

A COMPUTATIONAL MODEL FOR PROGRESSIVE CRACKING IN LARGE DAMS DUE TO THE SWELLING OF CONCRETE

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Abstract—The paper presents a computational model for the analysis of large concrete gravity dams subjected to severe damage due to internal actions. It has been observed in several operating concrete dams that the combined effects of water intrusion and concrete expansion produce a time-advancing deteriorating process that may endanger the global stability of the construction. The present study was undertaken to simulate numerically the observed phenomena and to enable to predict future developments. Tensile cracking of the concrete is modelled using an elastic-fracturing constitutive model. The model is able to simulate in a realistic manner the phenomena of primary and secondary crack initiation, elastic degradation, crack closing and reopening. The triggering of the volumetric expansion due to water intrusion is linked to the onset of cracking at each point, assuming that water enters the dam mostly through the cracks. Temperature and pore-water pressure effects are included using assumed distributions based on available field data. Construction joints are modelled using a frictional joint element, although a "constitutive alternative" is outlined. Close surveillance of the behaviour of a dam that presented this sort of problem was used to calibrate the numerical model and to confirm the obtained results.

1. INTRODUCTION

IT IS WELL established that many operating concrete dams present internal zones of extensive cracking or microcracking. The origin of cracking in concrete dams may be found in the phases of design, construction or in operation. The reasons for cracking may be many fold: thermal variations due to internal or external sources, seismic actions, irregular settlements of the foundation, internal chemical reactions during the setting, hardening and life of the concrete, etc. Whatever the reasons for cracking, one of its direct effects is to enhance the entrance of water into the body of the dam. The presence of water due to seepage is a regular fact in these constructions. However, the appearance of unexpected cracking and the increased amount of water inside the core of the dam may significantly reduce the predicted safety factor of the structure, in some cases, down to dangerous figures.

One of the most serious problems related to cracking in dams is the combined effect of cracking and water intrusion on the mechanical behaviour of concrete. The known phenomenon of concrete expansion in the presence of water plays a paramount role in the propagation of cracking inside the dam. The critical situation that may arise can be described in sequence (see Fig. 1): (a) Onset of cracking is produced in the up-stream wall due to the expansion of the concrete in direct contact with the water of the reservoir; (b) As water intrudes in the newly formed cracks, new areas of concrete begin to swell. This may force the opening of new cracks and the closing of those previously opened; (c) The combined effect of entrance of water + concrete expansion + opening of cracks may continue indefinitely. The process may be stopped by the drainage, if it is effective, since it enables further sections of the dam to come into contact with water.

The importance of the described process depends mostly on the magnitude of the concrete expansion, which in some cases is almost negligible. However, unexpectedly high values of concrete expansion in the presence of water may enhance the process described above and cause severe cracking inside the dam, to the point of changing radically its structural behaviour. Expansion of concrete in the presence of water is a well-known effect that causes continuous increase of strain in the concrete up to a maximum stabilized value. This effect, similar (and opposite) to shrinkage, depends on the physical and chemical properties of the mortar and the aggregate, the water-cement ratio, etc. Typical values of the maximum attainable volumetric strain are around 0.03%.

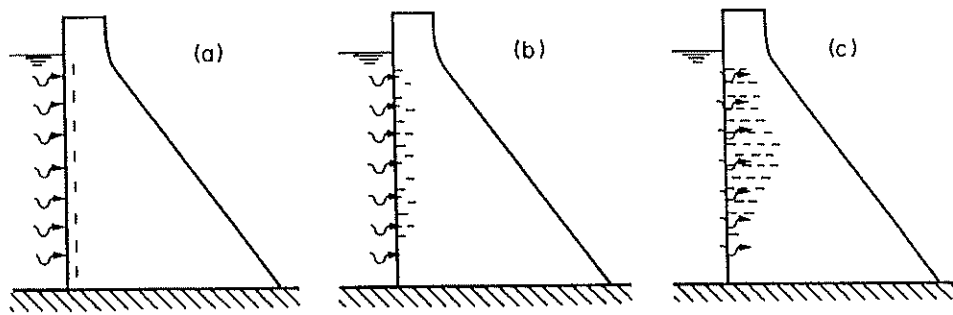


Fig. 1. Description of the phenomenon. (a) Concrete in direct contact with water begins to expand. Tensile strains are induced close to the up-stream wall. (b) Onset of cracking close to the up-stream wall. Water enters the dam through the opening cracks. (c) Extensive cracking inside the dam. Water reaches the plane of drainage.

The analysis of a process such as the one outlined above is complex. Some of the difficulties that arise can be summarized as follows:

(a) The *driving force* of the process is the intrusion of water inside the body of the dam, and the chemical reactions that take place in concrete when it comes into contact with water. Thus, the first analytical problem that arises is to simulate the *coupled time advancing* phenomena of water intrusion and evolution of swelling volumetric strain. A mechanism to define when water reaches a certain point inside the dam, triggering the chemical reactions, is needed. Furthermore, the evolution of volumetric strain with time must be considered.

(b) In order to achieve realistic results, the main features of the constitutive behaviour of concrete must be included in the model. In particular, the phenomenon of fragile fracture, elastic degradation, crack closing and reopening must be properly modelled. This presents all the analytical difficulties of an *extremely nonlinear* material behaviour. Additionally, large concrete dams present construction joints (both transversal and longitudinal) that influence greatly the structural behaviour of the dam. The frictional behaviour of such joints is also extremely nonlinear.

(c) There are other phenomena that may contribute significantly to the process, and should be, in some way, included in the model. Such is the case of the seasonal thermal oscillations that occur in the core of the dam, and the pore water pressure effect.

Classical methods for dam analysis are ineffective in tackling the kind of problem that has been described above. Only the decisive impulse that the numerical methods have experienced over the last decades allows structural analysts to study problems that pose the combined difficulties of a general geometry, a time-advancing phenomenon and a highly nonlinear material behaviour. In particular, the Finite Element Method has proved to be the most powerful tool to deal with such situations, and it will be used herein. However, even with the use of FEM, some simplifications can be introduced in order to achieve a relatively simple but effective model.

Due to the low permeability of concrete, a fully coupled analysis is not considered to determine the exact seepage flow inside the dam. Such a matter is approached in a simplified manner that will be described later. In a similar way, available field data about temperatures inside the dam is used to evaluate the distribution of temperatures and the seasonal thermal oscillations without actually solving the associated "thermal problem".

As has been explained above, the nature of the phenomenon to be studied is time advancing. However, because of the slowness of the process involved, all inertial and time-rate effects are neglected and a quasi-static analysis is performed. Thus, the advance on time produces variations on: (a) strains in the concrete in contact with the water and (b) temperatures in the core of the dam. Displacements, strains and stresses in the dam must be recomputed at every time step to reestablish the equilibrium that such variations have altered.

For the particular case of interest, five different items may be identified *a priori* as relevant to the problem: (a) material model for concrete, (b) expansion of concrete due to water intrusion, (c) thermal strains, (d) pore-water pressure effect and (e) construction joints. The model presented is able to include all these effects and evaluate the history in time of displacements, strains and stresses inside the dam. Details about the actual implementation of these five items in the model are given in the next section.

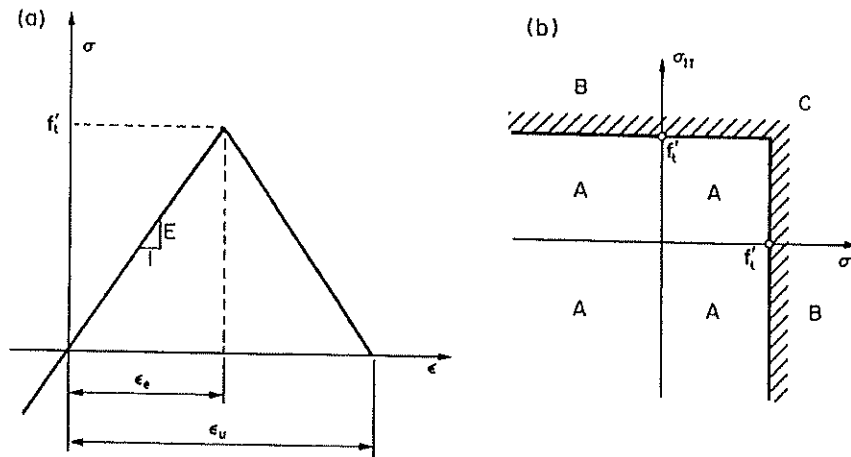


Fig. 2. Material behavior. (a) Stress-strain diagram. (b) Different areas of material state: elastic (A), singly-cracked (B) and doubly-cracked (C).

2. NUMERICAL MODEL

2.1. Material model for concrete

The mechanical behaviour of concrete under general solicitations is very complex and highly nonlinear. An "all purpose model" should consider phenomena such as cracking, yielding, crushing, creep, shrinkage, strain rate dependency, etc. Due to the particular stress states that are to be encountered in a gravity dam, with relatively low compressive stresses, plastic or viscoplastic phenomena such as yielding, crushing or creep will not be considered here. Therefore, an *elastic-fracturing model* will be adopted, the post-fracturing behaviour being the most important part of the model. A detailed description of the model is given elsewhere[1] and it will only be briefly summarized here. The model is formulated for 2D (plane stress/strain) and 3D applications[2, 3]. For simplicity, only some details of the 2D model are given here.

In the present work concrete is considered as an isotropic, elastic material for compressive and low tensile stress states. The maximum tensile stress criterion is employed to distinguish elastic behaviour under tension from tensile fracture (a representation of the fracturing criterion is shown in Fig 2b).

For a previously uncracked point, the principal stresses and their directions are evaluated. If the maximum principal stress exceeds a limiting value, a crack is assumed to form in a plane orthogonal to the offending stress. Thereafter, the behaviour of the concrete is no longer isotropic, material axes are assumed to appear, coinciding with the direction of the principal stresses (and strains) at the onset of cracking. Depending on the values of the principal stresses, one or two independent sets of cracks may appear at the same point. If two sets are formed at the same time, they will be orthogonal, and no further cracking will be allowed at that point. However, if only one set of cracks is formed, secondary cracking may occur under further loading. This second set of cracks is allowed to form at non-orthogonal directions to the primary set, as explained in[1]. The process is illustrated in Fig. 3.

After cracking, the stress normal to the crack direction must be released. Fracture mechanics considerations show that the shape of the softening branch of the stress-strain diagram must consider the "dimensions" of the cracking zone, in order to make the model objective with respect to the size of the elements used in the computational mesh. Details of how this affects the material model are given elsewhere[2-5].

The behaviour of cracked concrete under further loading or unloading, as it will occur as the combined effects of concrete swelling and cracking proceed in time, must be considered. Provisions are made in the model so that an active crack can close partially or fully and, eventually reopen. Such situations are shown in Fig. 4.

2.2. Expansion of concrete due to water intrusion

In order to be able to evaluate the volumetric strain due to the swelling of concrete at a given time, it is necessary to determine which points of the dam are "wet" and which are not. Because

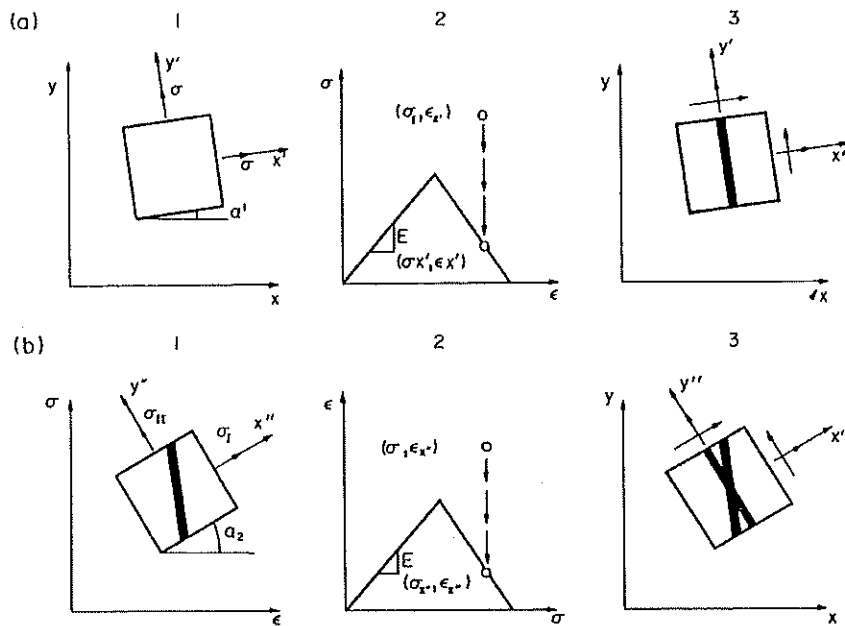


Fig. 3. Onset of cracking. (a) One set of cracks is formed. (b) Secondary set of cracks is formed. (1. Computation of principal stresses; 2. Release of tensile stress; 3. Definition of material system.)

of the low permeability of the concrete, it is reasonable to assume that water will enter the dam mainly through the opening cracks. This allows to avoid a full seepage analysis and to couple directly the appearance of water with the onset of cracking at a certain point. Accordingly, it will be assumed that each integration point starts tracing its corresponding free expansion strains vs time diagram at the same time step when cracking occurs.

The chosen strategy is as follows:

—The analysis starts considering that, after application of the gravity forces and hydrostatic pressures, only those points on the up-stream wall of the dam are in direct contact with water. Thus,

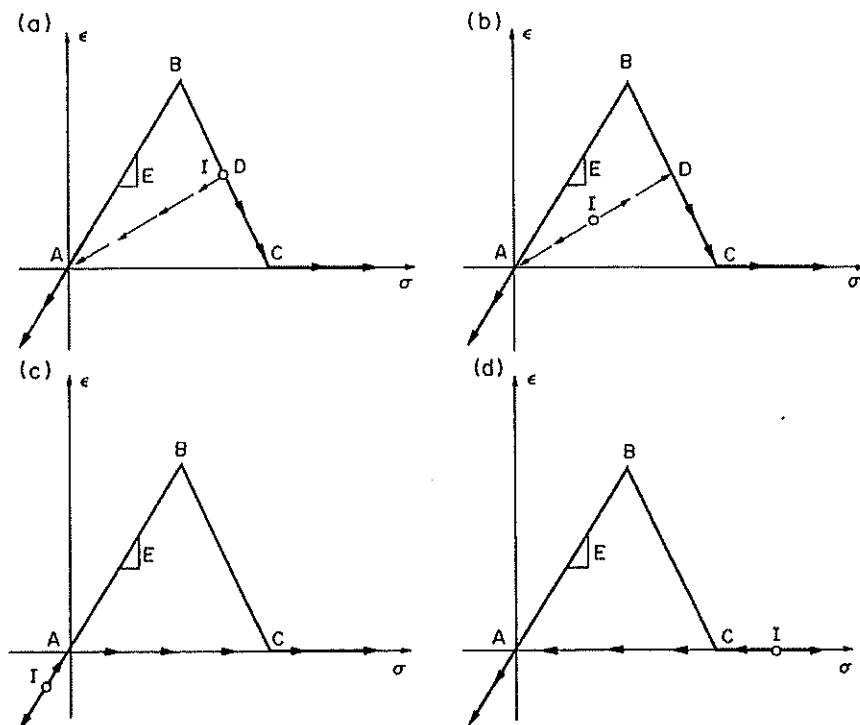


Fig. 4. Unloading-reloading behaviour of active cracks. (a) Loading-unloading of an opened crack. (b) Loading-unloading of a partially closed crack. (c) Reopening of a fully closed crack. (d) Closing a fully opened crack.

those points start to trace their corresponding expansion strain vs time diagram from the beginning of the analysis.

—The expansion of the up-stream wall will produce tensile stresses in the vicinity of the wall. This will trigger tensile cracking in those areas. As water intrusion is coupled to cracking, new points will start swelling as they crack.

—The process continues in time. Volumetric strains in one area may force new cracks to form in nearby areas, and opened cracks may be forced to close partially or fully.

It will be seen later that this mechanism is sufficient to make cracks and water “travel” from the up-stream wall to the plane of drainage, as it has been observed to happen in reality. Points located down-stream of the plane of drainage may crack but not swell (in the assumption that drainage is effective). This can be achieved simply by assigning a null expansion curve to those elements.

After some time, depending on the geometry and material properties of the dam, most of the “swelling points” reach the maximum attainable volumetric strain. The deformation process tends to slow down and, eventually, a stabilized state may be reached.

2.3. Seasonal thermal oscillations

Large concrete dams have an important thermal inertia, and significant differences in temperature can be measured in different points of the dam. The seasonal thermal oscillations produce variations of the volumetric strain, and accordingly, variations of the computed displacements and stresses. The thermal oscillations were included in the analysis, as they may force partial or full closing and reopening of existing cracks. Furthermore, differential thermal straining may be an added cause of cracking and water intrusion.

If temperature effects are to be included in the model different alternatives exist:

(a) A coupled time advancing problem dealing with displacements *and* temperatures can be performed. Initial conditions and time-varying boundary conditions must be specified for the thermal problem, and the model can compute for every time step the actual temperature distribution. This alternative is expensive, as the number of degrees of freedom in the problem is increased with the temperature field.

(b) The thermal problem can be solved separately, and temperature distributions can be stored to be used as input for the structural model. This alternative is more economic than the previous one, but still requires the solution of two different problems, and large data management. However, this is the only valid alternative if no field data is available about temperatures.

(c) If extensive temperature field data exists, a simplified method can be adopted to compute the time-varying temperature distribution inside the dam. This is the most economic alternative, and it was used in the present work.

From field measured data, and with the aid of a digitizer and an interpolation program, the amplitude of the temperature oscillations, the correspondent phase angle and the annual mean temperature were computed for each node of the finite element mesh. This data was further elaborated to produce isothermal contours at fixed intervals of a “typical year”. These contours could be confronted with those produced from the field data. Close agreement was found and this simplified method was used during the time-advancing analysis to evaluate temperatures (and in turn, thermal strains) at every point of the dam for each time step.

2.4. Pore water pressure effect

The analysis of a totally or partially saturated medium must be conducted in terms of effective stresses, that is, taking into account the effect of the pore-water pressure. Such analysis needs the determination of the distribution of pore-water pressure in the body of the dam. Again, in order to avoid the performance of a fully coupled analysis in terms of displacements and pressures, a simplified method was adopted. The method, common in dam analysis, consists in assuming an approximated distribution of pore-water pressure inside the body of the dam. This distribution was established so that along a transverse line across the section the pressure is equal to the hydrostatic pressure on the up-stream wall, and maintains its value up to the drainage. From there, it decays linearly down to the down-stream wall, where it vanishes. The distribution is shown in Fig. 5. With

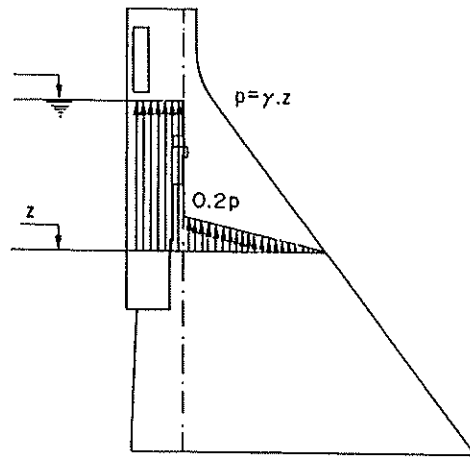


Fig. 5. Assumed distribution of pore-fluid pressure along a transverse line.

this simplified distribution of pore-water pressure, the effective stresses can be computed at each point of the dam, and the material behaviour is expressed in terms of these, rather than the total stresses.

2.5. Construction joints

Concrete dams present joints that, depending on their treatment and state, may influence importantly the structural behaviour. Transversal joints play an important role in a fully 3D analysis of the dam, allowing relative movement between different blocks of the dam. In a simplified 2D analysis, this effect cannot be included. However, longitudinal vertical construction joints can also exist, and they must be included both in 2D and 3D models.

The classical way of dealing with joints is making use of specially developed joint elements. Several joint elements have been recently proposed[6, 7], mostly in the context of rock mechanics. In this analysis, a frictional dilatant one-dimensional joint element has been used. The element is compatible with the standard solid 2D element used to model the body of the dam. Details of the formulation of the joint element and its mechanic behaviour are given in[6].

Usually joint elements are non-dimensional through the "thickness", and they present a constitutive equation that is formulated in terms of displacement vs stress. To avoid "overlapping" in compression, the normal component of the constitutive relations must present an asymptote. This is bound to produce severe numerical difficulties and very slow convergence rates. Additionally, different joint elements must be developed to "match" with the displacement field of different solid elements (linear and quadratic elements for 2D applications, and several more for 3D applications).

An alternative to the use of joint elements is to use standard element that behave as a joint, that is, to formulate the behaviour of the joint in terms of strain vs stress, and incorporate it as a special material, not as a special element *and* a special material. Obviously, once this joint material is developed, it does not require special treatment for different applications.

To formulate the behaviour of the joint in terms of strains, the element simulating the joint must be dimensional. This can be achieved considering a "smeared joint", rather than a "discrete joint" (see Fig. 6). Instead of considering the joint by itself, it is considered as a part of a composite material that represents the behaviour of the joint *and* some concrete adjacent to it. In this situation, the new element has a finite dimension, and relative movements between its nodes can be interpreted as "strains" in the standard way. Furthermore, when the smeared joint is "fully closed", its behaviour must tend to the compressive behaviour of the concrete, and it does not present any asymptote.

Obviously, the constitutive equation for this "smeared joint material" is orthotropic. Material axes must be fixed to determine the directions normal and tangential to the crack, and different behaviour must be established for these directions. This brings another advantage to this model: not only any element in the mesh can behave as a joint element, but the direction normal to the joint can be arbitrarily defined, in any set of global or local axes, with total independency of the orientation of the mesh.

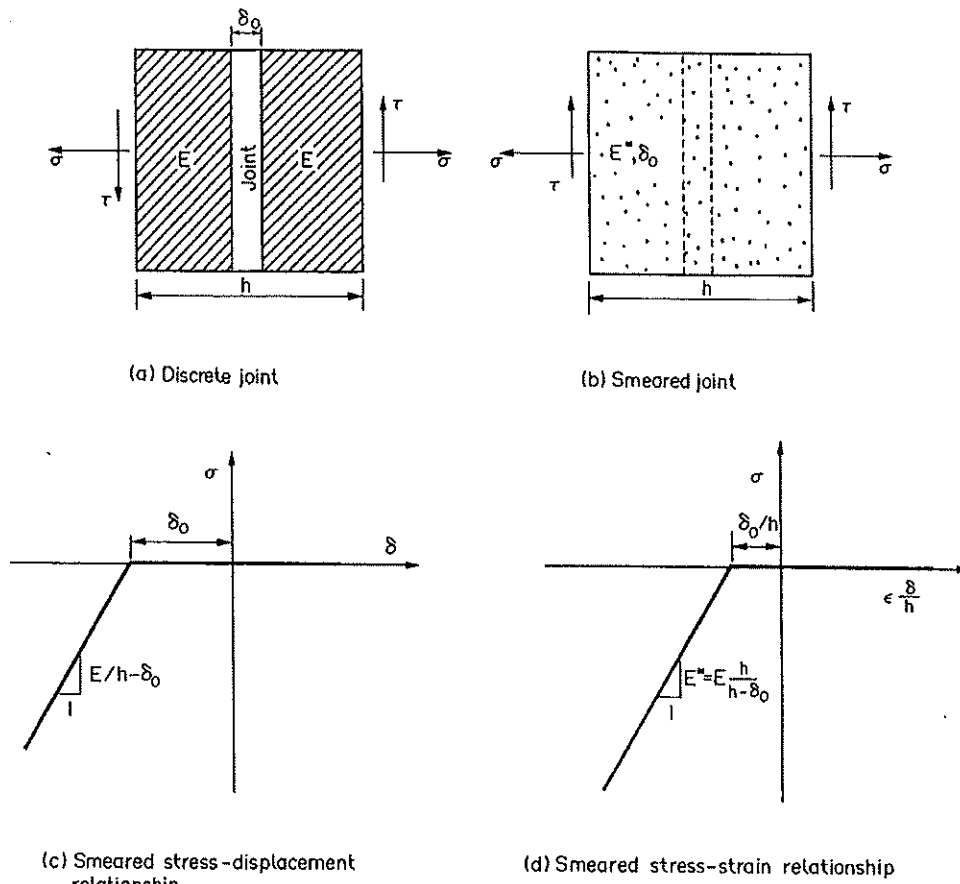


Fig. 6. Description of the "smeared" joint element. (a) Discrete joint idealization; (b) Smeared joint idealization; (c) Smeared normal stress vs normal displacement relationship; (d) Smeared normal stress vs normal strain relationship.

In the normal direction a "no tension" strain-stress relationship is used. A threshold value of compressive strain can be defined to simulate the effect of the initial opening gap of the joint. To make results objective respect to the size of the smeared joint, the stiffness modulus in that directions is scaled according to the normal length of the element. A von Mises or Drucker-Prager elasto-plastic model can be used to simulate the frictional behaviour of the joint, that is, to relate the normal compression to the tangential shear stress.

Note the similarities between these smeared joint model and the smeared crack model used for concrete. It must be pointed out that although some of the difficulties inherent to the joint elements are removed, the resulting smeared joint is also extremely nonlinear, and its will also cause difficulties in convergence. Numerical experiments are currently on course to compare the two models.

3. PRACTICAL APPLICATION

3.1. Description of the case

The case of interest in this study is a gravity dam in Spain 461 m long and 79 m high. The different construction blocks have a width of 14 m, 21 m and a singular one of 70 m, leaving transversal joints between the blocks. Construction of the dam started in 1962 and finished in 1967. Regular operation began in 1967 and even in those first years unexpected down-stream movements of the dam were observed, especially in the singular block close to the spillway. At that time, cracking of the concrete was detected on the up-stream wall and from the interior corridors. These events motivated an intense instrumentation and close surveillance of the dam, continued up to the present time. In 1984 this study was undertaken to model the known behaviour of the structure and to predict future developments. The study consists of two phases: firstly, a two-dimensional (2D) analysis of the most affected block (plane strain behaviour assumed), and secondly, a

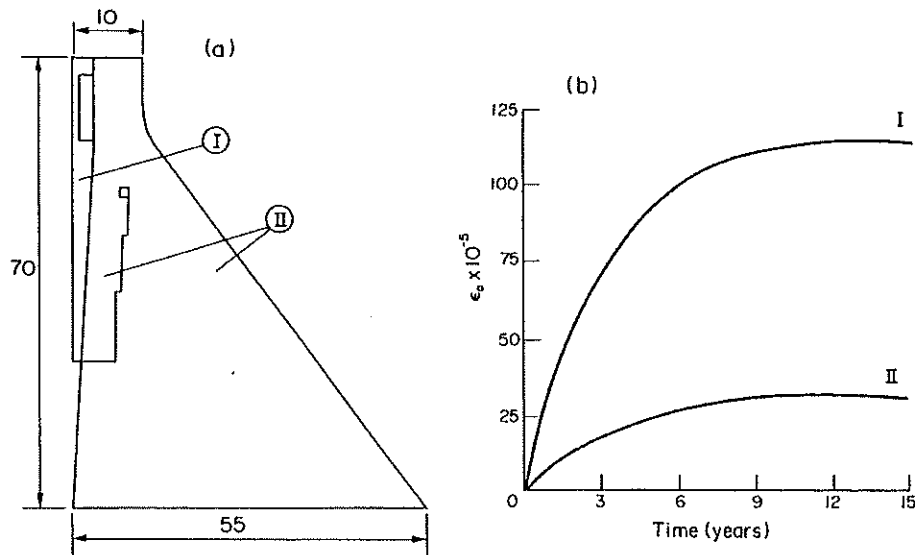


Fig. 7. Description of the problem. (a) Mechanical behaviour. (b) Free expansion vs time diagrams for the different zones of the dam (time in years).

three-dimensional (3D) analysis considering interaction of the blocks through the transversal construction joints.

3.2. 2D Analysis. Discussion of results

For the 2D analysis a typical cross section was chosen, and it is shown in Fig. 7(a). The section is divided in two zones: zone I (corresponding to the supports of the spillway) can be considered to be in a plane stress state; zone II (corresponding to the body of the dam) can be considered to be in a plane strain state. In the cross section is also considered a longitudinal vertical construction joint located approximately at the plane of drainage. Finally, the monitoring proved that the irregular displacements affect mainly the top 65 m of the dam, and the analysis is limited to that part.

Laboratory tests carried on samples extracted from different areas of the dam allowed to establish the free expansion vs time diagrams that are to be used as input in this study (see Fig. 7b). The greatest detected free expansion corresponds to zone I of Fig. 7(a), with a maximum considered value of 0.125%; while in zone II a "normal" expansion value of 0.03% was considered. Swelling of the concrete was not considered down-stream of the drainage.

(a) *Displacements.* Graphs showing displacement vs time are plotted in Fig. 8, corresponding to the horizontal and vertical displacements of the top of the construction. A comparison is shown in these graphs between results obtained with and without the effect of the pore-water pressure, and the measurements from the monitoring of the dam. The comparison between computed and measured displacements proves that the performed analysis is able to capture the main features present in the studied phenomenon. Three items can be observed: (1) The deformation process is stable. From the numerical analysis it can be expected that the "steady state" will be reached in the next three or four years; (2) The seasonal thermal oscillations have little influence on the main process. They appear as a superimposed oscillation on the general trend of the graphs; (3) Pore-water pressure does not play an important role in the general process.

It can be concluded that the main cause of the observed phenomenon is the combined effect of water intrusion + concrete expansion + concrete cracking. Thermal oscillations and pore-water pressure have only second-order effects. Additionally, if the hypothesis of the existence of an effective plane of drainage is confirmed, the analysis predicts the stabilization of the process.

(b) *Stresses and cracking.* Figure 9 shows the distributions of principal stresses and cracks at three different times during the analysis: after one, two and twenty years. The deformed shapes of the cross section for those times are also shown (displacements are amplified fifty times). After one year the situation is very close to the results after application of self-weight and hydrostatic pressure. Swelling and cracking only affect the part of the dam close to the reservoir. Comparing this with the results after two and twenty years it can be observed that: (1) The expansion of

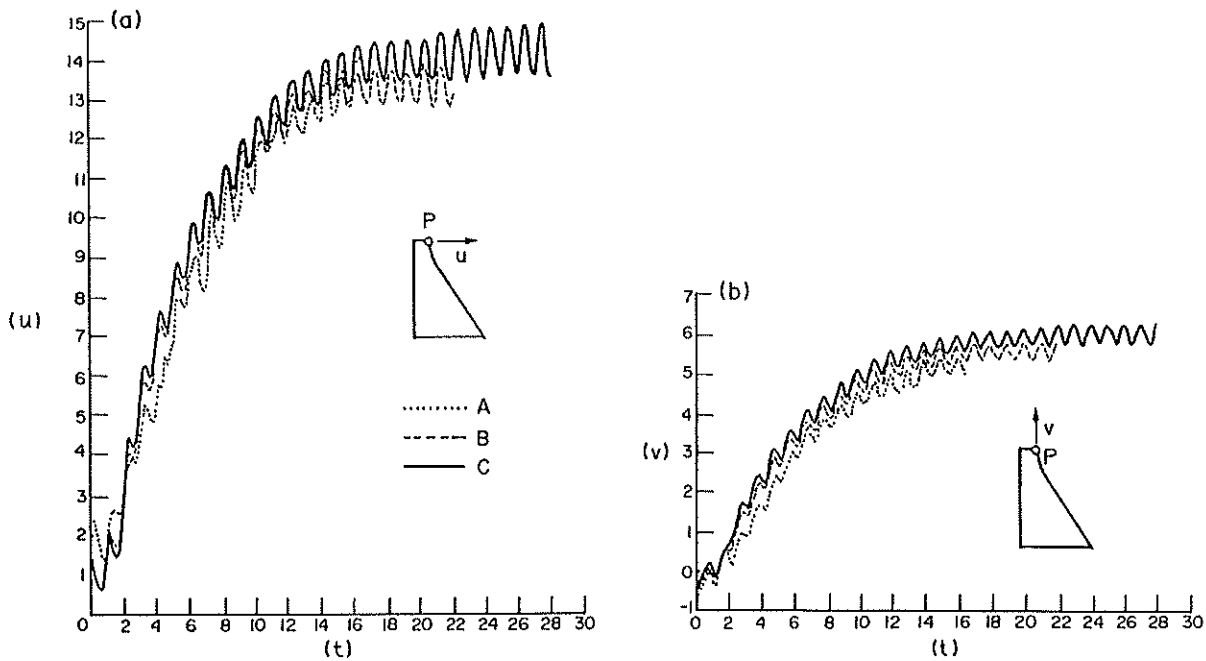


Fig. 8. Comparison between computed and measured displacements of top (down-stream) of the dam. (a) Horizontal displacement u (in cm); (b) Vertical displacement v (in cm). (A—Measured, B—computed without pore-water pressure effect, C—computed with pore-water pressure effect. t = time in years.)

concrete causes the appearance of an area with important tensile stresses. This area originates in the neighbourhood of the up-stream wall and its progresses with the advance of time towards the interior of the dam; (2) It is obvious an effect of growth and rotation towards the down-stream wall due to the expansion of the concrete in the up-stream wall; (3) The advance of microcracking with time, and the progressive closing of active cracks as time advances; (4) The evolution of the stress distribution towards a state where the up-stream and down-stream wall are subjected to strong compressions, while the core of the dam presents an extended area of residual tensile stresses.

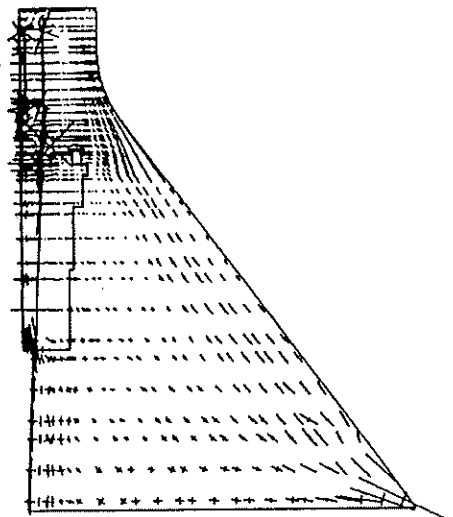
3.3. 3D Analysis. Preliminary results

With the insight of the problem gained from the 2D analysis, the full 3D problem was undertaken. As work is currently on-going, the results presented below are only preliminary. The objective of this first stage is to assess the behaviour of the model in 3D conditions. To that effect a simplified model of the dam was constructed (see Fig. 10). The mesh represents two adjacent construction blocks. There is a transversal joint between the blocks and a longitudinal joint along the body of the dam.

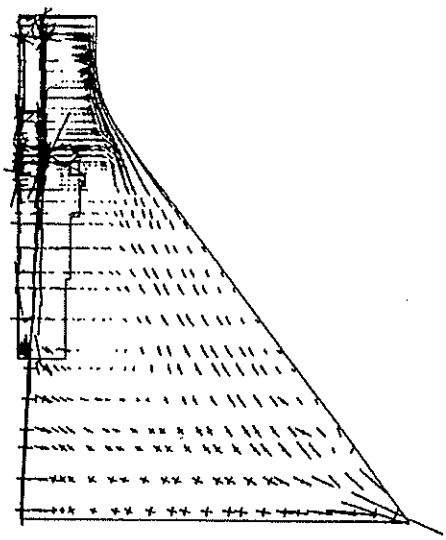
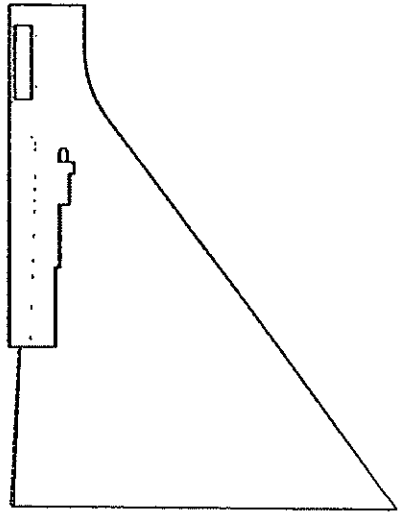
From the 2D analysis it can be concluded that temperature and pore-pressure effects are secondary. Thus, they are not initially considered in 3D. The problem is devised to test the mechanical behaviour of the concrete and the joints, and the "cracking-swelling" mechanism. To amplify the 3D effect and the nonlinear behaviour of the joints, swelling of the concrete is only "activated" in the up-stream wall of the right-hand side block. A maximum attainable value of 0.15% is considered for the free expansion strain.

Figure 11 shows the deformed shape of the model after twenty years (displacements amplified 200 times). The relative movement of the two blocks is evident. Figure 12(a) shows the displacement vs time curve for a point located at the top of the construction. The pattern of the curve follows that of Fig. 8 (without the seasonal oscillations). The initial part corresponds to the triggering of the "cracking-swelling" mechanism. A clear stabilization is observed at the end of the curve. Figure 12(b) shows the displacement profile along the coronation line, after twenty years. The jump in displacement across the transversal joint is clearly visible, and most of the deformation is localized close to that joint.

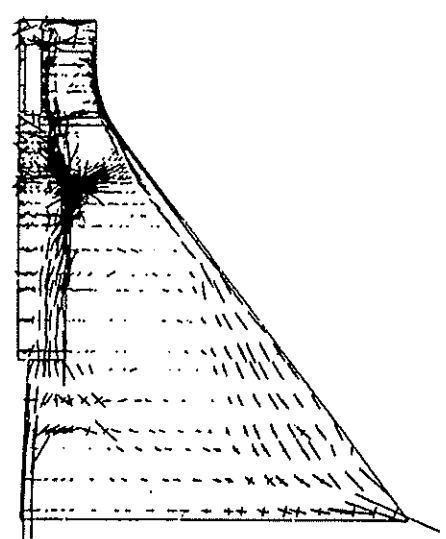
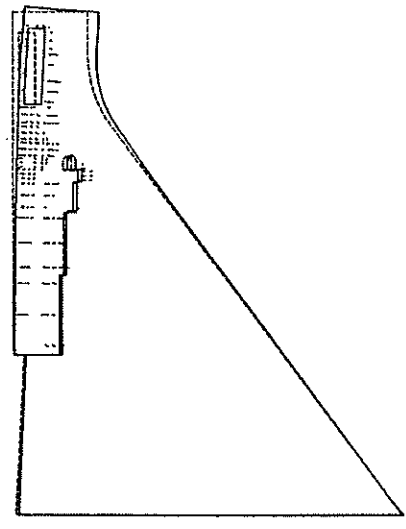
Figure 13 shows contours of cracking strain in the most affected part of the model (after twenty years). The effect of the transversal and longitudinal joints becomes clear in this plot, limiting the areas affected by fragile fracture of the concrete. Cracking takes place almost exclusively in the



(a)



(b)



(c)

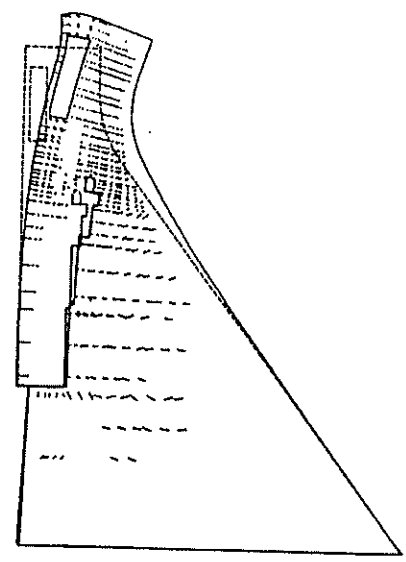


Fig. 9. Principal stresses, deformed shapes and microcracking: (a) after one year; (b) after two years; (c) after twenty years. (Displacements amplified fifty times.)

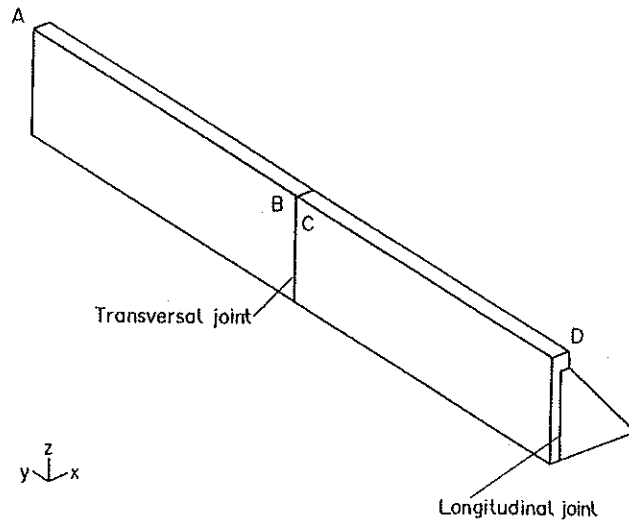


Fig. 10. Geometry for 3D analysis: location of the joints and significant points.

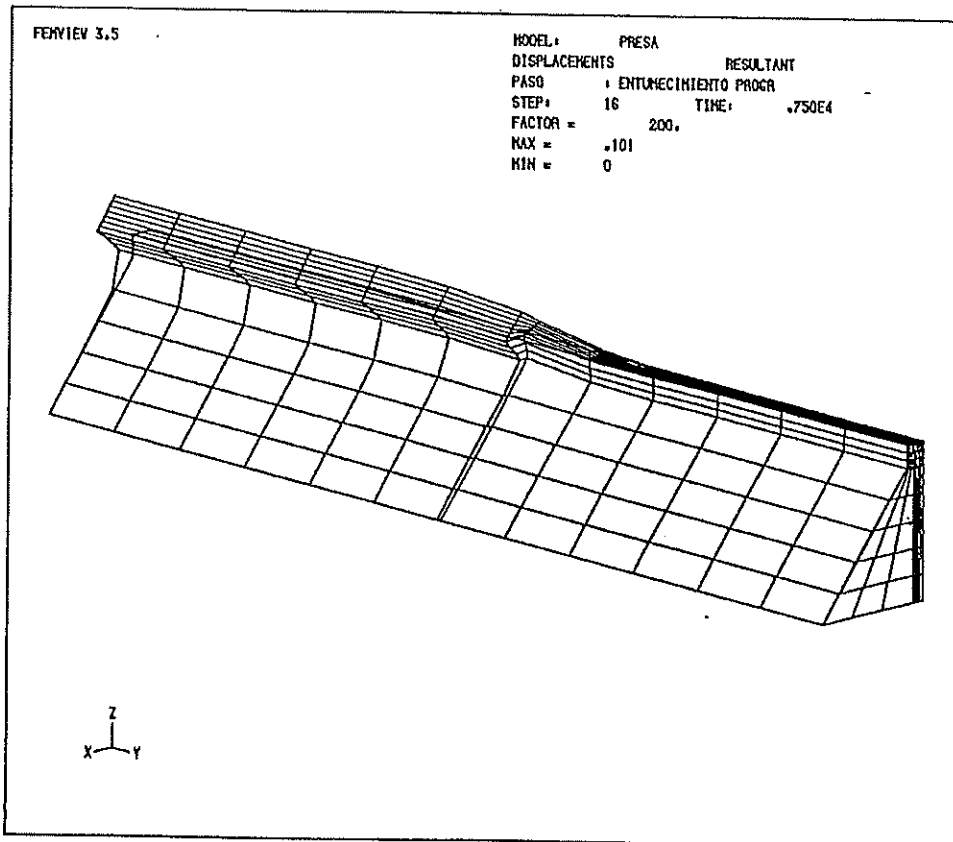


Fig. 11. Deformed mesh after twenty years (displacements amplified 200 times).

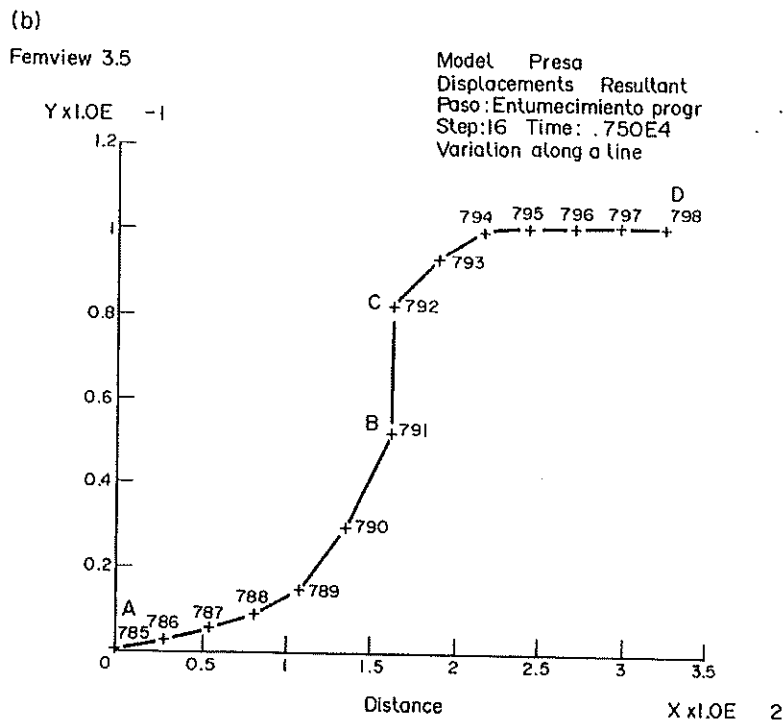
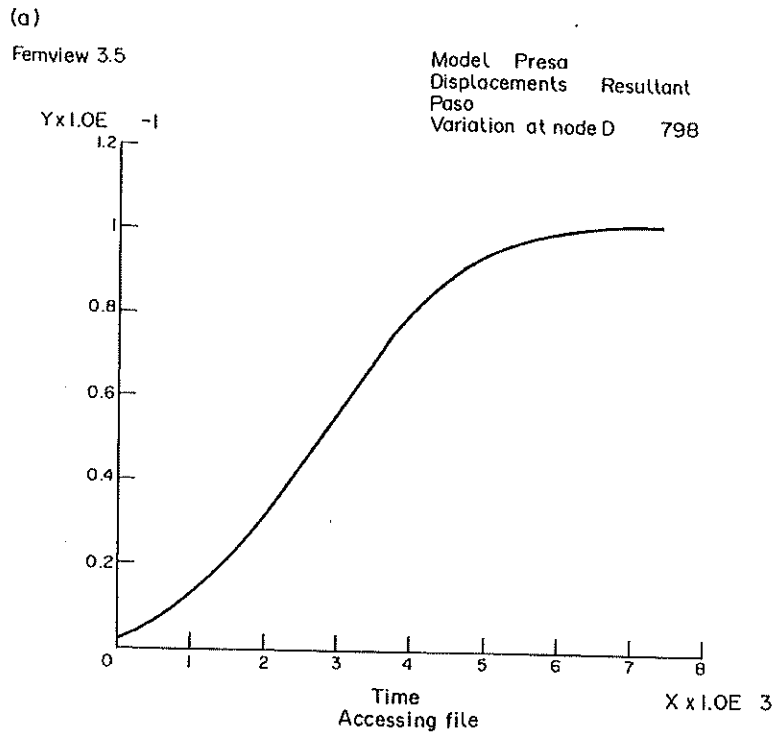


Fig. 12. (a) Displacement vs time for node D; (b) Displacements along nodal line A-B-C-D.

right-hand side block (the only one affected by swelling), and affects mainly the area around the top part of the longitudinal joint. This is reasonable, as the main deformation mode of the model corresponds to the upstream part slipping along the joint and inducing tensile stresses on the down-stream part of the dam.

4. CONCLUSIONS

A computational model for the analysis of large concrete dams suffering progressive cracking due to the swelling of concrete is presented. The model incorporates the most significant features of the phenomena involved: a mechanism for water intrusion, concrete expansion in presence of

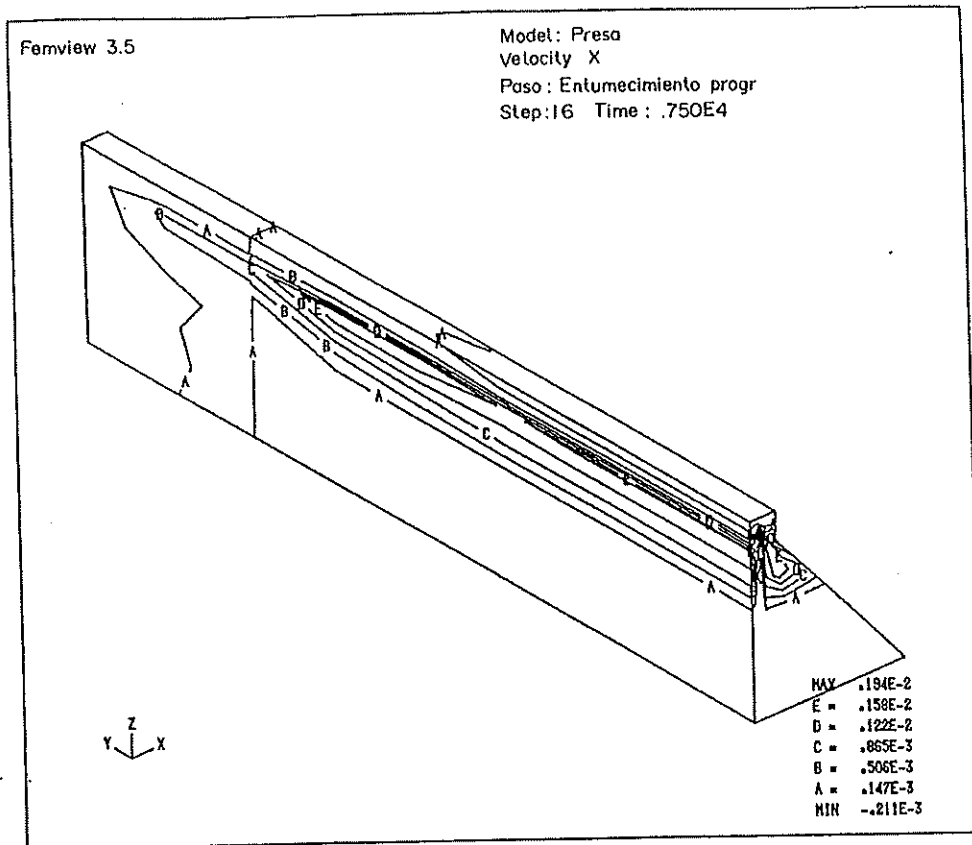


Fig. 13. Cracking strain contours in right-hand side block.

water, tensile fracture and seasonal thermal straining. Simplifications are proposed to avoid related seepage and thermal analyses. A "constitutive alternative" to the classical treatment of joints is outlined. Results are presented for the 2D analysis. They compare well with available field data. Preliminary results for a tentative 3D analysis are also given.

REFERENCES

- [1] E. Oñate, J. Oliver and G. Bugeda, Finite element analysis of the nonlinear response of concrete dams subjected to internal loads. *Finite Element Methods for Nonlinear Problems, Europe-US Symposium*, Trondheim, Norway (1985).
- [2] M. Cervera, Nonlinear analysis of reinforced concrete structures using three-dimensional and shell finite element models. Ph.D. Thesis C/Ph/93/86, University of Wales, Swansea (1986).
- [2] M. Cervera, E. Hinton and O. Hassan, Nonlinear analysis of reinforced concrete plate and shell structures using 20-noded isoparametric brick elements. *Comput. Struct.* **25**, 845-869 (1987).
- [3] L. Nilsson and M. Oldenburg, Nonlinear wave propagation in plastic fracturing materials—constitutive modelling and finite element analysis. *IUTAM Symposium on Nonlinear Deformation Waves*, Tallin, USSR (1982).
- [4] Z. P. Bazant and L. Cedolin, Fracture mechanics of reinforced concrete. *J. Engng Mech. Div. ASCE*, **106**, 1207-1306 (1980).
- [5] I. Carol and E. Alonso, A new joint element for the analysis of fractured rock. *Int. Cong. Rock Mech.*, Melbourne, Australia (1983).
- [6] R. E. Goodman and C. St. John, Finite element analysis for discontinuous rocks. *Numerical Methods Geotec. Engng*, McGraw Hill, New York (1977).

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