender sections may be susceptible to initial localized imperfection (**ω**), it is necessary to investigate the influence of uncertainty in **ω** on the accuracy of $\tau_{MN-\rho}$ (in conjunction with GNA). The investigation is conducted through probabilistic studies based on 3D models with random ω proposed in [29, 36].

5.1 Generation of 3D models with random ω and FE analysis

The Fourier series-based 3D model with random localized imperfection (ω), proposed in [29, 36], is used for probabilistic study. Localized imperfection of a generated surface comprises two components: transverse variation and longitudinal variation, as shown in Fig 14 (a), in which $f_1(x_i)$ and $f_2(x_i)$ are two functions that are decomposed into Fourier series with random coefficients.

The procedures of generating of 3D models with random ω for beam-columns are similar to that presented in [29]. A MATLAB script is written to generate random coefficient of Fourier series terms of function $f_1(x)$ and $f_2(x)$.The distribution of the generated random ω_{max} followed a log-normal distribution derived from experimental data of ω_{max}. Models with $\boldsymbol{\omega}_{max}$ higher than the allowable value are automatically found by MATLAB script and are discarded. The allowable value specified in EN-10219-2:2006 [43] is adopted. It is min{0.008b, 0.5}, where b is the side (straight side of the cross-section) length. There is slight difference between simply supported beam-columns and cantilever beam-columns. For simply supported beam-columns, Fourier series expansion of function $f_1(x)$ generated half-sine-wave (representing out-of-straightness) with amplitude of 0.001, as shown in Fig. 14(b). For cantilever beam-columns, Fourier series expansion of function $f_1(x)$ generated straight lines, as shown in Fig. 14(c), since the effects of out-of-straightness and out-of-plumbness are considered by applying notional loads (equivalent horizontal loads).

The developed Matlab program automatically created a Python script associated with an Input file operated in Abaqus. In conducting FE analysis through Abaqus, material properties and residual stresses were modelled as those presented in Section 3.

5.2 Beam-columns for probabilistic studies

The studied beam-columns are the same beam-columns presented in Section 4.6, but only one combined loading case is considered for each beam-column. All the studied beam-columns, shown in Fig.15, are susceptible to local buckling. The applied axial load (P) is factored load. For simply supported beam-columns, the applied end moment M_2 is equal to e*P; e=50mm (constant). For cantilever beam-columns, the applied horizontal load at the cantilever end is equal to 0.1P. Details of conducted analysis are shown in Table 5. For each beam-column, 100 models with random ω are produced. For each model, GMNIA-shell (with random ω) analysis is carried out to determine the ultimate axial load and ultimate end moment (referred to as $M_{u\text{-}rand}$). Thus, each beam-column has 100 $M_{u\text{-}rand}$ in all.

Table 5. Details of the conducted analysis

Method	Element	Localized imperfection (ω)	
		Shape	Amplitude(mm)
GMNIA	Shell	Idealized	ω_{max}=0.185
GNA-$\tau_{MN\text{-}\rho}$	beam	(Implicitly considered in $\tau_{MN\text{-}\rho}$)	
GMNIA	Shell	Random	0< ω_{max} ≤min{0.008b, 0.5}

5.3 Effect of uncertainty in ω on the accuracy of GNA-$\tau_{MN\text{-}\rho}$

The predicted results are shown in Table 6. Since the ultimate end moment (M_u) is directly proportional to the ultimate axial load (P_u), where M_u=e*P_u for the simply supported beam-columns and M_u=0.1P_u*L for the cantilever beam-columns, only M_u determined by different methods is shown in the table. In the table, $M_{u\text{-}GMNIA\text{-}S}$ is determined by GMNIA-shell with idealized ω (the lowest local buckling mode), and $M_{u\text{-}\tau MN\text{-}\rho}$ is determined by GNA-$\tau_{MN\text{-}\rho}$. The mean value of the $M_{u\text{-}rand}$ is denoted by μ($M_{u\text{-}rand}$).

Table 6. Statistical characteristics of the predicted ultimate end moments

M_u(kN*m)	Simply supported			Cantilever	
	a	b	c	a	b
$M_{u\text{-}GMNIA\text{-}S}$	6.0	21.9	76.8	4.75	30.83
$M_{u\text{-}\tau MN\text{-}\rho}$	5.2	23.5	77	4.03	32.15
μ($M_{u\text{-}rand}$)	5.9	22.2	79.6	4.66	29.90
COV($M_{u\text{-}rand}$)	0.07	0.05	0.12	0.09	0.17
μ ($M_{u\text{-}rand}$)/ $M_{u\text{-}GMNIA\text{-}S}$	0.98	1.01	1.04	0.98	0.97
μ ($M_{u\text{-}rand}$) / $M_{u\text{-}\tau MN\text{-}\rho}$	1.13	0.94	1.03	1.15	0.91

From the table, COVs (Coefficients of Variation) for the case c (simply supported beam-column) is 0.12, and COVs for the case b (cantilever beam-column) is 0.17. The two COVs demonstrate a relatively large extent of variability in relation to μ($M_{u\text{-}rand}$). One possible explanation is that the localized imperfection (ω) amplitude of the generated models is largely scattered. The mean-to-nominal ratios, μ ($M_{u\text{-}rand}$)/ $M_{u\text{-}GMNIA\text{-}S}$, for all the beam-columns are about 0.98-1.04, which indicates that for the studied beam-columns, localized imperfection can be statistically modelled as idealized shape times the deterministic value 0.185 (the mean value of the maximum ω).

The ratios of μ ($M_{u\text{-}rand}$) / $M_{u\text{-}\tau MN\text{-}\rho}$, for all the beam-columns are all about 0.91-1.15. It shows that prediction errors for GNA-$\tau_{MN\text{-}\rho}$ caused by uncertainty in ω are in an acceptable range for the studied beam-columns. This is because the results provided by GNA-$\tau_{MN\text{-}\rho}$ are generally close to those provide by GMNIA with idealized ω times the deterministic value 0.185 (detailed results are shown in Section 4.6), where the ultimate external moment of GMNIA with idealized ω times 0.185 can statistically represent the ultimate external moment of the studied beam-columns with random ω. It should be mentioned that, the prediction errors for GNA coupled with flexural stiffness reduction may be significant for some beam-column cases, in which the deterministic value of ω considered in the flexural stiffness reduction factor is not capable of statistically capturing the influence random ω on the ultimate capacity of these beam-columns.

6. Conclusion

For the accurate and safe in-plane stability design of cold-formed elements with non-compact and slender hollow sections (RHS and SHS), the flexural stiffness reduction formulations provide by Shen and Chacon [10], are extended to take local buckling effects and initial localized imperfection (ω) into consideration. For the determination of the flexural stiffness reduction factor of columns, beams and beam-columns, strength reduction factors, which depend on cross-sectional slenderness, are used to reduce the resistance of a gross section due to local buckling reduction. The accuracy of GNA with the extended flexural stiffness reduction factor for stainless steel elements (columns, beams and beam-columns) with non-compact and slender sections are verified. Predicted results by GNA with the extended flexural stiffness reduction factor using beam element are in close agreement with those determined by GMNIA using shell element.

Based on the 3D models with random ω, probabilistic studies on simply supported and cantilever beam-columns are conducted to evaluate the sensitivity of the extended flexural stiffness reduction factor (in conjunction with GNA) to random ω. It is found that, based on the studied beam-columns, uncertainty in ω result in prediction errors for GNA coupled with flexural stiffness reduction to some extent, but ignoring uncertainty in ω won't lead to significant errors for GNA coupled with flexural stiffness reduction. One possible explanation is the deterministic value of ω considered in the adopted flexural stiffness reduction factor is capable of statistically capturing the influence random ω on the ultimate capacity of these beam-columns.

In addition, the accuracy of the extended flexural stiffness reduction factor in conjunction with GNA for frame systems should be assessed. The applicability of the extended flexural stiffness reduction factor for slender open sections that are susceptible to lateral-torsional buckling should be studied further.

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Appendix

Table A.1. A brief summary of the collected samples

Reference	No. of samples	Stainless steel groups	Grade
B. Young and W.M. Lui, 2005 [44]	5	Duplex	EN1.4162
B.F. Zheng et al., 2016 [45]	4	Austenitic	EN1.4301
I. Arrayago. et al., 2016 [5]	12	Ferritic	EN1.4003
M.Theofanous and L.Gardner, 2009[46]	8	Duplex	EN1.4162
W.M. Lui et al., 2014 [47]	10	Duplex	EN1.4462
Y. Huang and B.Young, 2013 [48]	22	Duplex	EN1.4162
M. Bock et al., 2015 [49]	8	Ferritic	EN1.4003
I. Arrayago and E. Real, 2015 [50]	26	Ferritic	EN1.4003
O. Zhao et al.,2016[51]	24	Ferritic	EN1.4003
S.Afshan and L.Gardner,2013 52]	6	Ferritic	EN1.4003
	2	Ferritic	EN1.4509
O. Zhao et al.,2015 [53]	10	Austenitic	EN1.4301
	6	Austenitic	EN1.4571
	6	Austenitic	EN1.4307
	6	Austenitic	EN1.4404
	6	Duplex	EN1.4162
Total :161			

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