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MACRO-STEP-SIZE SELECTION AND MONITORING OF THE COUPLING ERROR FOR WEAK COUPLED SUBSYSTEMS IN THE FREQUENCY-DOMAIN

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Abstract. A rather general approach to establish a multiphysic simulation is referred to as non-iterative co-simulation or weak coupled simulation. The involved subsystems are coupled in a weak sense and thus stepwise extrapolation of the coupling signals is required. Extrapolation is associated with an error which may influence the dynamical behavior of the coupled system. This coupling error depends significantly on the coupling step-size, i.e. the macro-step-size, and is one of the most critical parameter of a non-iterative co-simulation. In practice, appropriate macro-step-sizes are determined by some numerical tests or chosen according to the experience of domain-specific engineers. But who assesses the results of a non-iterative co-simulation? Comparison between reference and co-simulation results in the time-domain is often practiced in case studies using complex subsystems to validate co-simulation performance and accuracy of the achieved results. But this approach is counter-productive and mostly not applicable in practice. In this work we consider the coupling process as single source of distortion and analyse it in the frequency-domain. As a consequence, a relation between the macro-step-size and the coupling signals is available which leads to a 'rule-of-thumb' for a adequate macro-step-size selection. In addition, the gained insight into the coupling process itself enables new possibilities to monitor the coupling error leading to the ability to assess the results of a weak coupled simulation. The proposed methodologies are examined using a complex mechatronic system describing a vehicle (multi-body system, MBS), which is controlled via an anti-lock braking system (ABS) during different scenarios.

1 INTRODUCTION

Engineers are highly interested in multiphysic simulations because of the improved possibilities to design and/or to analyse a complex system. Typically, separated components of the overall system, i. e. subsystems, are treated by domain-specific teams of engineers to handle the resulting system complexity. For such an approach distributed modelling of the subsystems using specific modelling languages and distributed simulation using tailored solvers is mandatory. Advantageously, this approach leads to maximum flexibility for virtual system design. But nowadays, the performance of the overall system becomes more and more important and thus, the entire (dynamical) behavior of the overall system has to be considered [7]. During the development process performing of holistic simulations of the overall coupled system is required.
Since about two decades an increasing demand for holistic system simulation is recognisable and significant efforts in devising standards were done, for instance see Modelisar [3]. On the one hand a monolithical simulation approach exists and on the other hand, coupled modular simulation of the subsystems is possible [2]. The idea behind a monolithical simulation approach is to import all subsystems - eventually including the accompanying numerical solver - into a single simulation tool, e. q. Matlab/Simulink[1] or Dymola[2]. The main drawback is obvious: Not every used simulation tool exhibits model-export functionalities and thus, an additional (huge) effort for porting the submodel is required. Even though mandatory, specific numerical solvers might not be available in the used (master) simulator. By the latter mentioned approach the subsystems are modelled and solved by tailored simulation tools and the required interactions are established via connection of the corresponding inputs and outputs. This approach is also referred to as co-simulation.
Interfacing and external control of the involved domain-specific simulation tools is necessary and coupling data, i. e. relevant inputs and outputs, are exchanged at predefined points in time for synchronisation purposes [1, 4, 7]. As disadvantageous fact, limited interfacing capabilities of the used simulation tools avoid the application of highly accurate iterative (implicit) coupling schemes [2]. Mostly, the application of non-iterative (explicit) coupling schemes is required. By the non-iterative coupling approach the accuracy significantly depends on the chosen coupling step-size, the so called macro-step-size. Thus, the variable or fixed macro-step-size is the most critical parameter of a non-iterative co-simulation. In practice, for large systems appropriate macro-step-sizes are determined by numerical tests ('trial & error' approach) or chosen according to the experience of domain-specific engineers. Additionally, estimates on the numerical error are very pessimistic and these measures are not adequate for the practical assessment of simulation results [1]. Furthermore, the result of a non-iterative co-simulation is typically evaluated by comparison to the result of a standalone simulation, i. e. a simulation without coupling

[1]http://www.mathworks.com
[2]http://www.dymola.com

errors. But this approach is only applicable for *simple* coupled problems and mostly used in scientific co-simulation studies where the possibility to compute a solution for reference is given. In contrast, for complex (real) co-simulations it is not possible to perform a standalone simulation and thus, there is no possibility to assess results of a non-iterative co-simulation.
The error behavior and bounds on accuracy are important aspects for virtual-prototyping using non-iterative co-simulation. In this work we consider the non-iterative coupling scheme as single source of distortion. Instead of analysing the overall complex system, we focus on the coupling process itself. From considerations in the frequency-domain it is possible to determine the influence of the couplings on the coupling signals. Meaning, small distortions of the coupling quantities intuitively result in small numerical errors. Keeping this idea in mind, a reduction of coupling errors will lead to more accurate co-simulation results [4].
This article is devoted to show the capabilities of a frequency-based description of the coupling process and is organised as follows: In Section 2 a short outline of the basic principle of the non-iterative coupling scheme is given and the required coupling process is modelled by a so called coupling element. In Section 3, based on considerations of the coupling element in the frequency-domain, a 'rule-of-thumb' for a suitable selection of the macro-step-size is proposed. In Section 4, to assess the quality of the results of a non-iterative co-simulation, this approach is enhanced. Finally, in Section 5, the proposed methodologies are verified by an example describing a multi-body system (MBS), which is controlled via an anti-lock braking system (ABS) during different scenarios.

2 THE COUPLING SCHEME MODELLED AS COUPLING ELEMENT

Using co-simulation, subsystems are solved independently by their implemented domain-specific numerical solvers with problem-specific fixed or variable step-sizes, i. e. the so called micro-step-sizes δT. For synchronisation purposes the individual subsystems are paused at discrete time instants to perform a data exchange (weakly coupled). The time intervals between consecutive coupling time instants are referred to as macro-time-steps ΔT. In particular, at coupling time instants input quantities of the subsystems are updated according to simulation results of the corresponding outputs of the connected subsystems [1, 7].
In the general case of bidirectional dependencies in between subsystems several coupling quantities are unknown which have to be estimated by extrapolation based on past coupling data. Thereby, an estimation error, i. e. a coupling error, is introduced. Typically, polynomial extrapolation techniques are applied which are based on past coupling quantities. Commonly used strategies are the zero-order-hold (ZOH), the first-order-hold (FOH) and the the second-order-hold (SOH) extrapolation approaches [1, 5]. Because of the non-iterative coupling scheme, no iterations are performed at each macro-time-step which results in an explicit character of the coupling scheme. Assuming that the subsystems are adequately (accurate) solved by the implemented numerical solvers, the required cou-

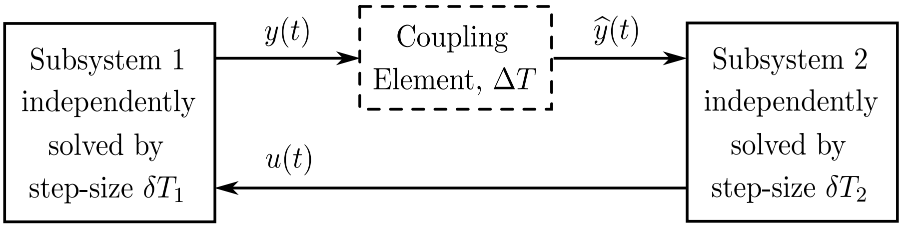

Figure 1: Sequential coupling of two subsystems and artificially introduced coupling element

pling yields as single source of distortion. From this abstract point of view, the progress of subsequent extrapolations of coupling quantities may be considered as an additional artificial introduced subsystem [4]. In this work, this subsystem is denoted as *coupling element*. Given a coupling signal $y(t)$, a piece-wise extrapolated function $\widehat{y}(t)$ is generated by the coupling element. Figure 1 depicts this abstraction. Two bidirectional coupled subsystems are illustrated. Because of a predefined *sequential scheduling* of the subsystems - Subsystem 2 is solved previous to Subsystem 1 - piece-wise extrapolation of the unknown coupling quantities $y(t)$ by $\widehat{y}(t)$ over the time interval of each macro-time-step ΔT is mandatory. The unintentional effect of the required extrapolation are represented by the artificially introduced coupling element. For co-simulation commonly used polynomial extrapolation techniques are modelled and analysed in [5]. In this context, the main coupling problems are possible aliasing effects due to sampling of the coupling signal, an unintentional introduced time-shift and discontinuities at coupling time instants. However, a coupling element represents the introduced coupling error and influences the entire behavior of the overall coupled system. Especially for linear and time invariant systems the analysis of the system behavior in the frequency-domain is possible. As an advantageous fact, the commonly used polynomial extrapolation techniques (ZOH, FOH and SOH) represent LTI-systems and may be analysed in the frequency domain by a transfer function $H(s)$:

$$H(s) = \frac{\widehat{y}(s)}{y(s)} \tag{1}$$

For instance, the transfer function describing the behavior of the zero-order-hold extrapolation (ZOH) is written as

$$H_{zoh}(s) = \frac{1 - e^{-s\Delta T}}{s\Delta T}, \tag{2}$$

where s denotes the *LAPLACE* variable. By this approach many different extrapolation schemes may by modelled and analysed in the frequency-domain. For the commonly used extrapolation techniques the corresponding transfer functions are outlined in [5]. The magnitude and the phase of the resulting frequency responses are illustrated in Figure 2 in dependence on the *normed* frequency $\omega\Delta T$. Obviously, over the illustrated interval of the normed frequency the zero-order-hold extrapolation technique leads to a significant frequency dependent phase-shift. This effect may be intuitively interpreted as a

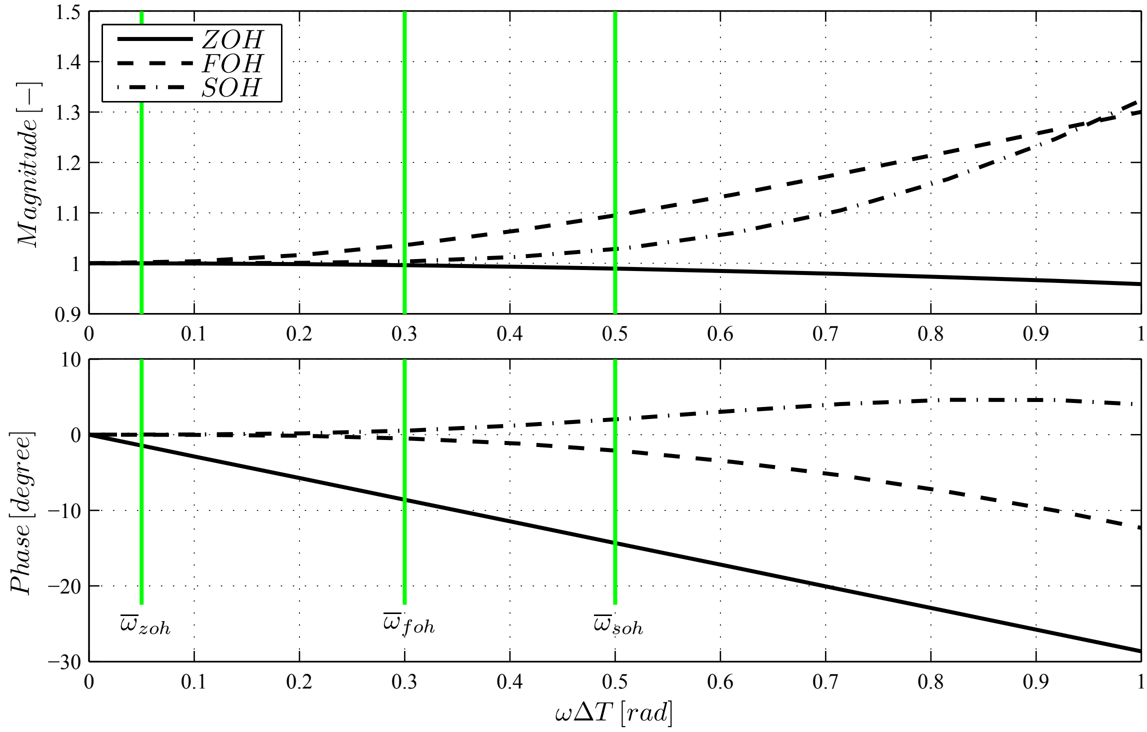

Figure 2: Magnitude and phase of coupling elements over the normed frequency $\omega\Delta T$ with depicted maximum normed bandwidths $\overline{\omega}_{zoh}$, $\overline{\omega}_{foh}$ and $\overline{\omega}_{soh}$ for ZOH, FOH and SOH extrapolation, respectively.

unintentional time-delay. Both higher order polynomial extrapolation techniques (FOH, SOH) exhibit a nearly ideal transfer behavior in the lower frequency range. In addition, an increasing normed frequency results in a significant modification of the amplitudes of the (harmonic) signal components of the coupling signal $y(t)$ and a frequency dependent phase-shift.

3 MACRO-STEP-SIZE SELECTION

Based on general considerations of the coupling elements in the frequency-domain a gained insight into the process of coupling is established. Concerning the magnitude and the phase characteristics of the modelled coupling elements, different coupling schemes may be assessed [5]. In fact, for ideal transfer behavior of the coupling element the following equation must be satisfied:

$$H(j\omega) = 1 + j0 \qquad \forall\omega \tag{3}$$

This ideal characteristic is impossible because of time-delays in the transfer functions, for example see (2). To compensate the introduced coupling error a non-causal system would be required which is not realisable [6]. But, by permitting small coupling errors, efficient normed bandwidths for the modelled coupling elements may be defined. As

Table 1: Heuristically defined maximum normed efficient bandwidth

Extrapolation	$\overline{\omega\Delta T}$
ZOH	0.05
FOH	0.3
SOH	0.5

illustrated in Figure 2, the maximum normed bandwidths denoted by $\overline{\omega}_{zoh}$, $\overline{\omega}_{foh}$ and $\overline{\omega}_{soh}$ are depicted for the three frequency responses of the ZOH, FOH and the SOH extrapolation scheme, respectively. The maximum normed bandwidths $\overline{\omega}$ are chosen according to the characteristics of the coupling elements with the focus on a negligible distortion of the coupling signal. For instance, a maximum amplification of the coupling signal of $|H(j\omega\Delta T)| < 1.03$ and a maximum phase-shift of $|\angle H(j\omega\Delta T)| < 3°$ seems to be adequate. Table 1 summarises the relations between the feasible efficient normed bandwidth of the coupling signal regarding the extrapolation techniques [5]. This table yields as 'rule-of-thumb' for macro-step-size selection and is determined heuristically based on considerations of the coupling elements in the frequency domain.

Of course, the definition of the efficient bandwidths also depends on the coupled system. Meaning, discontinuities at coupling time instants represent high frequency components in the coupling signal and may excite fast dynamics of the subsequent subsystem. Thus, the proposed 'rule-of-thumb' is valid for *nonstiff* coupled problems, if the coupling signals which correspond to the slow system dynamics are extrapolated.

4 VALIDATION OF THE COUPLING PERFORMANCE

Besides the selection of a suitable macro-step-size, also the quality of the coupling itself may be assessed [4]. Once again, this is enabled due to investigations of the modelled coupling elements in the frequency-domain. Based on the knowledge of the time varying maximum bandwidth of the coupling signals $\overline{\omega}(t)$ a relation for the evaluation of the couplings of a non-iterative co-simulation can be defined:

$$\overline{\omega}(t) < \frac{\overline{\omega\Delta T}}{\Delta T(t)} \tag{4}$$

In this inequality the macro-step-size depends on time ($\Delta T(t)$). This is necessary in general due to the application of (adaptive) macro-step-size control algorithms. Obviously, to proof relation (4) the extraction of information on frequency components of the coupling signal is required. This means, that co-simulation results can be qualitatively evaluated if the actual maximum bandwidth of the coupling signal may be extracted and an usable bandwidth of the corresponding coupling element is defined. Furthermore, a topological (location of the coupling element in the network of subsystems) as well as a temporal assignment of eventually occurring coupling errors according to (4) is enabled.

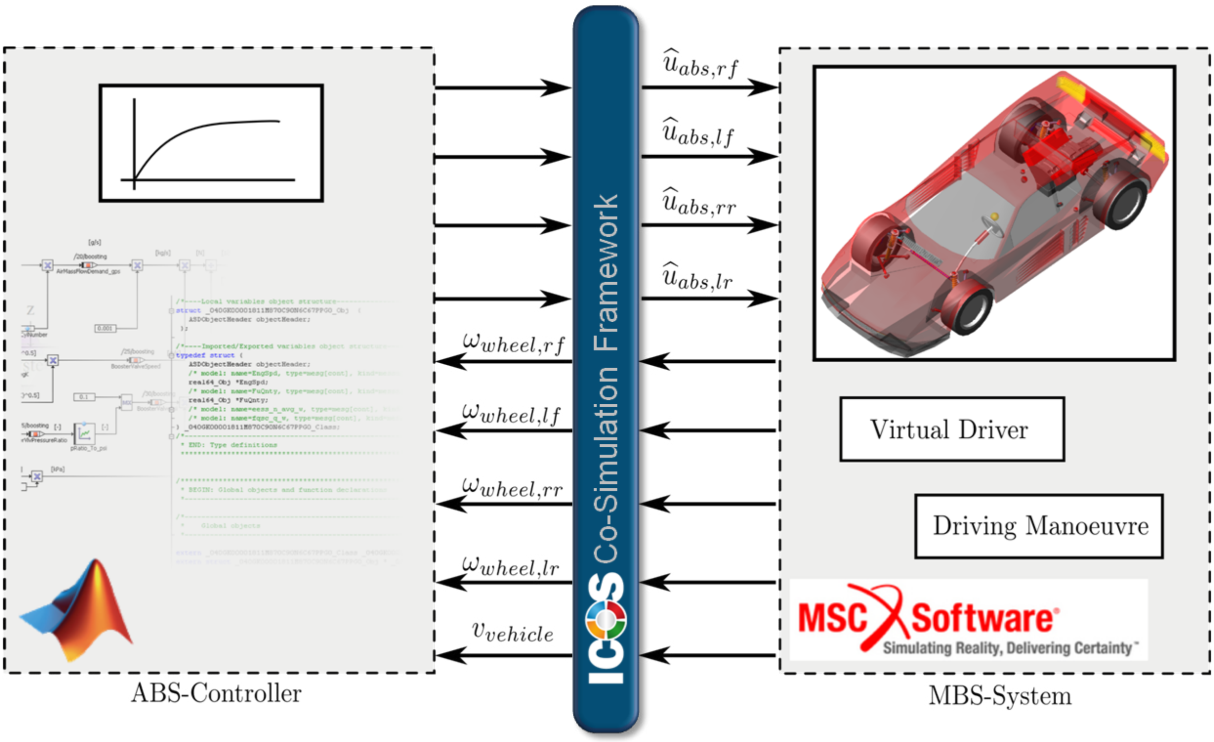

Figure 3: Co-simulation representation of the coupled subsystems

5 AN ANTILOCK BRAKING SYSTEM EXAMPLE

To evaluate the proposed approaches a rather complex multibody system designed for vehicle dynamic simulations is examined. Typically, simulations are carried out for different real world scenarios and predefined driving cycles, to analyse the dynamic vehicle behavior or to optimise the setup of the system. To enhance the engineering and analysis possibilities, components out of various domains are embedded into the multibody vehicle simulation leading to a multidisciplinary application [1]. Often, additional subsystems are modelled in other domain specific simulation tools. A huge effort is necessary to translate the subsystem into an appropriate modelling language or to embed the resulting code (using a export-functionality) into the whole vehicle simulation, using tailored interfacing capabilities. During the development process a separate translation has to be performed for each adaptation of the additional systems, which leads to a time consuming procedure and rapidly increasing development costs. In contrast, co-simulation approaches enable simultaneous modifications of the involved subsystems besides establishing the connection between different simulation tools.

5.1 Description of the coupled system

For the chosen evaluation example co-simulation is used to enhance a multibody vehicle model implemented in ADAMS/CAR[3] by an antilock braking system (ABS), which is realised using MATLAB/SIMULINK. Additionally, a virtual driver and the predefined driving manoeuvre are also configured in ADAMS/CAR. In particular, the virtual driver actuates the brake during a left-turn according to the lateral acceleration of the vehicle [4]. The co-simulation representation of the coupled system is sketched in Figure 3. The interactions in between the subsystems are established by the vehicle velocity $v_{vehicle}(t)$, the wheel speeds $\omega_{wheel,rf}(t)$, $\omega_{wheel,lf}(t)$, $\omega_{wheel,rr}(t)$, $\omega_{wheel,lr}(t)$ and the control signals $u_{abs,rf}(t)$, $u_{abs,lf}(t)$, $u_{abs,rl}(t)$, $u_{abs,lr}(t)$. Thereby, the subscripted characters (*l, r, f, r*) denote the topological position of the wheels mounted on the vehicle as *left*, *right*, *front* and *rear*, respectively. Both domain-specific subsystems (ABS-Controller, MBS-System) are coupled by the co-simulation platform ICOS[4] (Independent Co-Simulation), which satisfies efficient interfacing of the used simulation tools and handles the exchange of coupling data during co-simulation [8].
Beside the co-simulations a standalone simulation (a monolithical simulation) was carried out using a single *ADAMS/Car* model for evaluation purposes. In this case, the modelled controller subsystem was embedded into the *ADAMS/Car* subsystem using special model-export (real-time workshop from MATLAB) and model-import (general state equation in ADAMS/CAR) functionalities. Note, without the co-simulation approach, with every change of the ABS-Controller subsystem a model-export and model-import is required which results in a significant additional effort.

5.2 Non-iterative co-simulation

For this evaluation example three differently configured co-simulations are carried out. In all cases the subsystems are scheduled in sequential order. The multibody vehicle subsystem modelled in *ADAMS/Car* is solved previous to the ABS-Controller subsystem for each macro-time-step ΔT. This configuration requires some kind of piece-wise extrapolation of the inputs, i. e. the control signals, of the MBS-System, compare to Figure 1. For the three performed co-simulations the commonly used polynomial extrapolation techniques ZOH, FOH and SOH are used. Because of less information on the dynamics of the subsystems itself different constant macro-step-sizes are chosen for the extrapolation schemes. Further, according to the selected scenario (left-turn braking) only the control actions of the rear left wheel are illustrated in Figure 4.
Each plot compares the result of the monolithical simulation and the performed co-simulations using a specific extrapolation technique. The upper plot shows the application of the zero-order-hold (ZOH) extrapolation scheme. The results of the co-simulation

[3]http://www.mscsoftware.com
[4]http://www.v2c2.at/icos

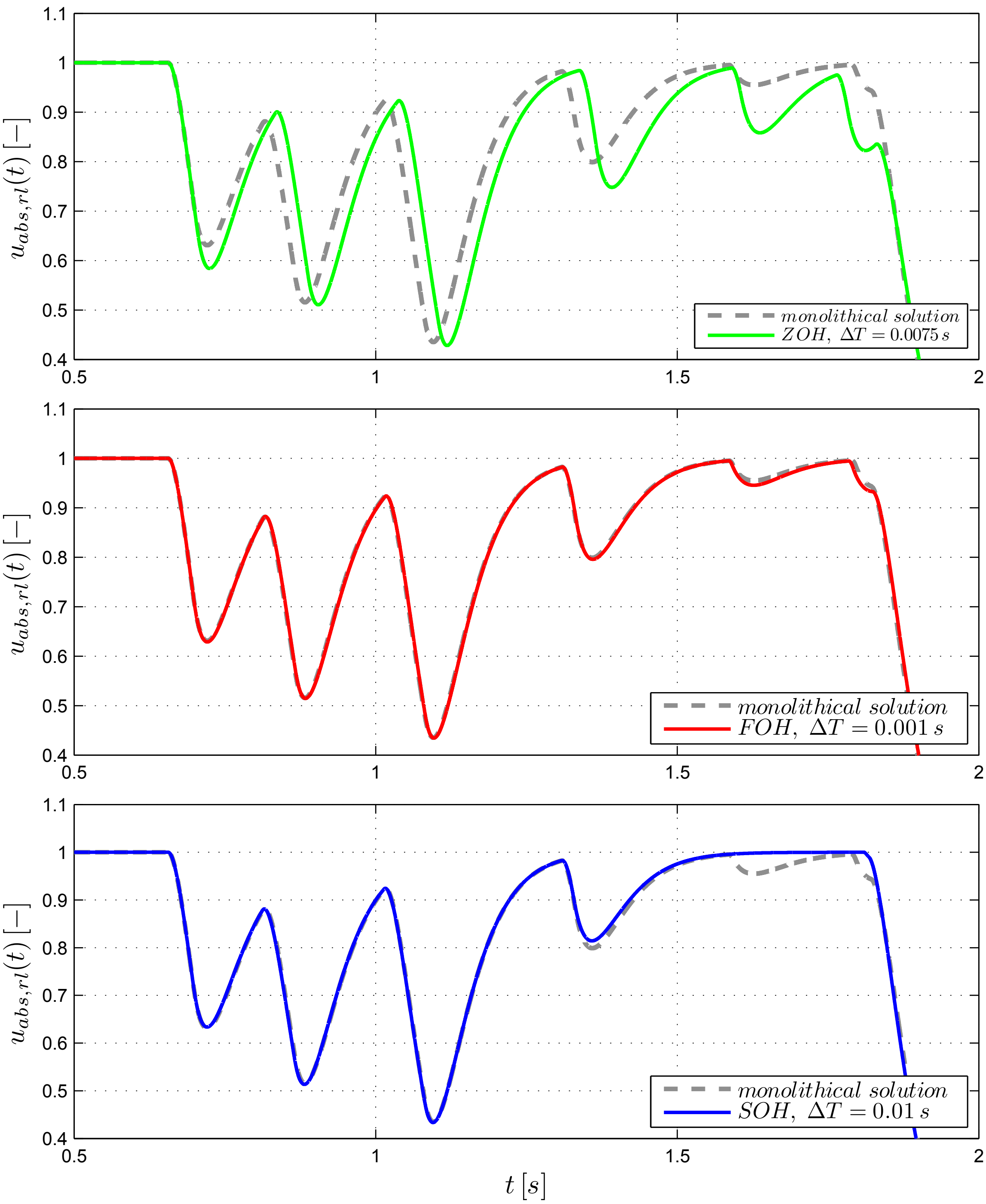

Figure 4: Co-simulation results according to different extrapolation schemes. Zero-order-hold (ZOH) with $\Delta T = 0.0075\,s$, first-order-hold (FOH) with $\Delta T = 0.001\,s$ and second-order-hold (SOH) with $\Delta T = 0.01\,s$.

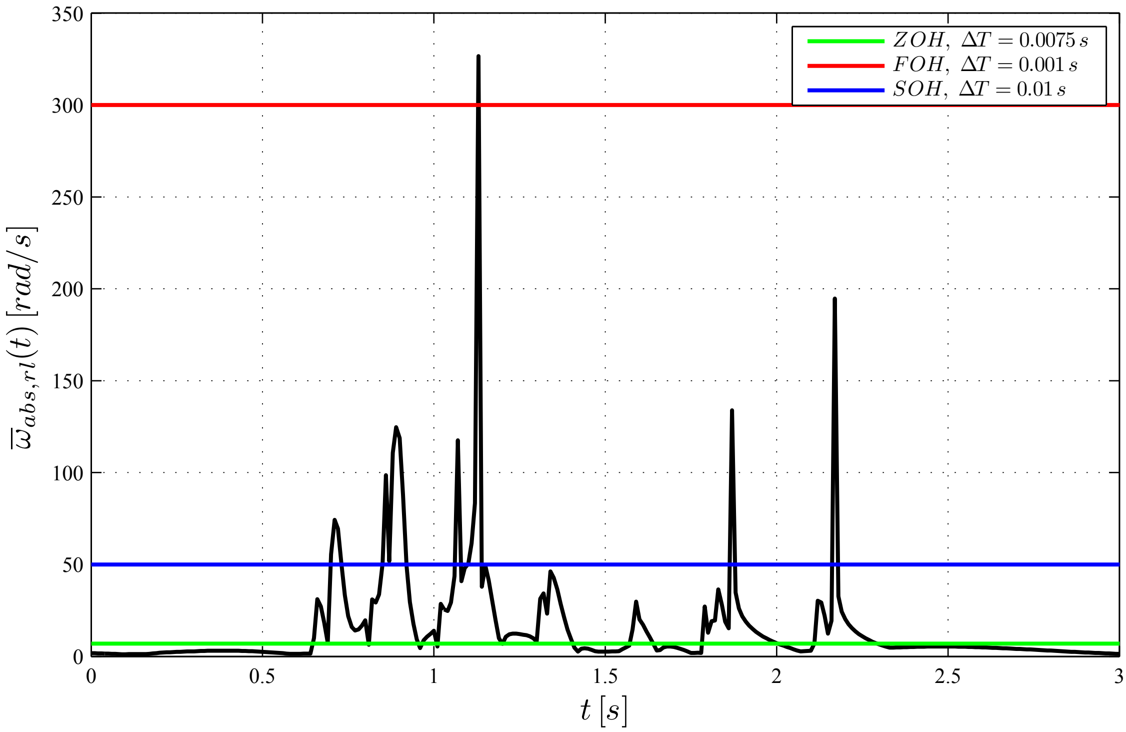

Figure 5: Maximum occurring instantaneous frequency $\overline{\omega}_{abs,hl}(t)$ of the coupling signal and depicted maximum bandwidths (horizontal lines) of the applied coupling elements

is computed using a constant macro-step-size of $\Delta T = 0.0075\,s$. Obviously, there are strong deviations between the depicted simulation results. Besides increased oscillations of the control signal also a time-shift is introduced. In contrast, the middle plot illustrates the results concerning first-order-hold (FOH) extrapolation. The macro-step-size is fixed to $\Delta T = 0.001\,s$. In this case, the results of the performed co-simulation are visually very similar to the result of the monolithical simulation. In the case of second-order-extrapolation (SOH, lower plot) with a macro-step-size of $\Delta T = 0.01\,s$, significant oscillations are eliminated in the time interval $t \in [1.5, 1.9]$. Eventually, these omitted oscillations may be important for the overall system design.

Different configurations of a co-simulation may lead to significantly different simulation results, as shown in Figure 4. One may expect that the SOH extrapolation scheme intuitively may lead to the best result of the performed co-simulations for this example. But in any case both, the applied macro-step-size and the used extrapolation scheme have to be accounted for coupling performance aspects. As an important remark: up to now, for the engineer it is impossible to assess co-simulation results without knowing a reference (monolithical) solution. However, to mitigate this significant drawback, the proposed approaches based on considerations in the frequency-domain are applied.

5.3 Performance of the coupling elements

The proposed 'rule-of-thumb' (Tab. 1) provide relations between the macro-step-size and the usable bandwidth of the coupling signal for commonly used extrapolation schemes. Knowing the macro-step-size ΔT the efficient bandwidths of the coupling elements can be determined using (4). The resulting efficient bandwidths of the three coupling elements are illustrated in Figure 5. Because of the chosen constant macro-step-sizes all efficient bandwidths of the applied couplings are constant over the simulation time. Additionally, in Figure 5 the maximum instantaneous frequency $\overline{\omega}_{abs,rl}(t)$ of the control signal $u_{abs,rl}$ is depicted. Obviously, the efficient bandwidths of the three coupling elements are significantly different. In particular, the ZOH scheme provides the smallest efficient bandwidth and many frequency components of the coupling signal are strongly modified. For the SOH scheme the efficient bandwidth is slightly enlarged, but also frequency components are distorted by the coupling process. Thus, by the application of ZOH ($\Delta T = 0.0075\,s$) and SOH ($\Delta T = 0.01\,s$) extrapolation significant distortions are introduced due to the required couplings. In contrast, the application of the first-order-hold extrapolation technique with a macro-step-size of $\Delta T = 0.001\,s$ leads to the largest efficient bandwidth. Only marginal frequency components are influenced and thus, accurate results of the co-simulation can be expected. As shown in Figure 4, only this coupling approach leads to meaningful simulation results. Thus, without knowing a reference solution, based on considerations in the frequency-domain the performance of the coupling elements according to the corresponding coupling signals are qualitatively assessable. The other way around, if the maximum occurring bandwidth of the coupling signal is a-priori known, a suitable macro-step-size can be defined based on Table 1. This macro-step-size ensures a sufficient bandwidth of the coupling element.

6 CONCLUSIONS

The non-iterative co-simulation approach requires marginal interfacing capabilities of the involved simulation tools. Thus, this coupling scheme is most often applicable. As a significant drawback, the required couplings introduce coupling errors. Coupling errors strongly depend on the macro-step-size and the used scheme for extrapolation of the coupling quantities. Concerning accuracy, bounds on the coupling error based on the approximation order of the used extrapolation technique are very pessimistic and therefore not applicable.

In our approach, we are focusing on the couplings itself instead of analysing the overall coupled system. From the abstract point of view the coupling scheme is considered as an additional subsystem, i. e. a coupling element. The analysis of the coupling element in the frequency-domain leads to a 'rule-of-thumb' to select the most critical parameter of a non-iterative co-simulation, the macro-step-size. Based on the resulting relations the assessment of the performance of the coupling elements is possible. In fact, results of a non-iterative co-simulation can be qualitatively assessed without a reference solution.

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