

NEPCE - A NEARLY ENERGY-PRESERVING COUPLING ELEMENT FOR WEAK-COUPLED PROBLEMS AND CO-SIMULATIONS

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Abstract. Application of the weak-coupled or non-iterative co-simulation approach is very popular in industry because of its simple synchronisation scheme of the involved subsystems, even though there are some significant drawbacks. The main problems are obvious: On the one hand the explicit character of this coupling strategy may lead to instability of the overall system solution and on the other hand the selection of an appropriate coupling step-size is challenging. Investigation of the weak-coupling effects is necessary to improve the performance as well as the usability of this coupling approach. Concerning non-iterative co-simulation, subsystems are solved once over coupling time steps and in the general case of bidirectional dependencies between subsystems, several coupling quantities are unknown. This requires some kind of extrapolation, leading to an explicit character of the weak coupling approach. If small coupling step sizes have to be applied, this strategy leads to a significant increase of overall simulation time.

In this work we analyse the coupling process as a standalone subsystem and on this basis we present a novel coupling strategy. From an energy flow point of view this strategy considers the transmission of generalised energy between coupled subsystems and ensures (nearly) energy conservation in the coupled overall system. Thus, this modified coupling-process is referred to as nearly energy-preserving coupling element (NEPCE). Besides the fundamental theory some examples demonstrate the improved performance of the weak coupling scheme leading to more accurate simulation results and to a reduced amount of simulation time as well.

1 INTRODUCTION

In the engineering and development process of large-scaled dynamic systems virtual prototyping is mandatory to avoid expensive re-designs in late design phases. In addition, often cross-domain (multiphysic) considerations are necessary to improve the system performance and to satisfy increasing customer needs. To handle the resulting complexity the system under investigation is typically splitted into subsystems [1, 5]. Those domain-specific system parts are analysed by domain specific engineers. Therefore, well-established simulation tools targeting specific requirements are used to cover the physical behavior of the separated subsystems. Each subsystem is modelled within a specific domain by an individual model description language and solved using a tailored solver. In consequence, to estimate the entire overall system behavior, a holistic system analysis is required. Besides the independent simulation of the separated subsystems also the interaction between the individual subsystems have to be considered [2].

In fact, there are two possibilities: Integration of all subsystem models into an overall system model using a single simulation tool or integration of the distributed modelled as well as simulated subsystems via a co-simulation approach [1]. The first mentioned approach may not be applicable, due to the lack of available model-export or -import functionalities, the lack of domain-specific modelling language characteristics or required numerical solvers. In contrast, the mentioned co-simulation approach seems to be suitable for a modular holistic system simulation. The subsystems are modelled using domain-specific modelling languages and are solved by tailored numerical solvers (multi-method) with individual problem-specific step-sizes (multi-rate). But as a disadvantageous fact, the couplings in between interacting subsystems have to be established without influencing the solution of the overall system behavior.

Concerning co-simulation, subsystems are solved independently over pre-defined time-intervals, the so called macro-time-steps, by the implemented numerical solvers. In the general case of bidirectional interacting subsystems at least two subsystems depend on each other. To solve co-simulation network internal loops iterative or non-iterative coupling schemes exist [5]. As shown in [1, 5], well-established iterative (implicit) coupling methodologies can solve network internal loops in an efficient way. Unfortunately they are not applicable to all problems, because of limited interfacing capabilities of the used simulation tools. For example, at each iteration, resetting of the involved subsystems as well as a reconfiguration of the simulation tools is mandatory. Furthermore, for fast simulations and for (soft) real-time applications iterative methodologies are not well suited because of the non-deterministic timing behavior. In contrast, by using the non-iterative (explicit) strategy the involved simulation tools are paused to perform a data exchange. Thus, this coupling methodology is applicable in general for co-simulation applications. As a disadvantageous fact, the non-iterative strategy requires extrapolation of unknown coupling quantities and thereby some kind of distortion, i. e. a coupling error, is introduced [2].

This coupling error depends strongly on the macro-step-size, smaller macro-step-sizes imply smaller errors, but they are not applicable in general [1, 5]. Thus, to mitigate the unintentional introduced distortion, a modification of the chosen extrapolation technique is necessary. Besides overall energy consistency further requirements on a compensation scheme are: Minor computational effort for fast applications and flexible adaptivity for on-line adjustments due to an adaptive control of the macro-step-size, resulting in an efficient easy-to-use coupling strategy for non-iterative heterogeneous co-simulation.

This article is organized as follows: In the Section 2 the non-iterative principle is explained. Based on a commonly used coupling scheme the occurring main problems are extracted. Therefore, a so called *nearly energy-preserving coupling element* (NEPCE) is introduced in Section 3. From an energy flow point of view a strategy to mitigate inherent coupling errors is proposed. Section 4 shows how the developed compensation scheme works. A realistic co-simulation example describing a hybrid electric vehicle is used to demonstrate the efficiency of the improved coupling element.

2 AN ARTIFICIAL SUBSYSTEM - THE COUPLING ELEMENT

In general, the subsystems of a co-simulation are solved independently for each macro-time-step. The subsystems are solved using the chosen domain-specific numerical solvers with problem-specific step-sizes. In the context of co-simulation, the steps of the numerical solvers are referred to as micro-time-steps δT . Therefore, in the general case of variable step size solvers, the L -th numerical evaluation of a subsystem is performed at the time instant $t_\delta^{<L>}$

$$t_\delta^{<L>} = \sum_{l=1}^L \delta T^{<l>}, \quad (1)$$

where $\delta T^{<l>}$ denotes the size of the l -th micro-time-step. Simultaneously, the same notation can be introduced to express the M -th coupling time instant $t_\Delta^{<M>}$:

$$t_\Delta^{<M>} = \sum_{m=1}^M \Delta T^{<m>} \quad (2)$$

The m -th macro-time-step is denoted by $\Delta T^{<m>}$ [2]. Subsystems are independently solved for each macro-time-step and at each coupling time instant coupling data is exchanged. If subsystems are interconnected in a closed loop (internal loop) this approach is not directly applicable, because of unknown coupling quantities. In fact, if bidirectional dependencies in between subsystems exist several inputs of the subsystems for the computation of the actual macro-time-step are unknown. For the non-iterative coupling scheme the unknown coupling quantities are extrapolated based on past coupling information [3, 5]. Synchronisation of the subsystems are only performed at the coupling time instants which leads to a *weak* coupling of subsystems. Thus, this coupling approach is referred to as weak coupling scheme.

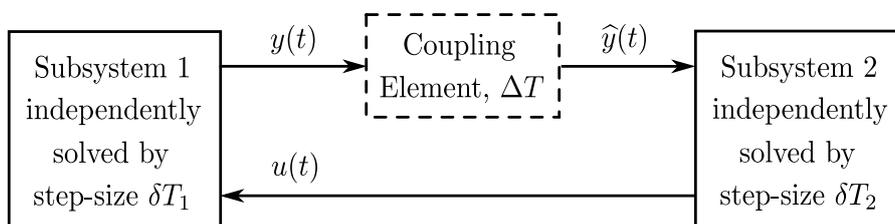


Figure 1: Sequential coupling of two subsystems by coupling step-size ΔT

To demonstrate the fundamental problem two interacting subsystems are sketched in Figure 1. Both subsystems directly depend on each other and thus, extrapolation of the required inputs (coupling quantities) is necessary. If Subsystem 2 is solved previous to Subsystem 1 - sequential scheduled - the input signal $y(t)$ of Subsystem 2 has to be estimated by $\hat{y}(t)$ through an extrapolation for the time interval of each macro-time-step $t \in (t_{\Delta}^{<M-1>}, t_{\Delta}^{<M>}]$, see Figure 2. The required extrapolation process influences the entire system behavior and can be considered as an additional artificially introduced subsystem. In this work the extrapolation process is referred to as *coupling element*.

As shown in Figure 2, based on past coupling data $y(t_{\delta}^{<L>})$ unknown coupling signals $\hat{y}(t)$ are extrapolated. Therefore, interpretation of the coupling data $y(t_{\delta}^{<L>})$ as discrete time function values of a continuous coupling signal $y(t)$ enables flexible handling of different step-sizes. Macro-step-sizes $\Delta T^{<m>}$ are completely decoupled according to fixed or variable micro-step-sizes $\delta T^{<l>}$ of the underlying numerical solvers. For extrapolation purposes sampling of the continuous sampling signal at coupling time instants is required and thus *aliasing*-effects may be introduced [3].

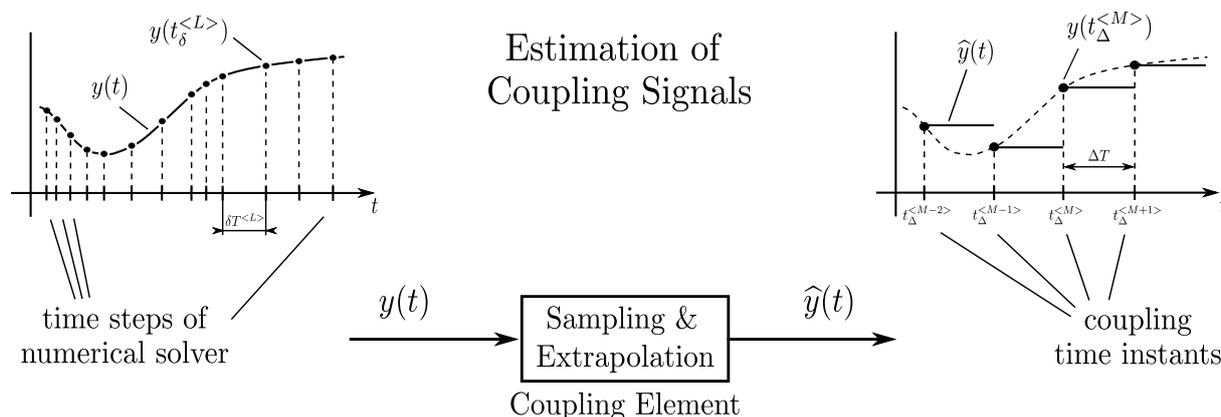


Figure 2: Estimation of coupling quantities at coupling time instants using a coupling element

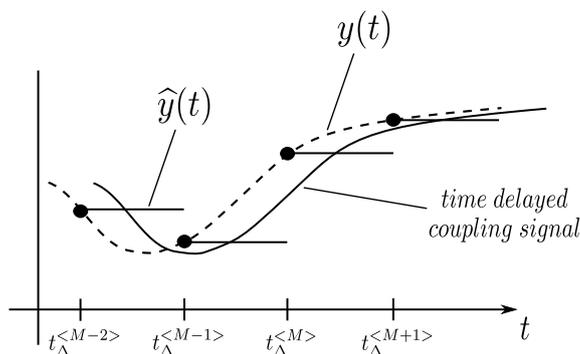


Figure 3: Zero-order extrapolation at coupling time instants and resulting time delayed coupling signal

2.1 Sample & Hold

Typically, in co-simulation applications extrapolation is performed signal-based via polynomial extrapolation techniques of low order [1]. The most commonly used extrapolation approach is the zero-order extrapolation scheme: Unknown inputs are treated as constant quantities over the interval of the actual constant macro-time-step¹ ΔT equal the value of the coupling signal $y(t_{\Delta}^{<M>})$ at the actual coupling time instant $t_{\Delta}^{<M>}$:

$$\hat{y}(t) = y(t_{\Delta}^{<M>}) \quad \text{with} \quad t_{\Delta}^{<M>} \leq t < t_{\Delta}^{<M+1>} \quad (3)$$

This extrapolation scheme results in a continuous time signal $\hat{y}(t)$ describing a piecewise constant function. Utilising the LAPLACE transformation and appropriate scaling regarding required sampling of the coupling signal leads to a transfer function $H(s)$

$$H(s) = \frac{\hat{y}(s)}{y(s)} = \frac{1 - e^{-s\Delta T}}{s\Delta T}, \quad (4)$$

which describes the behavior of the coupling element in the special case of zero-order extrapolation [3]. After substitution of the LAPLACE variable $s = j\omega$ and simple trigonometric transformations the frequency response of the coupling element may be written as follows:

$$H(j\omega) = \frac{2\pi \sin(\omega\Delta T/2)}{\pi\omega} e^{-j\omega\frac{\Delta T}{2}} = \underbrace{\overline{H}(\omega)}_{real} e^{-j\omega\frac{\Delta T}{2}} \quad (5)$$

As expected and represented by Equation (5), due to the zero-order extrapolation scheme a time-delay is introduced. In Figure 3, the effect of the simple coupling scheme is sketched. Given a continuous time coupling signal $y(t)$ a piece-wise constant function $\hat{y}(t)$ is generated by the coupling element. Obviously, discontinuities at coupling time instants occur and they represent high frequency components of the coupling signal. These

¹For the consecutive part of this article exclusively constant macro-step-sizes $\Delta T = \Delta T^{<m>} \forall m$ are considered for analysis.

discontinuities may influence the accuracy of the numerical solvers and may excite existing fast dynamics of the subsequent subsystem [3]. Assuming, that discontinuities do not influence the numerical accuracy and high frequency components in the coupling signal are suppressed by the subsystem, only a time-delayed coupling signal is relevant, see Figure 3.

This sample & hold extrapolation scheme limits the macro-step-size significantly when subsystems with high dynamics have to be considered and high accuracy is required. Especially in closed loop configurations, e. g. control systems, the artificially introduced time-delay influences significantly the dynamics of the coupled system and may lead to an unstable system behavior. As a consequence, small step-sizes have to be chosen resulting in a rapidly increasing overall simulation time.

3 A NEARLY ENERGY-PRESERVING COUPLING ELEMENT

The so called weak coupling scheme introduces an inherent coupling error due to the required extrapolation of unknown coupling quantities. Extrapolation is directly associated with an estimation error. From an energy flow consideration point of view, with each macro-time-step a *generalised* energy is produced or dissipated during modular simulation [2]. In this work *generalised* energy is considered, because in co-simulation applications two coupling signals ($u(t)$ and $y(t)$) may not represent power in the physical sense. Without coupling - performing a monolithical simulation - in the time interval of the m -th macro-time-step the generalised energy $E_{g,y}^{<m>}$ is transmitted:

$$E_{g,y}^{<m>} = \int_{t_{\Delta}^{<M-1>}}^{t_{\Delta}^{<M>}} u(t)y(t)dt. \quad (6)$$

As mentioned in Section 2, if bidirectional dependencies in between subsystems exist (Fig. 1), extrapolation of coupling signals is necessary. In this case the energy

$$E_{g,\hat{y}}^{<m>} = \int_{t_{\Delta}^{<M-1>}}^{t_{\Delta}^{<M>}} u(t)\hat{y}(t)dt \quad (7)$$

is transmitted, which leads to a deviation in the balance of transferred generalised energy:

$$E_{g,y}^{<m>} \neq E_{g,\hat{y}}^{<m>} \quad (8)$$

In the terminology of numerical simulation, the introduced coupling error represents a *local discretisation error* and is defined as

$$\epsilon(t) = y(t) - \hat{y}(t). \quad (9)$$

On the basis of (6), (7) and (9) the produced or dissipated amount of generalised energy $E_{g,\epsilon}^{<m>}$ in the time interval of the m -th macro-time-step can be determined:

$$E_{g,\epsilon}^{<m>} = E_{g,y}^{<m>} - E_{g,\hat{y}}^{<m>} \quad (10)$$

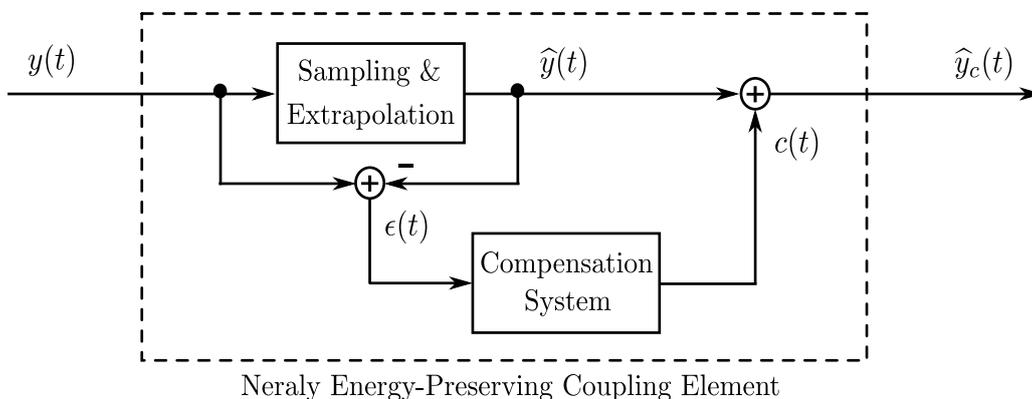


Figure 4: Introduced compensation system for coupling-error compensation

In contrast to the classical approach of adapting the step size of the numerical solver to meet predefined bounds on accuracy, this approach is not possible for non-iterative co-simulation. For such an approach resetting of the involved simulation tools is mandatory and not possible in general. Therefore, to compensate the coupling error, the extrapolated coupling signal $\hat{y}(t)$ could be modified by a correction signal $c(t)$ based on past coupling errors:

$$\hat{y}_c(t) = \hat{y}(t) + c(t) \quad (11)$$

Furthermore, to establish an energy-preserving (ideal) coupling the correction signal $c(t)$ has to satisfy the following relation:

$$E_{g,c}^{<m>} = \int_{t_{\Delta}^{<M-1>}}^{t_{\Delta}^{<M>}} u(t)c(t)dt \stackrel{!}{=} E_{g,\epsilon}^{<m>} \quad (12)$$

However, due to the non-iterative coupling scheme an exact compensation of the coupling error is not possible. As shown in Section 2, the coupling element introduces some kind of time-delay and for an ideal compensation a non-causal system - i. e. future coupling errors have to be known a priori - would be required. Because of this aspect it is not possible to realise an ideal correction signal. But the undisputed goal is to approximate an ideal compensation (12) and therefore a so called compensation system is introduced, see Figure 4. In the general case, this compensation system implements a control system with the aim of tracking the coupling error:

$$c(t) \stackrel{!}{=} \epsilon(t) \quad (13)$$

Based on past coupling errors (9) the compensation system determines a correction signal and the extrapolated coupling signal is modified (11). In fact, the compensation scheme leads to a coupling-error-based modification of the used extrapolation technique. This approach is rather general and applicable for different extrapolation schemes. Thus, for

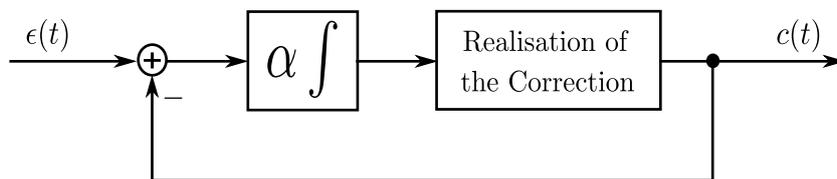


Figure 5: Feedback control compensation system

the class of nearly energy-preserving coupling elements on the basis of the proposed and fundamental considerations regarding generalised energy transmission, the abbreviation NEPCE is introduced [2].

3.1 Realisation of the correction signal

The proposed, nearly energy-preserving coupling scheme (NEPCE) modifies the extrapolated coupling signal by the use of a correction signal, which is determined based on past coupling data. Therefore, perhaps the most simplest choice for correction is the time-delayed coupling error signal (9):

$$c(t) = \epsilon(t - \Delta T^{<m>}) \quad (14)$$

Under the assumption of slowly varying coupling signals the coupling error of the previous step approximates the actual coupling error and therefore, this compensation approach is applicable to mitigate unintentional introduced distortions. But there are some problems concerning the realisation of the correction signal. First, in the case of time varying macro-step-sizes $\Delta T^{<m>} \neq \Delta T^{<m+1>}$ a variable time-delay has to be realised. In this case, it is difficult to interpret the resulting behavior of the compensation scheme. As a second aspect, the continuous time correction signal must be represented in form of discrete function values, i.e. a quasi-continuous approximation of the continuous time signal is necessary. In both cases many samples are required to represent the correction signal. Simulation tools may not be able to handle the increased amount of coupling data at coupling time instants because of limited interfacing capabilities.

A possible simplification is the consideration of the integrated coupling error over the time interval of the last macro-time-step. Under the already mentioned assumption² of slow varying coupling signals a piece-wise constant correction signal may be used to compensate the coupling error, which is defined by

$$c(t) := \frac{\alpha}{\Delta T^{<m+1>}} \int_{t_{\Delta}^{<m-1>}}^{t_{\Delta}^{<m>}} \epsilon(t) dt \quad \text{with} \quad t \in (t_{\Delta}^{<m>}, t_{\Delta}^{<m+1>}]. \quad (15)$$

This compensation scheme is illustrated in Figure 5. The coupling error is integrated and used to realise a piece-wise constant correction signal. Instead of resetting the integrator at

²By the use of sufficient small macro-step-sizes, this assumption is fulfilled for every continuous time coupling signal in general.

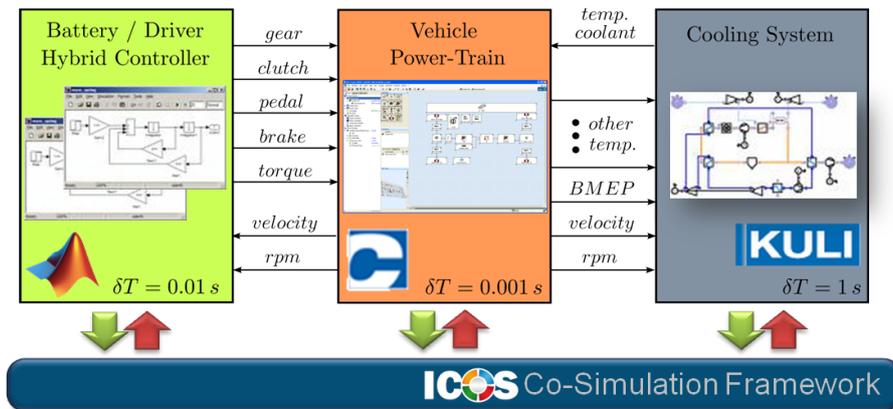


Figure 6: Co-simulation representation of the coupled subsystems

each coupling time instant a feedback path is introduced. Stability of the resulting control system as well as the performance of the proposed compensation system is adjusted by the tuning factor $\alpha \in [0, 2]$.

4 INDUSTRIAL EXAMPLE

For evaluation purposes a coupled system describing a hybrid electric vehicle (HEV) is considered. A hybrid electric vehicle simulation example is well suited for co-simulation because different engineering domains (electrical, mechanical, thermal) are involved. Typically, each subsystem is analysed using a specific simulation tool. For this example, a hybrid drive-train (AVL CRUISE, $\delta T = 0.001$ s) is coupled with a cooling subsystem (ECS KULI, $\delta T = 1.0$ s) and a third subsystem, which implements a virtual driver, a battery system for energy storage and a hybrid control unit (MATLAB, $\delta T = 0.01$ s). The co-simulation representation of the coupled system is sketched in Figure 6. Beside the use of problem-specific micro-step-sizes δT coupling of the three subsystems is performed using the co-simulation platform ICOS [6]. In particular, to establish the interactions in between the subsystems unknown coupling quantities are estimated by the zero-order-hold (ZOH) extrapolation scheme.

Three different non-iterative co-simulations were carried out and the results are shown in Figure 7. Because of a predefined sequential scheduling of the involved subsystems (the vehicle is solved first) the inputs to the vehicle are estimated. This leads to piece-wise constant signals of the *acceleration pedal* (upper plot) and a required *hybrid torque* (middle plot). In the lower plot, as not estimated output signal the vehicle velocity is illustrated. For this example it is not possible to determine a monolithical (standalone) simulation and thus, the results of the co-simulation according to $\Delta T = 0.01$ s are considered as a good numerical approximation of the exact solution. However, two co-simulations are performed with a macro-step-size of $\Delta T = 0.04$ s with and without application of the proposed compensation scheme. As expected, a significant coupling error is introduced

without compensation. Due to the bidirectional dependencies in between the subsystems a strong deviation of the entire system behavior is recognisable. Especially the deviation in the velocity of the vehicle lead to a different behavior of the implemented virtual driver (*acceleration pedal* and *brake*). In contrast, with the application of the proposed nearly energy-preserving coupling scheme (NEPCE) the simulation results are very close to the approximation of the exact solution ($\alpha = 1.5$).

Using the discussed coupling signals several definitions of generalised energy are possible. The results are demonstrated in Figure 8. The upper plot depicts the generalised energy $E_{g,pedal}(t)$ based on the *vehicle velocity* and the estimated *acceleration pedal* signal. In the lower plot, the generalised energy $E_{g,torque}(t)$ according to the *vehicle velocity* and the

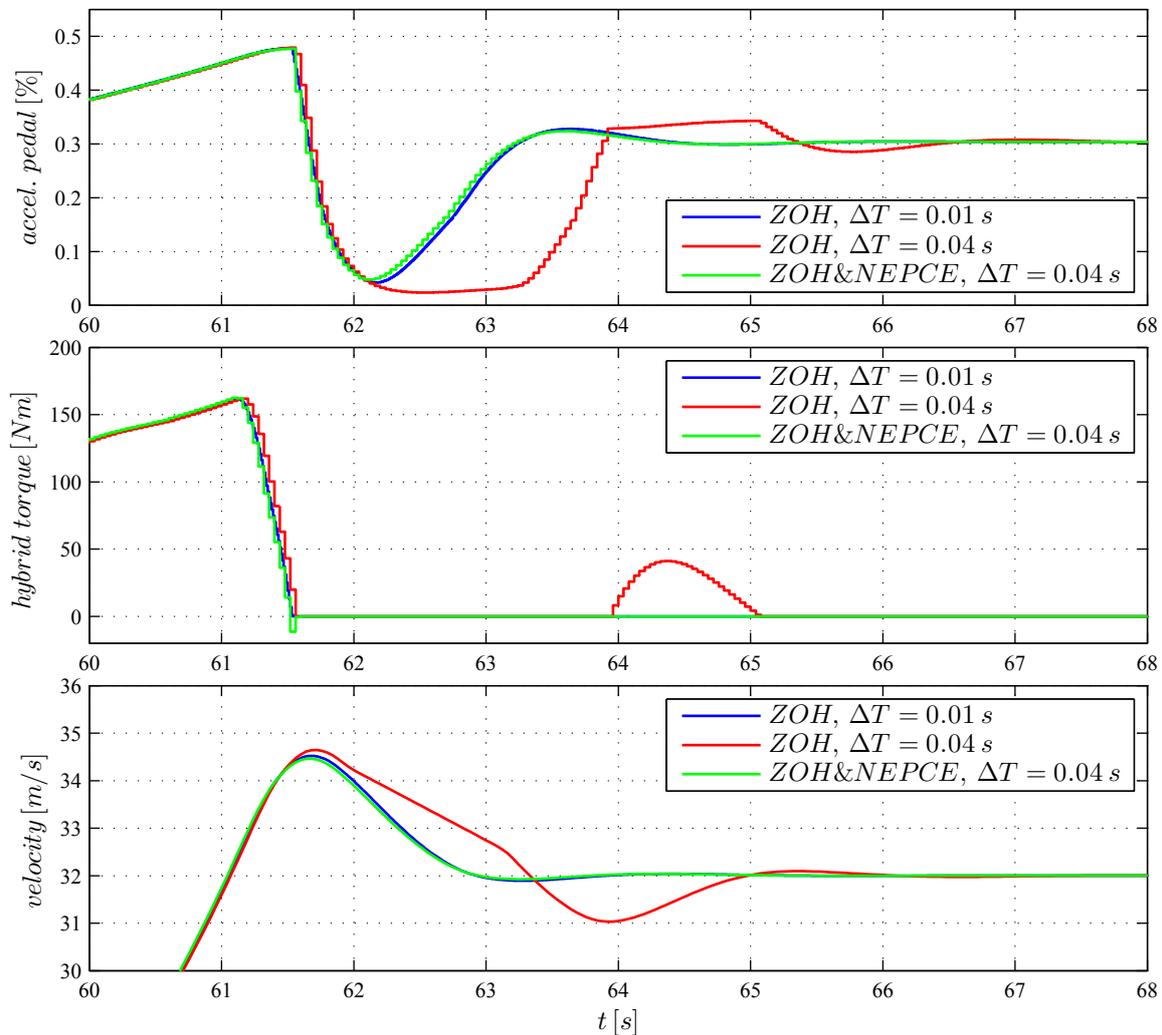


Figure 7: Co-simulation results for different zero-order-hold (ZOH) extrapolation schemes. Demonstration of the proposed correction scheme NEPCE on co-simulation results.

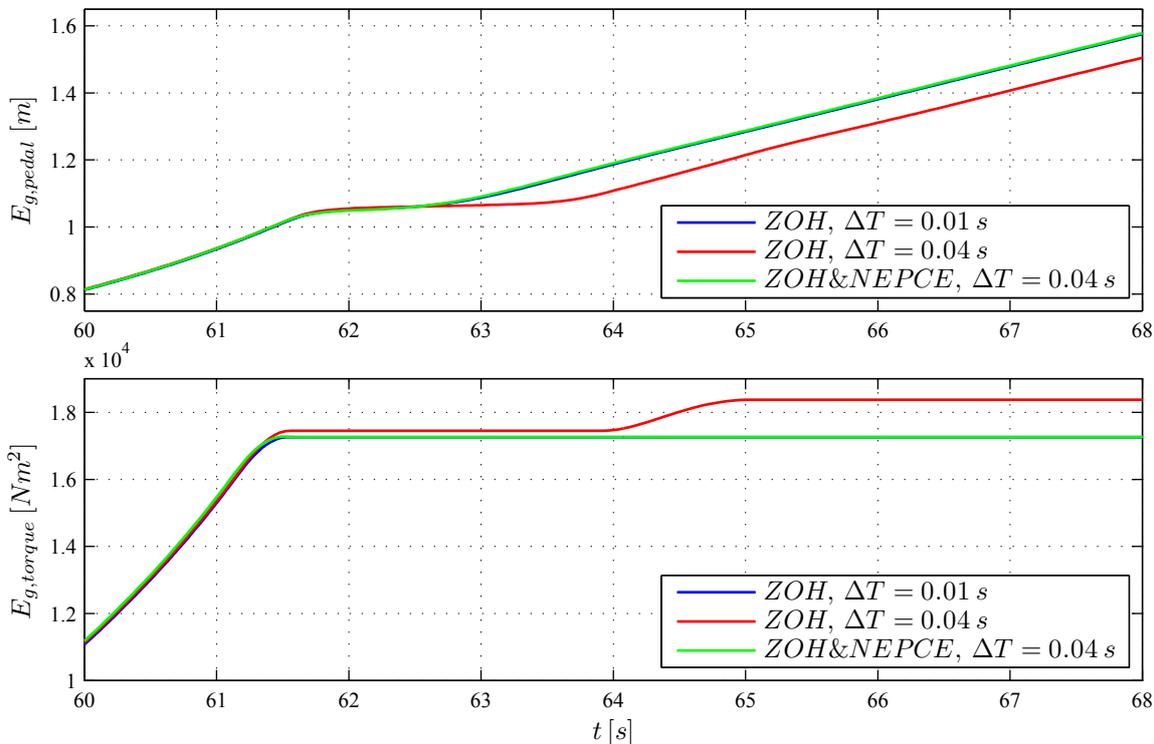


Figure 8: Generalised energy according to specific coupling signals

estimated *hybrid torque* signal is illustrated. Obviously, for both interpretations, without a compensation strategy generalised energy is lost or added due to the coupling process. This leads to significant deviations and a strongly differing numerical solution, compare to Figure 7. However, the proposed coupling scheme (NEPCE) compensates coupling errors and leads to highly accurate simulation results, even though non-iterative co-simulation.

5 CONCLUSIONS

The modular treatment of subsystems in terms of modelling and simulation enables the engineering of complex multiphysic systems and yields flexibility during the product development process. As a drawback, bidirectional interaction between interconnected subsystems cause some problems and thus, an unintentional coupling error is introduced. Modelling of the coupling process by a coupling element leads to an additional (artificial) subsystem. Beside possible aliasing effects and discontinuities (high frequency components) at coupling time instants a time-shift of the coupling signal occurs intuitively. For exact compensation a non-causal system is mandatory, which is not realisable in general. Thus, in this work we present a novel strategy to compensate immanent coupling errors. From an energy flow consideration point of view between interacting subsystems a (nearly) energy-preserving coupling element (NEPCE) is proposed. The improvement

concerning the classical coupling element is demonstrated by an industrial co-simulation example. The accuracy of the co-simulation results is significantly increased.

Remark: The presented theory describes a novel approach of error compensation for non-iterative co-simulation and weak coupled problems. Protected by a pending European patent (EP 2 442 248 A1) since 2010 this approach is implemented within the co-simulation platform ICOS "Independent Co-Simulation" [6], which is developed at the Virtual Vehicle Research Center.

6 ACKNOWLEDGEMENTS

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