Rate dependent effects in an isotropic damage constitutive model for concrete

O. Manzoli
M. Cervera
X. Oliver

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Centro Internacional de Métodos Numéricos en Ingeniería
Gran Capitán s/n, 08034 Barcelona, España

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INTRODUCTION

The basic purpose of this work is to present an alternative formulation for computing the rate-dependent behavior the concrete in the context of the continuum isotropic damage model developed by Faria and Oliver (1993) for large scale computations.

A more efficient time integration of the resulting viscous-damage constitutive equations is obtained by defining a new law for the inviscid damage evolution in connection with the improvement of the numerical scheme used to compute its viscous regularized form.

The second section is devoted to present the consistent inviscid isotropic damage model. Two damage parameters are used in this model to distinguish damage due to tension from damage due to compression so that the "Unilateral Effect" for the concrete can be simulated.

In section 3, the rate-dependent formulation is introduced from the viscous regularization of the inviscid damage model proposed with a structure analogous to that of viscoplasticity of the Perzyna Type. In addition, two alternatives for the damage flow function are presented.

In section 4 the numerical implementation of the rate-dependent formulation is addressed, leading to an almost closed form algorithm for a displacement-based incremental finite element code.

Finally, some numerical analyses are presented in section 5, in order to show some characteristics of the model and demonstrate its capacity to describe the relevant features of concrete by an adequate calibration of the model parameters from experimental data.

RATE INDEPENDENT MODEL

2.1- Damage parameter and effective stress concept

The internal damage parameter is introduced in theory of Continuum Damage Mechanics in order to describe the progressive degradation suffered by the mechanical properties of a material when submitted to a particular load. This variable can be interpreted as a measure of the state of material defects due to the propagation of internal microcraks, ranging from 0 (undamaged material) to 1 (completely collapsed material) [Lemaitre (1985)].

The damage concept is easier to be understood by considering a surface element referred to a particular point of a material volume. Assuming that this surface area, denoted by S, is large enough to contain a representative number of defects, the damage parameter can be related to the measure of the state of damage in this surface according to the following definition:

$$d = \frac{S - \bar{S}}{S} = 1 - \frac{\bar{S}}{S} \qquad (2.1.1)$$

where $\bar{S}$ is the effective resting area (disregarding the microvoids).

In connection, a effective stress definition, $\bar{\sigma}$, associated with the effective area, $\bar{S}$, can be introduced so that

$$\sigma\, S = \bar{\sigma}\, \bar{S} \qquad (2.1.2)$$

where σ is the usual Cauchy stress considered acting on the nominal area S.

From the damage parameter definition (1), the effective stress can be expressed as

$$\bar{\sigma} = \frac{\sigma}{1-d} \qquad (2.1.3)$$

It is clear that the scalar damage variable d as defined above is a directional quantity referred to a surface outward normal direction $\boldsymbol{n}$. Then, for a general tridimensional stress state, the relation between the effective stress tensor and the homogenized Cauchy stress tensor becomes:

$$\bar{\sigma} = \boldsymbol{M}^{-1} : \sigma \qquad (2.1.4)$$

with $\boldsymbol{M}$ being a fourth-order tensor.

In the isotropic damage case, where the cracks are assumed to be equally distributed in all directions [Lemaitre (1984)], $\boldsymbol{M}$ is reduced to a simply scalar damage parameter, leading to

$$\bar{\sigma} = \frac{1}{1-d}\,\sigma \qquad (2.1.5)$$

from what is clear that $\bar{\sigma}=\sigma$ when the damage is null and tends to infinity when the damage approaches 1 (S approaches 0).

2.2- Non-linear constitutive law for the concrete

By the following hypothesis of strain equivalence [Lemaitre (1978)]:

"the strain associated with a damaged state under the applied stress is equivalent to the strain associated with its undamaged state under the effective stress"

a standard elastic-linear constitutive law can be used to describe effective stress-strain relations for the material points located on the effective area, i.e. :

$$\bar{\sigma} = \boldsymbol{D}_o : \varepsilon \qquad (2.2.2)$$

Here, $\boldsymbol{D}_o$ is the usual fourth-order linear-elastic constitutive tensor and ε is the second-order strain tensor .

Consequently (by using the already expressed dependency between $\bar{\sigma}$ and σ),

$$\sigma = (1 - d)\,\bar{\sigma} = (1 - d)\,\boldsymbol{D}_o : \varepsilon = \boldsymbol{D}\,\varepsilon \qquad (2.2.2)$$

It is important to notice that for an elastic-linear stress-strain assumption in the effective space, the non linear overall behavior is essentially driven by the damage evolution and the damage can be interpreted as a measure of the reduction on initial stiffness of the material.

2.3- Damage evolution

In order to characterize the progressive degradation of the mechanical properties of the material, a consistent damage evolution law will be introduced.

A damage criterion formulated in a effective stress space can be defined in the following form [Simo and Ju(1987)]:

$$g(\bar{\tau}(\bar{\sigma}), r) = \bar{\tau}(\bar{\sigma}) - r \leq 0 \qquad (2.3.1)$$

where $\bar{\tau}$ ($\bar{\sigma}$) is the scalar equivalent effective stress given by a defined norm of the effective stress tensor and r is the scalar current damage threshold. The damage criterion (1) defines a set of damage surfaces whose form is determined by the norm choice for $\bar{\tau}$ ($\bar{\sigma}$) and the size is controlled by the increasing values of r.

The condition (1) can be expressed by the following equivalent form:

$$g'(\bar{\tau}, r) = G(\bar{\tau}) - G(r) \leq 0 \qquad (2.3.2)$$

where G(.) is a suitable monotonic increasing scalar function.

The evolution of the damage variable d is defined here by the following rate equations :

$$\dot{d} = \dot{\mu}\ \frac{\partial g'(\tau,r)}{\partial r} = \dot{\mu}\ \frac{\partial G(r)}{\partial r} \qquad (2.3.3)$$

$$\dot{r} = \dot{\mu}$$

where $\dot{\mu}$ is the damage consistency parameter that defines the damage loading or unloading conditions according to the Kuhn-Tucker relations [Simo and Ju (1987)]:

$$\dot{\mu} \geq 0 \ ; \ g' = 0 \ ; \ \dot{\mu}\ g'(\tau,r) = 0 \qquad (2.3.4)$$

from that, it can be concluded that:

- if $g' < 0$ (damage criterion is not satisfied), ($\dot{\mu} = 0$, no damage evolution exists);
- if $\dot{\mu} > 0$ (damage is increasing) ,

$$g' = 0 \Rightarrow \tau - r = 0 \Rightarrow \dot{\tau} = \dot{r} = \dot{\mu}$$

which leads to the following explicit expression for the threshold:

$$r_t = \max\ [r_o\ ,\ \max(\tau_t\ )]\ \ ,\ \ \tau_t \in [0\ ,\ t] \qquad (2.3.5)$$

where r_o is the initial damage threshold, assumed to be a material property.

The evolution of the damage defined by (3) is at variance with that employed by Simo and Ju [Simo and Ju (1987); Faria (1993)] as

$$\dot{d} = \dot{\mu}\ \frac{\partial g'(\tau,r)}{\partial \tau} = \dot{\mu}\ \frac{\partial G(\tau)}{\partial \tau} \qquad (2.3.6)$$

$$\dot{r} = \dot{\mu}$$

that is equivalent to the equations (3) for inviscid cases, but it will be shown that the latter form can lead to an improper damage evolution in the viscous regularization used for the rate-dependent formulation.

From the rate equations (3) with the initial condition of null damage, it is easy to conclude that

$$\dot{d} = \dot{r}\ \frac{\partial G(r)}{\partial r} = \dot{G}(r) \Rightarrow d = G(r) \qquad (2.3.7)$$

Hence, the damage state is explicitly expressed as function of the current threshold given by eq. (5). Therefore, according to the damage parameter definition, the function G(.) must be also defined to ensure that

$$0 \leq d \leq 1$$

2.4- The unilateral damage model

By using two independent scalar damage parameters related to tensile and compressive stress states, d^+ and d^-, the model becomes able to take into account the different damage behavior of the concrete under tensile and compressive stress. The use of two parameters ensures that the stiffness degradation for tensile stress does not affect the stiffness for compressive stress and vice versa ("unilateral effect" [Mazars & Pijaudier-Cabot(1989))], a feature desired for the situation where changes in the stress sign during the load history are expected.

For this, the effective stress tensor will be decomposed into tensile and compressive parts according to [Faria & Oliver(1993)]

$$\bar{\sigma}^{+} = <\bar{\sigma}> = \sum_{i=1}^{3} <\bar{\sigma}_i> p_i \otimes p_i$$

$$\bar{\sigma}^{-} = >\bar{\sigma}< = \sum_{i=1}^{3} <-\bar{\sigma}_i> p_i \otimes p_i \qquad (2.4.1)$$

where $\bar{\sigma}_i$ denotes the i-th principal stress from tensor $\bar{\sigma}$ and the symbol <.> is the MacAuley bracket.

Then, the damage criterion and the evolution law for tensile and compressive damage can be taken as before; i.e.:

for tension:

$$g^{+} = \tau^{+}(\bar{\sigma}^{+}) - r^{+} \leq 0 \quad ; \quad d^{+} = G^{+}(r^{+}) \quad ; \quad r^{+} = \max\ [\ r_o^{+}\ ,\ \max(\tau^{+})]$$

for compression:

$$g^{-} = \tau^{-}(\bar{\sigma}^{-}) - r^{-} \leq 0 \quad ; \quad d^{-} = G^{+}(r^{-}) \quad ; \quad r^{-} = \max\ [\ r_o^{-}\ ,\ \max(\tau^{-})]$$

The damage evolution functions, $G^{+(-)}(.)$, the thresholds, $r^{+(-)}$, and the norms, $\tau^{+(-)}$, used here are defined in the appendix A.

The homogenized Cauchy stress tensor σ can be finally expressed by [Faria (1993)]

$$\sigma = (1 - d^{+})\ \bar{\sigma}^{+} + (1 - d^{-})\ \bar{\sigma}^{-} \qquad (2.4.2)$$

RATE DEPENDENT MODEL

3.1 - Rate dependent formulation

It is verified that under high rate of loading the concrete exhibits a significant increase of "strength" and a decrease of nonlinearity on the stress-strain response curves that can be interpreted as a consequence of the retarding of microcracking growth at high strain rates [Suaris and Shah(1984)].

To model this effect, a viscous regularization of the inviscid rate-independent damage model is introduced, in a similar form to the classical viscoplastic regularization of the Perzyna type [Simo and Ju (1987)], rendering for tension and compression damage, the following rate equations:

$$\dot{d}^{+(-)} = \vartheta^{+(-)} <\varphi(\tau^{+(-)} - r^{+(-)})> \frac{\partial g^{\prime +(-)}}{\partial r}(\tau^{+(-)}, r^{+(-)}) \qquad (3.1.1.a)$$

$$\dot{r}^{+(-)} = \vartheta^{+(-)} <\varphi(\tau^{+(-)} - r^{+(-)})> \qquad (3.1.1.b)$$

where $\vartheta^{+(-)}$ is the fluidity parameter, supposed to be a material property, $\varphi^{+(-)}$ is a dimensionless scalar function that represents the viscous damage flow function.

Note that the equations (1) are obtained from the rate-independent eq. (3.3.3) by replacing $\dot{\mu}$ by $\vartheta <\varphi(\tau\text{-}r)>$ and as before, the damage parameter can be expressed explicitly as,

$$\dot{d} = \dot{r}\,\frac{\partial G(r)}{\partial r} \;\Rightarrow\; d = G(r) \tag{3.1.2}$$

Then, a scheme for integrating in time is only necessary to evaluate the current threshold form (1.b).

It can be seen that this assumed rate-dependent formulation is a general form for the damage evolution, so that:

- with a infinity value for the fluidity parameter, the inviscid rate-independent damage evolution law is recovered;
- with a zero value for the fluidity parameter, the damage variable does not present evolution and the elastic response is obtained.

Since the equations (1) guarantee non-decreasing damage parameters, it can be demonstrated that the viscous damage model presented satisfies the condition of non negative dissipation required for a energetic consistent model.

The viscous damage flow function may assume the following form :

$$\varphi(\tau - r) = r_o\,\left(\frac{\tau - r}{r_o}\right)^a \tag{3.1.3}$$

or alternatively,

$$\varphi(\tau - r) = r_o\left(\frac{\tau - r}{r}\right)^a = r_o\left(\frac{\tau}{r} - 1\right)^a \tag{3.1.4}$$

Note that in the above assumption a new parameter a is introduced so that the completely characterization for the viscous damage is affected by the two parameters (ϑ and a) that can be estimated from uniaxial rate stress-strain tests.

If the same viscous regularization is applied on the rate equations employed by Faria & Oliver (3.6), it is obtained:

$$\dot{d} = \vartheta <\varphi(\tau - r)> \frac{\partial G(\tau)}{\partial \tau} \quad (3.1.5.a)$$

$$\dot{r} = \vartheta <\varphi(\tau - r)> \quad (3.1.5.b)$$

From which it is impossible to obtain an explicit expression for *d* as before and in this case, a time integration scheme must be employed in both equations (3.1.5.a) and (3.1.5.b) to evaluate the threshold and the damage parameter.

3.2- Numerical implementation

In this section a consistent algorithm for the numerical time integration of the viscous elastic damage constitutive equations is presented in the context of the finite element method.

Box 1 presents an algorithm performed to update the damage parameters and thresholds in order to enable the evaluation of the stress tensor corresponding to a given strain tensor for each time step $[t_n,t_{n+1}] \subset \mathbb{R}_+$, in a displacement-based incremental finite element code. The analysis of the algorithm revels that for rate-independent procedures no iteration is needed and the time integration is obtained from the explicit expressions for thresholds and damages parameters, eq.(2.3.5) and (2.3.7).

For the rate dependent analysis, the current threshold may be evaluated by adopting a generalized mid-point rule for integrating equation (3.1.1.b) , i.e.,

$$r_{n+1} = r_n + \Delta t \ \vartheta <\varphi(\tau_\alpha - r_\alpha)> \quad (3.2.1)$$

where $\Delta t = t_{n+1} - t_n$. τ_α and r_α are the equivalent stress and the threshold given by a linear combination of these variables at the start and end of the increment, so that :

$$\tau_\alpha = (1-\alpha)\tau_n + \alpha\tau_{n+1}$$
$$r_\alpha = (1-\alpha)r_n + \alpha r_{n+1} \quad (3.2.2)$$

Then, substituting (2) into (1), renders :

$$r_{n+1} = r_n + \Delta t\ \vartheta <\varphi((1-\alpha)(\tau_n - r_n) + \alpha(\tau_{n+1} - r_{n+1}))> \quad (3.2.3)$$

Notice that for $\alpha=1$, the equation (3) corresponds to a backward-Euler difference scheme. It can be demonstrated that algorithm (3) is unconditionally stable for $\alpha\geq0.5$ and second order accurate for $\alpha=0.5$, which allows the use of larger time step intervals in a rate dependent analysis.

Its is obvious that the explicit determination of r_{n+1} in the equation (3) is possible only for some simple assumptions for the viscous damage function $\varphi(.)$ and for the adopted assumptions (3.1.3) or (3.1.4), this is obtained for small integer values of the exponent a. In a general case, the equation (3) may be solved by means of a iterative Newton-Raphson method that ensure a fast rate convergence.

For this purpose, the equation (3) must be rewritten as :

$$f(r_{n+1}) = -r_{n+1} + r_n + \Delta t\ \vartheta <\varphi((1-\alpha)(\tau_n - r_n) + \alpha(\tau_{n+1} - r_{n+1}))> = 0 \quad (3.2.4)$$

so that, the problem now is to find the root of the equation (4) by a iterative Newton-Raphson procedure given by :

$$r_{n+1}^{i+1} = r_{n+1}^{i} - \frac{f(r_{n+1}^{i})}{f'(r_{n+1}^{i})} \quad (3.2.5)$$

where $r^{i+1}{}_{n+1}$ indicates an improved approximation to the exact root obtained from the previous $r^{i}{}_{n+1}$ approximation and f' denotes the first derivative of f respect to r_{n+1}. The iteration procedure (5) initiates for i=0, with $r^{0}{}_{n+1}=r_n$, and finishes when a convergence criterion is obtained. This algorithm is summarized in Box 2.

Box 1 - Algorithm for the rate dependent procedure

INPUT: r_o^+ , r_n^+ , r_o^- , r_n^- , D_o , σ_n , $\Delta\varepsilon$

OUTPUT : σ_{n+1} , r_{n+1}^+ , r_{n+1}^+ , d_{n+1}^+ , d_{n+1}^+ , σ_{n+1}

(1) if $t_n = 0$ set

$$r_n^+ = r_o^+$$
$$r_n^- = r_o^-$$

(2) Evaluate $\sigma_{n+1} = \sigma_n + D_o : \Delta\varepsilon$

(3) Split σ_{n+1} into σ_{n+1}^+ and σ_{n+1}^- according to (4.1).

(4) Compute equivalent stresses $\tau_{n+1}^+ = \tau^+ (\sigma_{n+1}^+)$ and $\tau_{n+1}^- = \tau^- (\sigma_{n+1}^-)$

(5) Check for $\tau_{n+1}^+ > r_n^+$

YES : Check for $\vartheta^+ \geq 0$ (rate dependent case) ?

YES : update threshold r^+ according to the integration scheme of the rate equation (3.1.1.b) described in box 2.

NO : update threshold according to $r_{n+1}^+ = \tau_{n+1}^+$

NO : set $r_{n+1}^+ = r_n^+$

(5) Check for $\tau_{n+1}^- > r_n^-$?

YES : Check for $\vartheta^- \geq 0$ (rate dependent case) ?

YES : update threshold r^- according to the integration scheme of the rate equation (3.1.1.b) described in box 2.

NO : update threshold according to $r_{n+1}^- = \tau_{n+1}^-$

NO : set $r_{n+1}^- = r_n^-$

(6) Evaluate damage variables

$$d_{n+1}^+ = G^+(r_{n+1}^+)$$
$$d_{n+1}^- = G^-(r_{n+1}^-)$$

(7) Compute final Cauchy stress tensor :

$$\sigma_{n+1} = (1 - d_{n+1}^+)\, \sigma_{n+1}^+ + (1 - d_{n+1}^-)\, \sigma_{n+1}^-$$

Box 2 - Algorithm for Threshold updating scheme (Newton-Raphson procedure)

INPUT : r_n , τ_n , τ_{n+1} , α , Δt , ϑ, *TOLER*

OUTPUT: r_{n+1}

(1) Set

$$i = 0$$

$$r^{i}_{n+1} = r_n$$

(2) Evaluate f and $f' = \dfrac{\partial f}{\partial r_{n+1}}$ from (4)

(3) Compute:

$$r^{i+1}_{n+1} = r^{i}_{n+1} - \frac{f}{f'}$$

(4) Evaluate the error:

$$\varepsilon = \frac{r^{i+1}_{n+1} - r^{i}_{n+1}}{r^{i}_{n+1}}$$

(5) Check the convergence: $\varepsilon \leq TOLER$?

YES: Convergence attained. Set

$$r_{n+1} = r^{i+1}_{n+1}$$

NO: Further iteration needed. Set $i = i+1$ and GO TO step (2).

NUMERICAL APPLICATIONS

4.1- Tension compression cyclic test

This first example is presented to demonstrate the capacity of the model to be adjusted to accommodate the characteristic tensile and compressive behavior of the concrete for inviscid situations.

Figure 1 shows a uniaxial stress-strain curve for a cyclic imposed loading , plotted in a non-dimensionalized mapped form (f x e) obtained by dividing the stress from the respective linear-elastic stress thresholds and the strain from the respective linear-elastic strain thresholds, i.e.,

$$f = \frac{\sigma^{+(-)}}{f_o^{+(-)}} \quad ; \quad e = \frac{\varepsilon^{+(-)}}{\varepsilon_o^{+(-)}} = \frac{\varepsilon^{+(-)}}{\dfrac{f_o^{+(-)}}{E_o}}$$

with E_0 being the initial elastic secant modulus of the concrete.

The parameters that define this idealized concrete behavior are given by (see appendix 1):

$$A^{+} = 0.677$$
$$A^{-} = 0.000$$
$$B^{-} = 0.890$$

The load sequence and the response analysis is described as follow :

(1) - Path OBCO - The undamaged material experiments tensile loads and exhibits a linear-elastic behavior up to tensile threshold (B) and a softening branch thereafter (B-C). The irreversible tensile damage produced in the branch B-C can be evidenced by the reduced elastic modulus observed during the discharge C-O.

(2) - Path ODEO - The material submitted to a compressive loading recovers the initial undamaged stiffness (the *Unilateral Effect* is verified) and exhibits a linear-elastic behavior until the compressive damage threshold is attained. After this, the material experiments a non linear hardening and a subsequent softening effect (D-E). The discharge E-O demonstrate the irreversible compressive damage occurred during the non linear branch D-E.

(3) - The latter incursions in the tensile regimen OCFO and consecutively in compressive regimen OEGO, demonstrate the total independence between the compressive and tensile damage state when the load passes to tension from compression and viceversa.

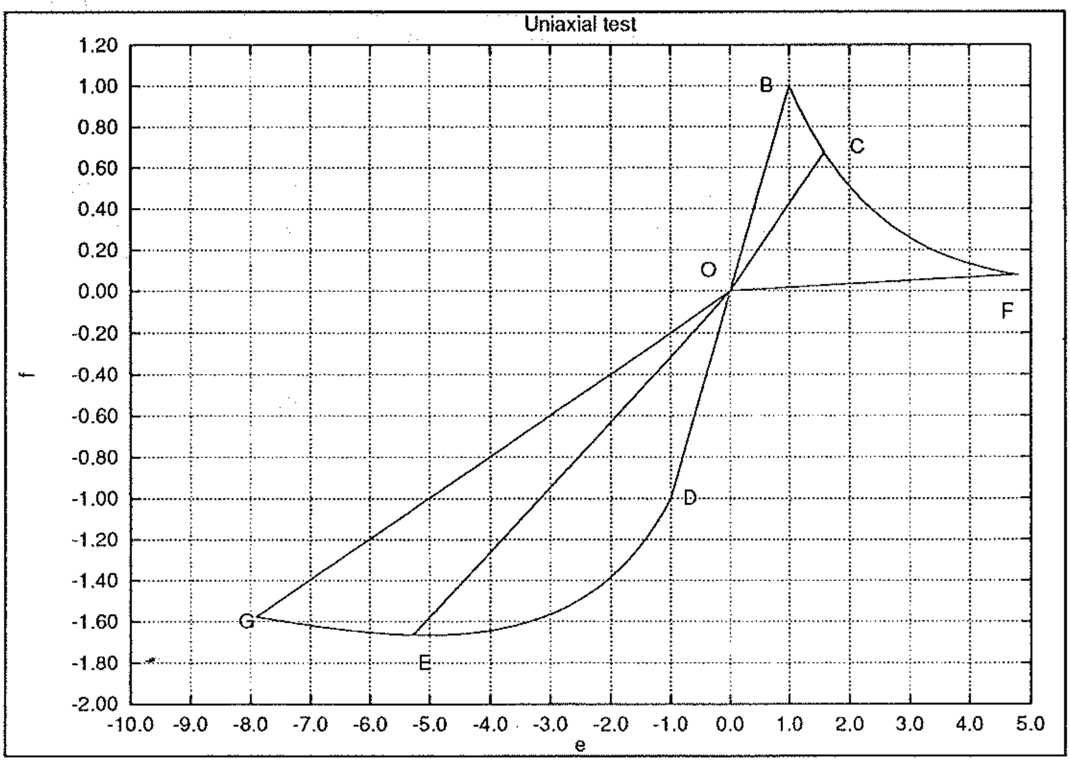

Figure 1 - Uniaxial cyclic test.

4.2- Rate dependency effect for the Koyna Dam concrete

For the rate dependent analysis, it will be used the following material parameters calibrated to describe the behavior of the concrete of Koyna Dam [Galindo (1993)]:

$$E = 31.64 \times 10^6 \ KN/m^2 \ ; \qquad \nu = 0.2 \ ;$$

$$f_o^+ = 2000 \ KN/m^2 \ ; \qquad f_o^- = 12000 \ KN/m^2 \ ;$$

$$G_f = 0.25 \ KJ/m^2 \ ; \qquad A^+ = 0.677 \ ;$$

$$A^- = 0.000 \ ; \qquad B^- = 0.890 \ ;$$

Figures 1 and 2 illustrate the stress-strain curves obtained for uniaxial tension at different strain rates ($v = \dot{\varepsilon}$), for each of the viscous damage flow functions defined in equations (3.1.3) and (3.1.4).

It can be noted that a decrease in the nonlinearity of the stress-strain curves is manifested by a increasing in the peak strength as the strain rate is increased. It can be also seen that the increasing of the dissipated energy as the strain rate is more accentuated for the second case.

The viscous parameters $\vartheta^{+(-)}$ and $a^{+(-)}$ which define the rate-dependent behavior have been calibrated to match the analytical response to the experimental *peak strength ratio x strain-rate* curves obtained by Suaris and Shah (1985), as shown in figure 4. The peak strain ratio is referred to the relation between the dynamic and the static peak strength. The values of the selected are:

$$\vartheta^+ = 640 \ s^{-1} \ ; \qquad \vartheta^- = 40000 \ s^{-1} \ ;$$

$$a^+ = 5 \ ; \qquad a^- = 5 \ ;$$

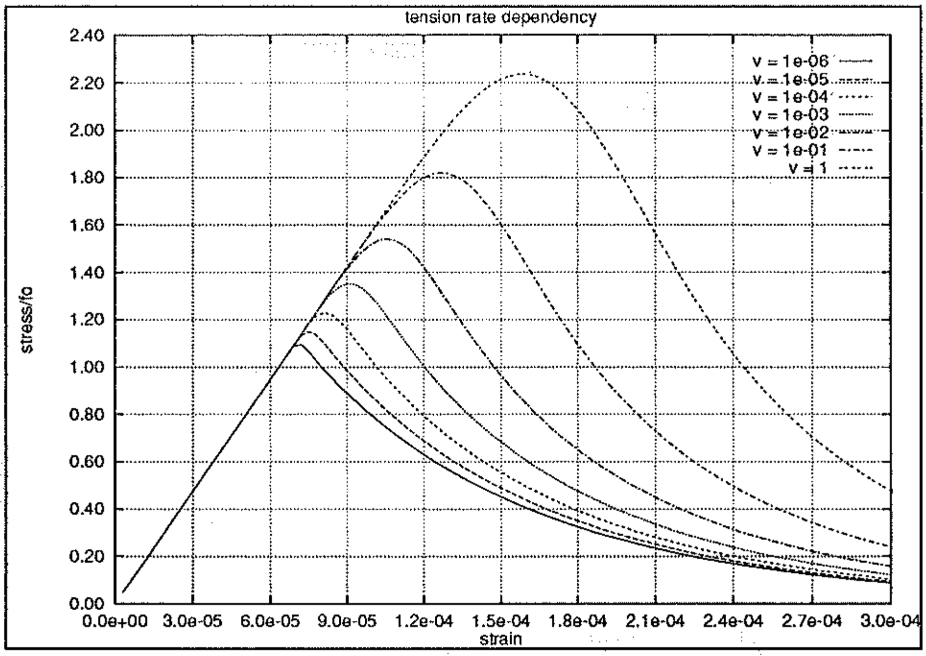

Figure 2 - Rate dependent effect for viscous damage flow (3.1.3)

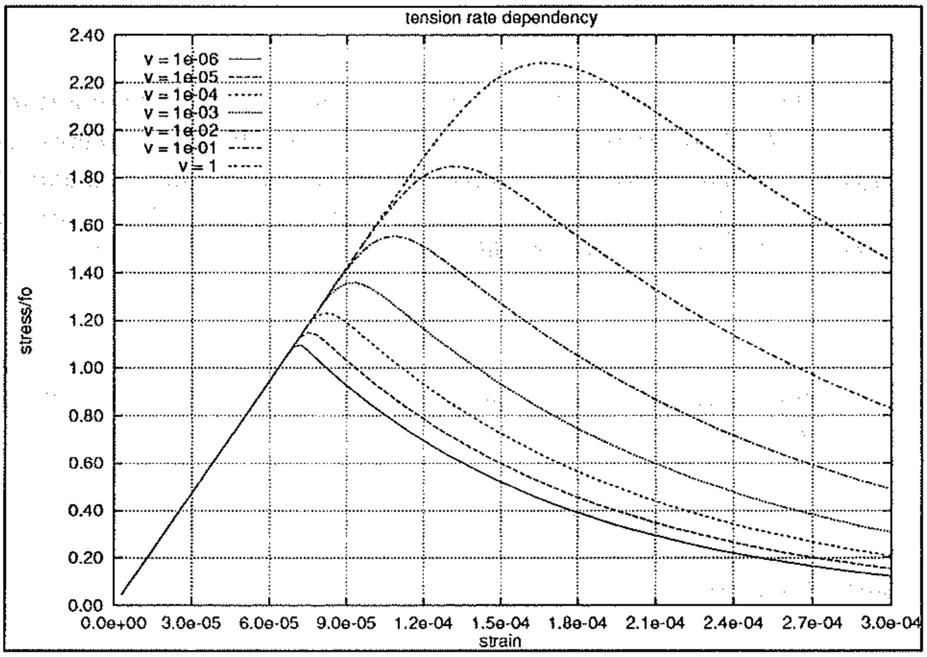

Figure 3 - Rate dependent effect for viscous damage flow (3.1.4)

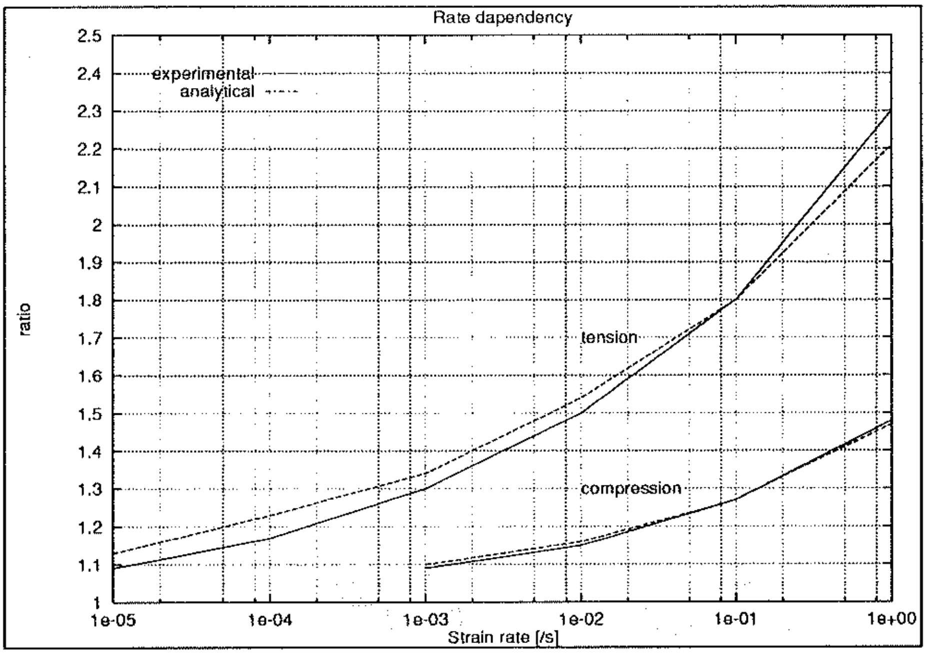

Figure 4 - Peak strength ratio x strain-rate [Suaris & Shah(1985)]

4.3 - Time integration performance

The performance obtained by using the general mid point rule with α=0.5 and α=1.0 (forward Euler) to integrate the rate damage evolution equations for a tensile loading ($\dot{\varepsilon}=10^{-1}$) are shown in the figures 5 and 6. As expected, the first algorithm leads to a more accurate results and it is able to capture the peak strength with less number of steps (or with greater time steps intervals).

4.4 - The viscous damage model presented by Faria & Oliver(1993) and Simo & Ju (1987)

As seen in section 3.1, the viscous regularization of the damage evolution used by Faria & Oliver (1993) and Simo & Ju (1987)] leads to following rate dependency law:

$$\dot{d} = \vartheta <\varphi(\tau - r)> \frac{\partial G(\tau)}{\partial \tau}$$

$$\dot{r} = \vartheta <\varphi(\tau - r)>$$

If the functions $G(.)$ and $\varphi(.)$ adopted before are also used here, the model presents the improper performance illustrated in figure 7 where it is shown that the damage parameter tends to a non null value when strain goes to infinity, so that the curves recover the elastic ascendent form. This effect is given by the fast descending to the zero value of the derivative of G(.) in the first equation conducing to the constant values of the damage before attaining zero value.

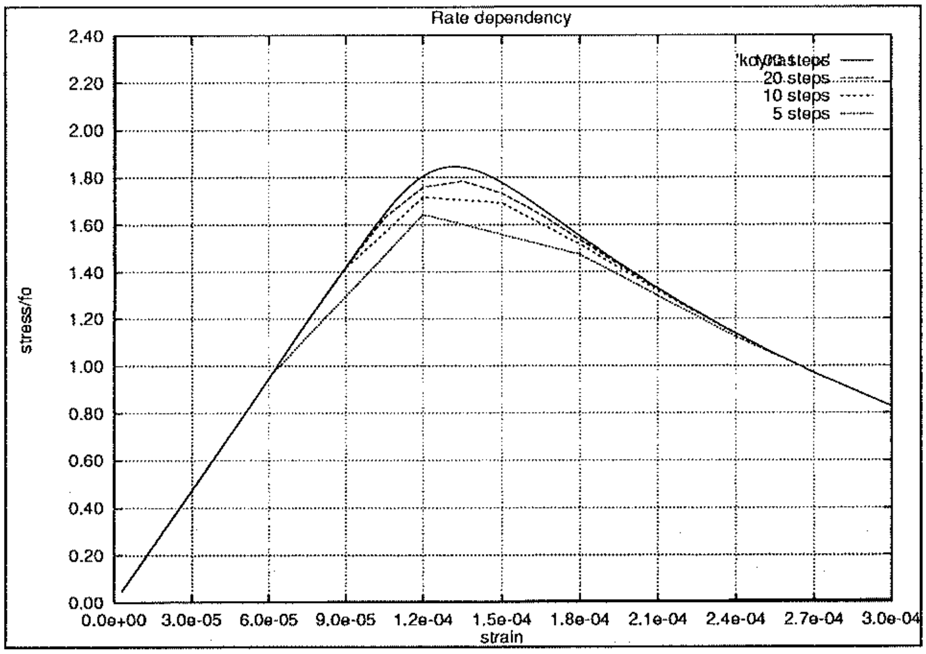

Figure 5 - Time integration performance for α=1 (Forward Euler)

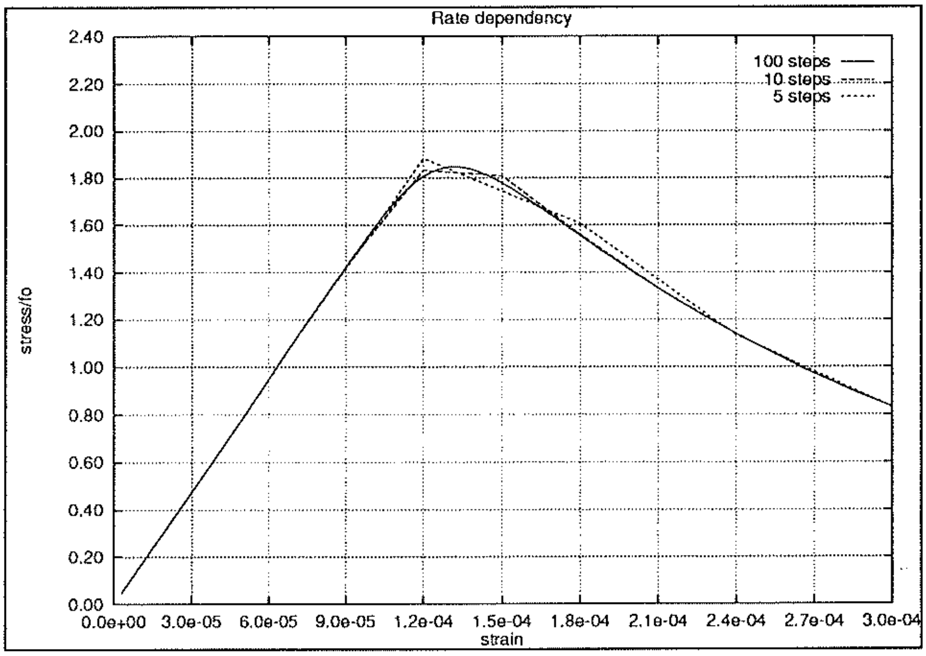

Figure 6 - Time integration performance for α=0.5.

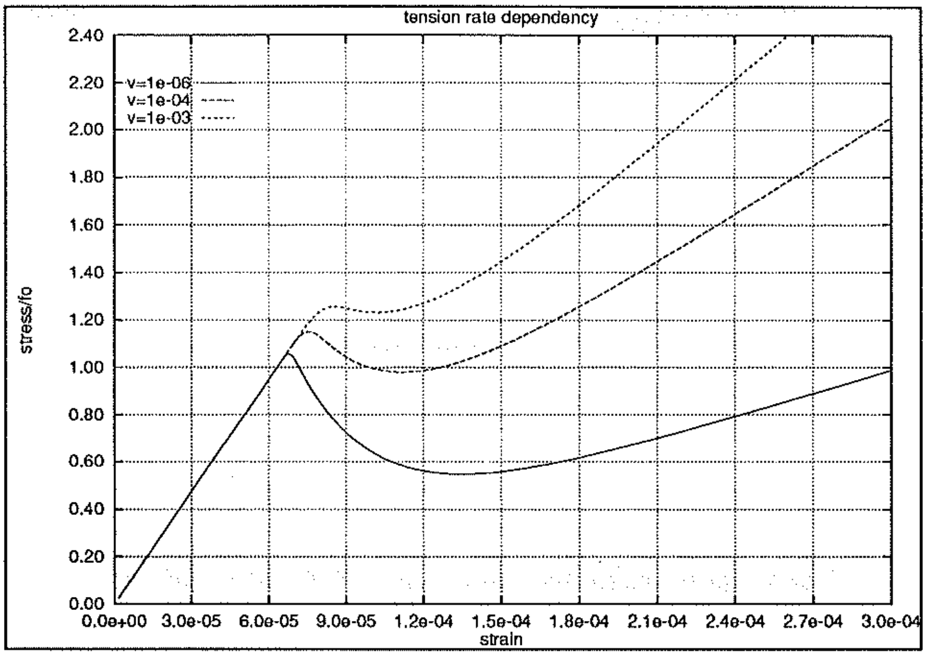

Figure 7 - Viscous damage model (Faria&Oliver and Simo&Ju)

APPENDIX A

A.1 - Equivalent norm of the positive part of stress tensor - τ^+

The scalar norm for the positive part of the stress tensor is assumed as the energy norm given by [Simo and Ju(1987)]:

$$\bar{\tau}^+(\sigma^+) = \sqrt{\sigma^+ : D_o^{-1} : \sigma^+}$$

A.2 - Equivalent norm of the negative part of the stress tensor - τ^-

Inspired on the Drucker-Pragger criterion, the norm for the negative part is given as [Faria & Oliver(1993)]:

$$\bar{\tau}^-(\sigma^-) = \sqrt{\sqrt{3}\,(K\ \sigma^-_{oct} - \tau^-_{oct})}$$

where K is given as :

$$K = \sqrt{2}\,\frac{1-R_o}{1-2R_o}$$

with R_o being the following ratio:

$$R_o = \frac{f^-_{o2D}}{f^-_{o1D}}$$

f_{o1D} and f_{o2D} denote the maximum σ_3 elastic stresses obtained for the 1D and 2D tests.

A.3 - Initial tensile and compressive thresholds - r_o^+, r_o^-

The initial threshold for the damage criterion is given for the norms defined above of the uniaxial elastic threshold stress, i.e.:

• for tension :

$$r_o^+ = \sqrt{f_{o1D}^+ \frac{1}{E} f_{o1D}^+} = \frac{f_{o1D}^+}{\sqrt{E}}$$

• for compression

$$r_o^- = \sqrt{\frac{f_{o1D}^-}{\sqrt{3}}(K - \sqrt{2})} = \sqrt{\sqrt{\frac{2}{3}} \frac{R_o}{1-2R_o} f_{o1D}^-}$$

A.4 - Tensile damage parameter evolution law - $G^+(.)$

For the tensile damage parameter it is assumed the following form :

$$G^+(r^+) = 1 - \frac{r_o^+}{r^+} e^{A^+(1 - \frac{r^+}{r_o^+})}$$

As can be noted, the assumed function presents zero value for $r^+ = r_o^+$ and unit value for r^+ tending to infinity.

The parameter A^+ is obtained by the relating between the total dissipated energy on a 1D tensile process and the concrete fracture energy, G_f, so that [Faria & Oliver(1993)]:

$$A^+ = \left(\frac{G_f\, E}{l^*(f_{o1D}^+)^2} - \frac{1}{2} \right)^{-1}$$

with l^* being the characteristic length of the finite element considered [Oliver (1989)].

A.5 - Compressive damage parameter evolution law - $G^{-}(.)$

For compressive damage it is assumed :

$$G^{-}(r^{-}) = 1 - \frac{r_o^{-}}{r^{-}}(1 - A^{-}) - A^{-}e^{B^{-}(1 - \frac{r^{-}}{r_o^{-}})}$$

from which is possible verify that this function also provides values in the interval [0,1]. The parameters A^{-} and B^{-} are determined to fit the 1D compressive stress-strain test. From this parameter is possible to produce the hardening effect on the concrete submitted to compression, as well the softening which occurs after the compressive strength is attained (see figure 1).