V International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2013
S. Idelsohn, M. Papadrakakis and B. Schrefler (Eds)

SEMI-ANALYTICAL APPROACH FOR RATCHETING STRAIN AND WEAR PREDICTION IN CASE OF LINE ROLLING CONTACT

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Key words: Rolling contact fatigue, Wear, Coupled problems, Mazzu, Inverse algorithm

Abstract. The most studies in the field of rolling contact fatigue and wear consider that these phenomenons are independent, but this paper shows strong coupling of both in the case, when ratcheting mechanism is the driving one. There is presented the application of semi-analytical approach, introduced by A. Mazzu [1], for two dimensional rolling contact case. The model is based on non-linear kinematic and isotropic hardening rule of Chaboche and Lemaitre [2]. Mazzu´s approach has been modified to correctly predict ratcheting and wear rate for different values of contact pressure. A correlation between the parameter σ_L (yield stress) and the value of the maximum contact pressure p_0 has been found. All performed simulations are based on rolling contact fatigue tests realized on author´s department [3].

1 INTRODUCTION

Rolling contact is in practice relatively common phenomenon, no wonder, that this phenomenon is investigated worldwide. Some authors have in recent years published papers, which are focused on evolution of surface and subsurface ratcheting or wear due to rolling contact. This problem can be to a certain degree solved by numerical methods, but the main disadvantage is their computational demand, related to impossibility of simulating thousands or millions of cycles. On the other side, analytical or semi-analytical methods are giving us a possibility of simulating a high number of cycles.

Wear and rolling contact fatigue are one of the most common damage phenomena, which are occurring on components subjected to cyclic and repeated loading. On the basis of realized twin-disk experiments on author´s department [3], there was a possibility to explore the correlation between the parameter σ_L (yield stress) and maximum contact pressure for the case of line contact, concretely for maximum pressure values of 800, 1000 and 1200 MPa. For these purposes, there were utilized semi-analytical model of A. Mazzu [1] and inverse algorithm [4, 5].

2 WEAR ASSESSMENT WITH AIM OF MAZZU APPROACH

Mazzu´s approach [1] is based on non-linear kinematic hardening rule, introduced by Lemaitre and Chaboche [2] and is subsequently used for ratcheting prediction under contact surface in case of rolling/sliding two-dimensional contact. The basis of the model is the assumption that τ_{xz} is the only stress component, which is responsible for ratcheting and plastic flow in contact.

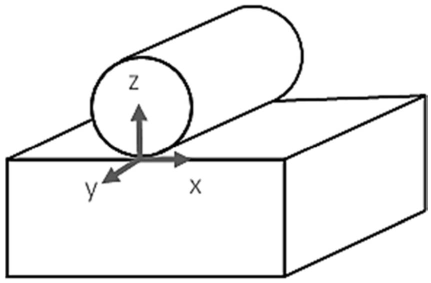


Figure 1: Coordinate system

This hypothesis is, according to [1], reasonable in the material layer of subsurface flow, where the kinematic hardening process is mainly govern by shear stress $\tau_{xz,}$ referring to the coordinate system presented in Fig.1. The reason is in the variation of the shear stress component τ_{xz}, which varies within larger range compared to other deviatoric stress components whatever the friction coefficient is. In the Mazzu´s model [1], elastic domain can be expressed in the following way:

$$F = \left|\sqrt{3} \cdot \tau_{xz} - X_{xz}\right| - \left(R + \sigma_L\right) \leq 0 \tag{1}$$

where:
σ_L – initial tensile yield stress [MPa],
X_{xz} – back-stress expressing kinematic hardening [MPa],
R – isotropic hardening variable [MPa]

Occurrence of plastic strain will induce variation in the isotropic and kinematic hardening term respectively. The back-stress variation principle is given by following equation:

$$dX_{xz} = C \cdot \frac{d\gamma_{xz}}{\sqrt{3}} - \gamma \cdot X_{xz} \cdot \left|\frac{d\gamma_{xz}}{\sqrt{3}}\right| \tag{2}$$

where:
C – material parameter [MPa],
γ – material parameter [-]

The isotropic term variation is expressed by:

$$R = R_{\infty 1} \cdot \left[1 - \exp\left(-b_1 \cdot \frac{\gamma_{xz}}{\sqrt{3}}\right)\right] + R_{\infty 2} \cdot \left[1 - \exp\left(-b_2 \cdot \frac{\gamma_{xz}}{\sqrt{3}}\right)\right] \tag{3}$$

where:
$R_{\infty 1}$, $R_{\infty 2}$ – material parameters [MPa], which states the limit value of R: $R_\infty = R_{\infty 1} + R_{\infty 2}$
b_1, b_2 – material parameters [-]

In general, R_∞ will be greater than zero if material hardens and less than zero if material softens. If the plastic flow if sufficiently large, the isotropic term R tends asymptotically to R_∞ and therefore can be considered constant. We can get the solution in closed-form for the back-stress variable by integration of equation (2), thus:

$$X_{xz} = \nu \cdot \frac{C}{\gamma} + \left(X_{xz}^{0} - \nu \cdot \frac{C}{\gamma} \right) \exp\left[-\frac{\nu \cdot \gamma}{\sqrt{3}} \cdot \left(\gamma_{xz} - \gamma_{xz}^{0} \right) \right] \tag{4}$$

from where we can finally derive relation for plastic shear strain increment:

$$\Delta\gamma_{xz} = -\frac{\sqrt{3}}{\nu \cdot \gamma} \cdot \ln\left[\frac{X_{xz} - \nu \cdot (C/\gamma)}{X_{xz}^{0} - \nu \cdot (C/\gamma)} \right] \tag{5}$$

where:
ν – multiplier, which is defined as ν=1 for loading and ν=-1 for unloading
X^{0}_{xz} – initial value of backstress variable [MPa],
γ^{0}_{xz} – initial value of plastic shear strain component [-]

Finally, from equation (1) can be expressed relation for shear stress as a function of back-stress and isotropic variable, respectively:

$$\tau_{xz} = \frac{1}{\sqrt{3}} \cdot \left[X_{xz} + \nu \cdot (R + \sigma_L) \right] \tag{6}$$

2.1 Shear band cracking mechanism

Shear band cracking mechanism, published by Donzella and Mazzu [6], describes wear as an independent simultaneous phenomenon, which is progressively removing material layers from surface and influences plastic strain accumulation under the contact surface. If is in particular depth reached the critical strain, then all material layers above will be considered as removed material, see Fig.2.

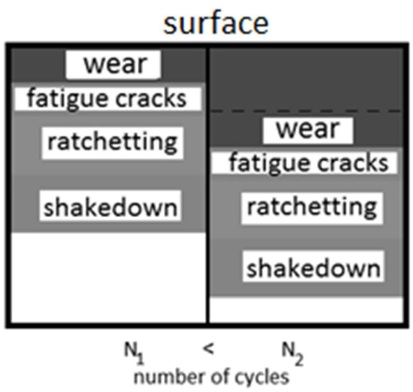


Figure 2: Shear band cracking mechanism

According to [6] there are four layers in material subjected to contact loading: wear layer, layer with fatigue cracks, layer with ratcheting presence and layer with shakedown presence. These layers are moving towards the depth and thus simulating the wear process.

3 INVERSE ALGORITHM FOR MATERIAL PARAMETERS IDENTIFICATION

Identification process of material parameters introduced by Donzella [6] is time and financially challenging. Therefore authors of presented paper were trying to find more effective procedure, which will also utilize experimental dates gained so far.

In our case, the identification of material parameters for finding the correlation between the maximum contact pressure and yield stress has been realized with aim of inverse algorithm. The algorithm is due to the nature of the task incorporating the random number generator. In each step there is, according to initial settings of the algorithm, generated a set of material parameters, which is then used for calculating the worn layer after specified number of cycles. The user can specify the number of reference steps on which has to be calculated worn layer after specified number of cycles. There are usually used three or four reference values, gained from proven linear relationship between diameter loss and realized cycles [6] from the experiment.

According to input parameters such as initial material parameters, defined scatter of material parameters, selected number of reference values of diameter loss with corresponding number of cycles for each cycle, there is made specific number of computations per one cycle, which is dependent on chosen number of reference steps. In each reference step, the program is calculating diameter loss after reference number of cycles and we are also taking into account the number of cycles, if the reference diameter loss has been reached earlier than after reference number of cycles.

After each cycle, which consists of prescribed number of reference steps, the error value is computed and if it is smaller than its previous value, the generated parameters are stored. The error value is computed by following way:

$$error^i = \frac{1}{3} \cdot \sum_{k=1}^{4} \left(2 \cdot \frac{\left| N_{ref\ k} - N^k \right|}{N_{ref\ k}} + \frac{\left| h_{Wref\ k} - h_W{}^k \right|}{h_{Wref\ k}} \right), \tag{7}$$

where:
$N_{ref\,k}$ – reference number of cycles of k-th step [-]
$h_{Wref\,k}$ – reference value of diameter loss for k-th step [mm]
N^k – number of cycles, needed for reaching $h_{W\,k}$ [-]
$h_W{}^k$ – diameter loss after for $N_{ref\,k}$ number of cycles [mm]

It is worth to mention, that through the weight of two is taken greater emphasis on the algorithm´s calculated number of cycles to achieve N_k, which is needed to achieve diameter loss $h_W{}^k$. The Fig.3 simply shows flowchart of inverse algorithm.

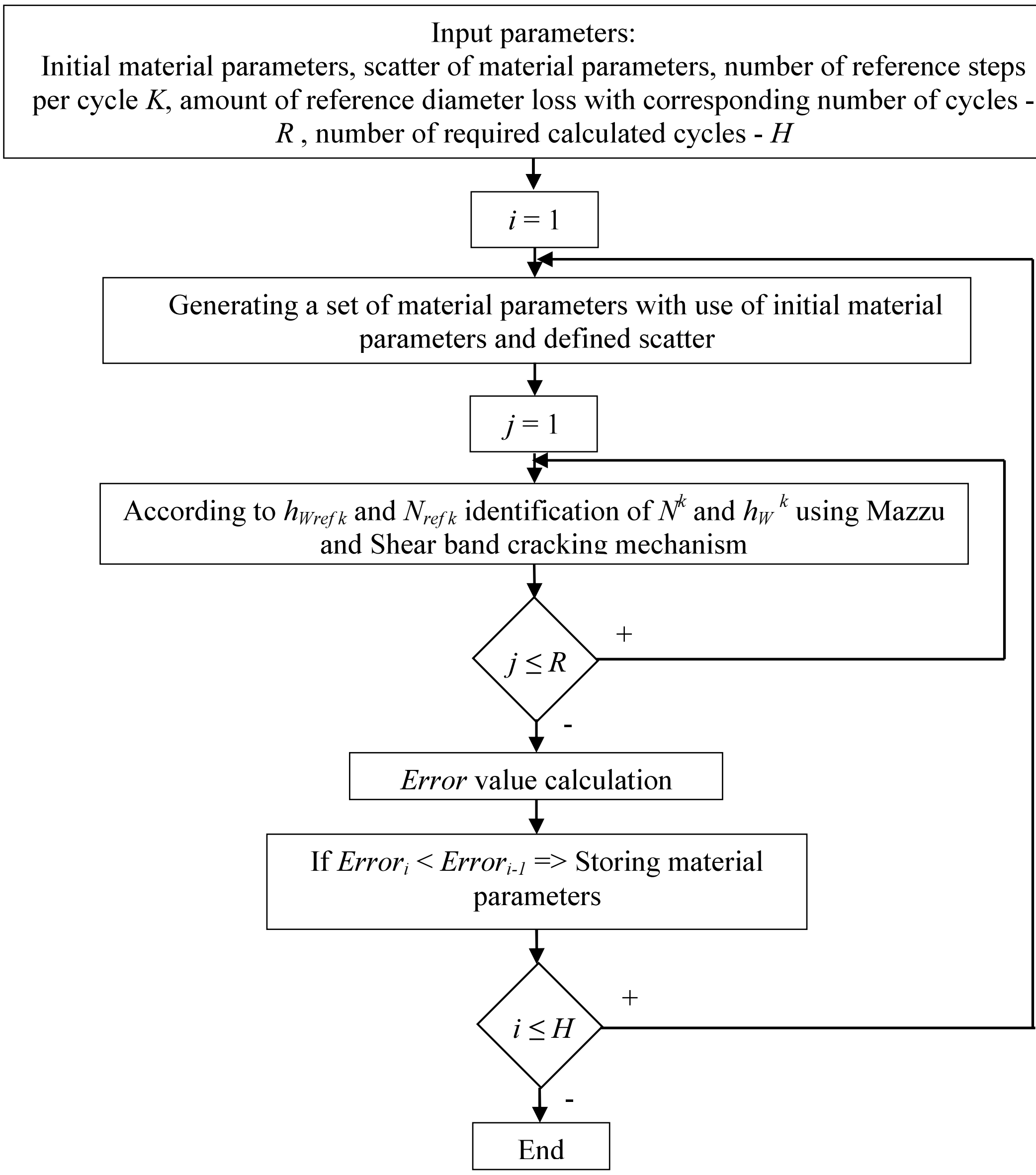


Figure 3: Flowchart of inverse algorithm

4 PERFORMED COMPUTATIONAL STUDY

The main aim was to analyze correlation between the parameter σ_L (yield stress) and maximum contact pressure with aim of Mazzu/Shear band cracking algorithm and inverse algorithm. Experimental data came from three realized twin-disc experiments with R7T wheel material. The experimental conditions are stated in table 1.

Table 1: Summary of parameters of realized experiments with R7T wheel material

Experiment no.	Maximum contact pressure p_0 [MPa]	Average value of friction coefficient f [-]	Slip ratio s [%]	Test duration/Number of cycles N [-]
1	800	0,24	0,75	100000
2	1000	0,35	0,75	100000
3	1200	0,33	0,75	100000

On the basis of proven linear relationship between wear and realized cycles [5], authors were allowed to apply the linear extrapolation in order to get the wear data for cycles in range from 250000 to 600000. Reason of choise the reference cycles in mentioned range consisted in detected stabilized deformation profile under the contact surface for stated conditions under which the experiments were carried out. The table 2 contains reference values of diameter loss with corresponding number of cycles, gained with help of linear extrapolation from performed experiments.

Table 2: Reference values of diameter loss with corresponding number of cycles for performed computations

p_0 = 800 MPa	$h_{Wref\ 1}$ [mm]	0, 2	$N_{ref\ 1}$ [-]	250000
	$h_{Wref\ 2}$ [mm]	0,36	$N_{ref\ 2}$ [-]	450000
	$h_{Wref\ 3}$ [mm]	0,48	$N_{ref\ 3}$ [-]	600000
p_0 = 1000 MPa	$h_{Wref\ 1}$ [mm]	0, 25	$N_{ref\ 1}$ [-]	235000
	$h_{Wref\ 2}$ [mm]	0,45	$N_{ref\ 2}$ [-]	420000
	$h_{Wref\ 3}$ [mm]	0,6	$N_{ref\ 3}$ [-]	560000
p_0 = 1200 MPa	$h_{Wref\ 1}$ [mm]	0, 25	$N_{ref\ 1}$ [-]	225000
	$h_{Wref\ 2}$ [mm]	0,45	$N_{ref\ 2}$ [-]	400000
	$h_{Wref\ 3}$ [mm]	0,61	$N_{ref\ 3}$ [-]	540000

In the first step, there have been identified material parameters for experiment no. 1, i.e. for maximum contact pressure 800 MPa. Following tables show us initial and optimized material parameters for numerical calculation with p_0 = 800 MPa as well as output data (Table 4).

Table 3: Initial and optimized material parameters for experiment no. 1(p_0 = 800 MPa)

	Initial material parameters	Optimized material parameters
C_0 [MPa]	150000	174636
γ_0 [-]	8	5,54
σ_{L0} [MPa]	150	172
$R_{\infty 10}$ [MPa]	60	98
b_{10} [-]	0,03	0,0129
$R_{\infty 20}$ [MPa]	30	47
b_{20} [-]	0,0003	0,0005

Table 4: Output data obtained from inverse algorithm for experiment no. 1: (p_0 = 800 MPa)

$h_{Wref\ 1}$ [mm]	0, 2	$h_W^{\ 1}$[mm]	0,2	$N_{ref\ 1}$ [-]	250000	$N^{\ 1}$[-]	236477
$h_{Wref\ 2}$ [mm]	0,36	$h_W^{\ 2}$ [mm]	0,36	$N_{ref\ 2}$ [-]	450000	$N^{\ 2}$[-]	387633
$h_{Wref\ 3}$ [mm]	0,48	$h_W^{\ 3}$ [mm]	0,48	$N_{ref\ 3}$ [-]	600000	$N^{\ 3}$[-]	529633
		error [-]			0,2066		

In the next two computations, concretely for p_0 = 1000 MPa and p_0 = 1200 MPa, the amount of optimized parameters has been restricted to only one parameter, which was the yield stress σ_L. Despite the optimization of only one parameter, concretely σ_L, there has been still achieved relatively small error value. Table 5 and 6 below contain gained results and also output data from inverse algorithm. Graph 1 shows us the dependance of yield stress on maximum contact pressure.

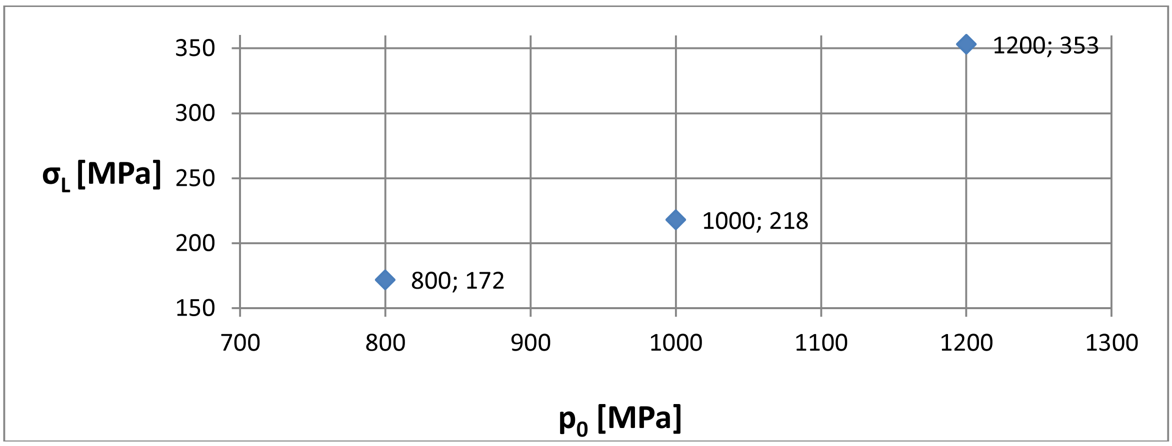
Table 5: Output data obtained from inverse algorithm for experiment no. 2 (p_0 = 1000 MPa)

Yield stress σ_L [MPa]		Initial value			Optimized value		
		180			218		
$h_{Wref\ 1}$ [mm]	0, 25	$h_W^{\ 1}$[mm]	0, 25	$N_{ref\ 1}$ [-]	235000	$N^{\ 1}$[-]	223214
$h_{Wref\ 2}$ [mm]	0,45	$h_W^{\ 2}$ [mm]	0,45	$N_{ref\ 2}$ [-]	420000	$N^{\ 2}$[-]	385344
$h_{Wref\ 3}$ [mm]	0,6	$h_W^{\ 3}$ [mm]	0,6	$N_{ref\ 3}$ [-]	560000	$N^{\ 3}$[-]	506897
		error [-]			0,1516		

Table 6: Output data obtained from inverse algorithm for experiment no. 3 (p_0 = 1200 MPa)

Yield stress σ_L [MPa]		Initial value			Optimized value		
		305			353		
$h_{Wref\ 1}$ [mm]	0, 25	$h_W^{\ 1}$[mm]	0, 25	$N_{ref\ 1}$ [-]	225000	$N^{\ 1}$[-]	170681
$h_{Wref\ 2}$ [mm]	0,45	$h_W^{\ 2}$ [mm]	0,45	$N_{ref\ 2}$ [-]	400000	$N^{\ 2}$[-]	303078
$h_{Wref\ 3}$ [mm]	0,61	$h_W^{\ 3}$ [mm]	0,61	$N_{ref\ 3}$ [-]	540000	$N^{\ 3}$[-]	409696
		error [-]			0,4833		

Graph 1: Dependance of yield stress on maximum contact pressure



CONCLUSION

A semi-analytical approach, introduced by A. Mazzu [1], has been used to assess the correlation between the yield stress and the value of maximum contact pressure for the case of line contact. It was presented in the previous work [7], that the cyclic plasticity model including simple Armstrong-Frederick kinematic hardening rule can correctly describe the ratcheting behavior of the wheel steel. On the other hand, such simple constitutive model cannot capture correctly cyclic strain curve of the material. Based on incremental theory of plasticity [2], it is necessary to incorporate a memory surface to the constitutive model to better describe the stress-strain behaviour of the material. Results of performed computations show, that the parameter σ_L corresponding to yield stress have to increase in Mazzu approach with increasing the Herzian contact pressure, which corresponds to level of loading of a material point. The memory surface seems to be the simplest way to improve the models capability, while preserving the coupling of ratcheting and wear in the calculations.

Acknowledgment

This chapter has been elaborated in the framework of the IT4Innovations Centre of Excellence project, reg. no. CZ.1.05/1.1.00/02.0070 supported by Operational Programme 'Research and Development for Innovations' funded by Structural Funds of the European Union and state budget of the Czech Republic. Authors would like to appreciate the assistance of BONATRANS GROUP a.s. Bohumín company during the preparation and realization of experiments.

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