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NON-SYMMETRICAL BE-FE PARTITIONED FORMULATION FOR ACOUSTIC FLUID-STRUCTURE INTERACTION PROBLEMS

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Abstract. This work presents a non symmetric Finite-Boundary Element Tearing and Interconnecting (FE-BETI) formulation for acoustic FSI problems, where the finite element method is used to model the structure, while the acoustical fluid domain is represented by the boundary element method. The method interconnects fluid and structure domains using the localized Lagrange multipliers, allowing the use of non-matching meshes on the interfaces. Furthermore, the methodology proposes a preconditioned projected biconjugate gradient solver, that presents good scalability properties in the solution of large problems.

1 INTRODUCTION

This paper extends the recently proposed nsBETI [1] formulation to acoustics-FSI problems, where the finite element method is used to model the structure, while the acoustics fluid domain is represented by the boundary element method. This non-overlapping domain decomposition technique uses the classical non-symmetrical acoustics boundary element formulation, instead of a symmetric Galerkin boundary element one [2]. The method interconnects fluid and structure domains using the localized Lagrange multipliers [3, 4, 5, 6, 7, 8, 9, 10, 11], which also allow considering non-matching meshes on the interfaces. Furthermore, the methodology uses a preconditioned Bi-Conjugate Gradient

[1]The authors would like to dedicate this work to the memory of Prof. Ramón Abascal García (1956-2013). We mourn his untimely death as we loose a great engineering educator, researcher and above all, a good person.

Stabilized (Bi-CGSTAB) algorithm, which presents a very good scalability in the solution of large problems.

2 ACOUSTIC FSI PARTITIONED FORMULATION

A FEM structure and a BEM fluid domain are considered, so the total virtual work of the system δW_T can be expressed as the addition of the virtual work done by the FEM structure domain δW^s, the BEM fluid domain δW^f and the interface coupling contribution δW_c,

$$\delta W_T = \delta W^s + \delta W^f + \delta W_c \tag{1}$$

2.1 Structure domain

The virtual work of a flexible structure, which is susceptible to the dynamic of the fluid, δW^s, is described by the principle of virtual work for a continuum body of domain Ω_s and surface Γ_s that, assuming small displacements,

$$\delta W^s = \int_{\Omega_s} \boldsymbol{\sigma}_s : \nabla \delta \mathbf{u}_s d\Omega - \int_{\Omega_s} (\omega^2 \rho \mathbf{u}_s + \mathbf{b}_s) \cdot \delta \mathbf{u}_s d\Omega - \int_{\Gamma_s} \mathbf{t}_s \cdot \delta \mathbf{u}_s d\Gamma \tag{2}$$

where $\mathbf{u}_s$ are the structural displacements, $\boldsymbol{\sigma}_s$ the Cauchy stress tensor, $\mathbf{t}_s$ the applied surface tractions and $\mathbf{b}_s$ the body forces. Finally, ω and ρ are the angular frequency of the displacement oscillation and the density of the structures, respectively.

Next, the substructure is discretized using the classical FEM approximation, where the assembly of element contributions by the direct stiffness method leads to the semidiscrete equations of motion:

$$\delta W^s = \delta \mathbf{d}_s^T \{(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{d}_s - \mathbf{f}_s\} \tag{3}$$

where $\mathbf{K}$ is the stiffness matrix, $\mathbf{M}$ is the mass matrix, $\mathbf{d}_s$ is the vector of nodal displacements and $\mathbf{f}_s$ is the applied nodal forces. Equation (3) can be written in a more compact form

$$\delta W^s = \delta \mathbf{d}_s^T \{\bar{\mathbf{K}} \mathbf{d}_s - \mathbf{f}_s\} \tag{4}$$

defining $\bar{\mathbf{K}} = (\mathbf{K} - \omega^2 \mathbf{M})$.

2.2 Fluid domain

The governing equation for the linear acoustic fluid domain Ω_f is known as the Helmholtz equation and can be written as

$$\Delta p + k^2 p = 0 \tag{5}$$

where Δ represents the Laplace operator, p is the acoustic pressure, $k = \omega / c$ is the wave number, and ω and c are the angular frequency of the pressure oscillation and the speed of sound traveling in the fluid, respectively. On the boundary Γ_f can be prescribed the following boundary conditions:

- Neumann boundary condition

$$\frac{\partial p}{\partial n} = -i\rho\omega v_n \tag{6}$$

- Rigid boundary

$$\frac{\partial p}{\partial n} = 0 \tag{7}$$

where n denotes the unit normal at the surface point, ρ is the density of medium, v_n represents the velocity on the boundary surface and $i = \sqrt{-1}$.

The BEM formulation for acoustic problems is well known and can be found in many classical texts like [12], where the Helmholtz equation (5) is transformed into a boundary integral equation. First, Helmholtz equation is written in a weak form using a weighted residual approach with the following fundamental solution as the weight function

$$G(\mathbf{x}, \mathbf{y}) = \frac{e^{-ik|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|} \tag{8}$$

In the expression above, $|\mathbf{x} - \mathbf{y}|$ is the distance between the collocation point $\mathbf{x}$ and the source point $\mathbf{y}$. Applying Green's second theorem on the weighted residual and defining the collocation points at the boundary, the resulting boundary integral equation can be expressed as

$$C(\mathbf{x})p(\mathbf{x}) + \int_{\Gamma_f} p(\mathbf{y})\frac{\partial G(\mathbf{x}, \mathbf{y})}{\partial n} d\Gamma = \int_{\Gamma_f} G(\mathbf{x}, \mathbf{y})\frac{\partial p(\mathbf{y})}{\partial n} d\Gamma \tag{9}$$

where $C(\mathbf{x})$ is a coefficient which depends on the position of point $\mathbf{x}$: $C(\mathbf{x}) = 1$ for an internal point, $C(\mathbf{x}) = 1/2$ for $\mathbf{x}$ on a smooth boundary Γ_f, and $C(\mathbf{x}) = 0$ for a external point.

Equation (9) can be rewritten, taking into account the Neumann boundary condition (6), as

$$C(\mathbf{x})p(\mathbf{x}) + \int_{\Gamma_f} p(\mathbf{y})\frac{\partial G(\mathbf{x}, \mathbf{y})}{\partial n} d\Gamma = -\int_{\Gamma_f} G(\mathbf{x}, \mathbf{y}) i\rho\omega v_n(\mathbf{x}) d\Gamma \tag{10}$$

The boundary Γ_f is divided into N_e elements, $\Gamma_e \in \Gamma$, so: $\Gamma_f = \bigcup_{e=1}^{N_e} \Gamma_e$ and $\bigcap_{e=1}^{N_e} \Gamma_e = \emptyset$. The fields p and v_n are approximated over each element Γ_e using shape functions, as a function of the nodal values

$$p = \sum_{j=i}^{m} N_j p_j = \mathbf{N}_f \mathbf{p} \quad v_n = \sum_{j=i}^{m} N_j v_{nj} = \mathbf{N}_f \mathbf{v} \tag{11}$$

p_j and v_{nj} being the nodal values acoustics pressure and particle normal velocity at node j, respectively, and $\mathbf{N}_f$ being the shape functions approximation matrix. A discrete linear equations set is obtained when we substitute Eq. (11) into Eq. (10) and point $\mathbf{x}$ is chosen to be all the boundary nodes

$$C_i\delta_{ij}p_j + \sum_{e=i}^{N_e}\int_{\Gamma_e}\frac{\partial G(\mathbf{x},\mathbf{y})}{\partial n}N_j p_j d\Gamma_e = -\int_{\Gamma_e} i\rho\omega G(\mathbf{x},\mathbf{y})N_j v_{nj} d\Gamma_e \tag{12}$$

being δ_{ij} is the Kronecker δ-symbol. Equation (12) can be written in a matrix form

$$\mathbf{Hp} = \mathbf{Gv} \tag{13}$$

where

$$\mathbf{H} = C_i\delta_{ij}p_j + \sum_{e=i}^{N_e}\int_{\Gamma_e}\frac{\partial G(\mathbf{x},\mathbf{y})}{\partial n}N_j d\Gamma_e \quad \mathbf{G} = -\int_{\Gamma_e} i\rho\omega G(\mathbf{x},\mathbf{y})N_j d\Gamma_e \tag{14}$$

The particles normal velocity at every node can be computed as

$$\mathbf{v} = i\omega\mathbf{d}_f \tag{15}$$

so Eq. (13) can be expressed in terms of the nodal acoustic pressure and particles normal displacement:

$$\mathbf{Hp} = i\omega\mathbf{G}\mathbf{d}_f \tag{16}$$

The virtual work of a BE fluid domain can be computed using a weak statement for static equilibrium reduced to the boundary, using the Clapeyron formula,

$$\delta W^f = \int_{\Gamma_f}(\mathbf{p} - \bar{\mathbf{t}})\cdot\delta\mathbf{u}_f d\Gamma \tag{17}$$

combined with the discretized BE equation (16). Note that symbol δW is used instead of $\delta\Pi$ to express virtual work, emphasizing that in general this variational statement does not derive from an energy functional.

Equation (17) is discretized using the BE mesh to obtain a discrete approximation of the virtual work:

$$\delta W^f = \delta\mathbf{d}_f^T(\int_\Gamma \mathbf{N}^T\mathbf{N}, d\Gamma)(\mathbf{p} - \bar{\mathbf{t}}) = \delta\mathbf{d}_f^T\mathbf{M}(\mathbf{p} - \bar{\mathbf{t}}) \tag{18}$$

with

$$\mathbf{M} = \int_{\Gamma_b}\mathbf{N}^T\mathbf{N}\,d\Gamma \tag{19}$$

Substituting the discrete tractions $\mathbf{p}$ coming from the BE equations (16) a final expression for the discrete variation is obtained

$$\delta W^f = \delta\mathbf{d}_f^T\mathbf{M}\{\mathbf{H}^{-1}i\omega\mathbf{G}\mathbf{d}_f - \bar{\mathbf{t}}\} \tag{20}$$

As it has been done in previous section, Eq.(20) is written in a more compact form

$$\delta W^f = \delta\mathbf{d}_f^T\{\bar{\mathbf{A}}\mathbf{d}_f - \mathbf{f}_f\} \tag{21}$$

defining $\bar{\mathbf{A}} = i\omega\mathbf{M}\mathbf{H}^{-1}\mathbf{G}$.

3 LOCALIZED LAGRANGE MULTIPLIERS

The virtual work for the interface frame δW_c can be also evaluated applying the variational-based formulation proposed by Park and Felippa [3, 4], and González et al. [14]. The virtual work variation of the total system δW_T consists of those of both FE structure and BE fluid, δW^s and δW^f, plus of the interface frame δW_c. This formulation enforces the kinematical positioning of the frame in a weak sense with the following expression

$$W_c = \int_{\Gamma_c} \{\lambda_n^s(u_n^s - u_f) + \{\lambda_n^f(u_n^f - u_f)\} d\Gamma \tag{22}$$

where both integrals are extended to the boundary interface Γ_c. The localized Lagrange multipliers and the displacements on the structure interface are represented by $(\lambda_n^s,\, u_n^s)$, and the fluid localized Lagrange multipliers and displacements by (λ_n^f, u_n^f). Finally, the frame displacements are represented by u_f.

Equation (22), can be written in a matrix form as follows

$$\delta W_c = \delta\{\boldsymbol{\lambda}_s^T(\mathbf{B}_s^T\mathbf{d}_s - \mathbf{L}_s\mathbf{u}_f)\} + \delta\{\boldsymbol{\lambda}_f^T(\mathbf{B}_f^T\mathbf{d}_f - \mathbf{L}_f\mathbf{u}_f)\} \tag{23}$$

$\mathbf{B}_s$ and $\mathbf{B}_f$ being the boolean matrices defined in previous section, used to extract the interface displacements of the structure and fluid, respectively, and $\mathbf{L}_f$ [5, 14] is a matrix whose terms are obtained evaluating the frame shape functions at the interface structure and fluid nodes projection over the frame: P_j^s and P_j^f (see Fig.1).

The total virtual work of the coupled BEM-FEM-Frame system can now be expressed as

$$\begin{aligned} \delta W_T = \delta\mathbf{d}_s^T\{\bar{\mathbf{K}}\mathbf{d}_s + \mathbf{B}_s\boldsymbol{\lambda}_s - \mathbf{f}_s\} + \delta\mathbf{d}_f^T\{\bar{\mathbf{A}}\mathbf{d}_f + \mathbf{B}_f\boldsymbol{\lambda}_f - \mathbf{f}_f\} \\ +\delta\boldsymbol{\lambda}_s^T\{\mathbf{B}_s^T\mathbf{d}_s - \mathbf{L}_s\mathbf{u}_f\} + \delta\boldsymbol{\lambda}_f^T\{\mathbf{B}_f^T\mathbf{d}_f - \mathbf{L}_f\mathbf{u}_f\} \\ \delta\mathbf{u}_f^T\{\mathbf{L}_s^T\boldsymbol{\lambda}_s + \mathbf{L}_f^T\boldsymbol{\lambda}_f\} \end{aligned} \tag{24}$$

The stationary point of the above virtual work expression leads to the following coupled equilibrium equation set:

$$\begin{bmatrix} \bar{\mathbf{K}} & \mathbf{0} & \mathbf{B}_s & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{A}} & \mathbf{0} & \mathbf{B}_f & \mathbf{0} \\ \mathbf{B}_s^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_s \\ \mathbf{0} & \mathbf{B}_f^T & \mathbf{0} & \mathbf{0} & \mathbf{L}_f \\ \mathbf{0} & \mathbf{0} & \mathbf{L}_f^T & \mathbf{L}_s^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{d}_s \\ \mathbf{d}_f \\ \boldsymbol{\lambda}_s \\ \boldsymbol{\lambda}_f \\ \mathbf{u}_f \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s \\ \mathbf{f}_f \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{25}$$

If we have N_p fluid and structure partitions, equation (25) can be written as

$$\begin{bmatrix} \mathbf{K} & \mathbf{B} & \mathbf{0} \\ \mathbf{B}^T & \mathbf{0} & \mathbf{L} \\ \mathbf{0} & \mathbf{L}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \boldsymbol{\lambda} \\ \mathbf{u}_f \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \tag{26}$$

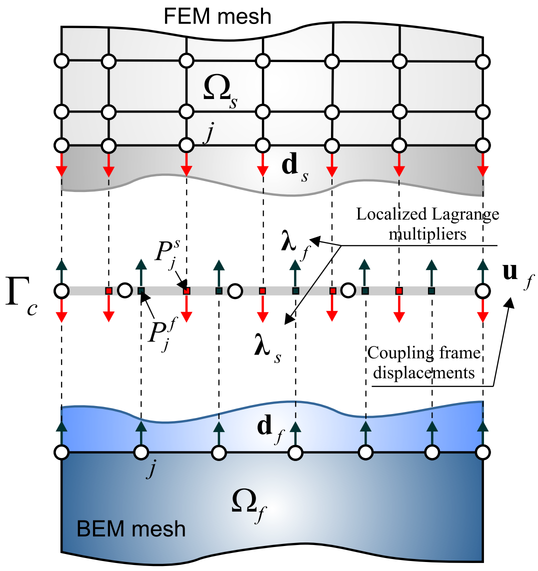


Figure 1: Fluid Structure BEM-FEM system with intercalated frame and localized Lagrange multipliers.

where

$$\mathbf{B} = \mathbf{diag}\left[\mathbf{B}^{(1)} \dots \mathbf{B}^{(N_p)}\right] \quad \text{with} \quad \mathbf{B}^{(p)} = \begin{cases} \mathbf{B}_s^{(p)} & \text{(FEM)} \\ \mathbf{B}_f^{(p)} & \text{(BEM)} \end{cases} \tag{27}$$

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}^{(1)} \\ \vdots \\ \mathbf{L}^{(N_p)} \end{bmatrix} \quad \text{with} \quad \mathbf{L}^{(p)} = \begin{cases} \mathbf{L}_s^{(p)} & \text{(FEM)} \\ \mathbf{L}_f^{(p)} & \text{(BEM)} \end{cases} \tag{28}$$

$$\boldsymbol{\lambda} = \begin{Bmatrix} \boldsymbol{\lambda}^{(1)} \\ \vdots \\ \boldsymbol{\lambda}^{(N_p)} \end{Bmatrix} \quad \text{with} \quad \boldsymbol{\lambda}^{(p)} = \begin{cases} \boldsymbol{\lambda}_s^{(p)} & \text{(FEM)} \\ \boldsymbol{\lambda}_f^{(p)} & \text{(BEM)} \end{cases} \tag{29}$$

Eliminating $\mathbf{d}$ from the first row of (26) using the relation

$$\mathbf{d} = \mathbf{K}^{-1}(\mathbf{f} - \mathbf{B}\boldsymbol{\lambda}) \tag{30}$$

the following flexibility matrix equation is obtained

$$\begin{bmatrix} \mathbf{F}_{bb} & \mathbf{L} \\ \mathbf{L}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda} \\ \mathbf{u}_f \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix} \tag{31}$$

being $\mathbf{F}_{bb} = \mathbf{B}^T\mathbf{K}^{-1}\mathbf{B}$ and $\mathbf{b} = \tilde{\mathbf{B}}^T\mathbf{K}^{-1}\mathbf{f}$.

4 ITERATIVE SOLUTION ALGORITHM FOR THE INTERFACE PROBLEM

In this section the solution strategy is presented to solve the flexibility system obtained for the FSI localized Lagrange multipliers formulation, together with some studies of convergence and scalability.

4.1 LLM coupled system

The algorithm uses a decomposition of the interface solution vector in the form:

$$\boldsymbol{\lambda} = \mathcal{P}_{\mathbf{L}} \boldsymbol{\lambda}_d \tag{32}$$

with symmetric projector

$$\mathcal{P}_{\mathbf{L}} = \mathbf{I} - \mathbf{L}(\mathbf{L}^T\mathbf{L})^{-1}\mathbf{L}^T \tag{33}$$

such as $\mathcal{P}_{\mathbf{L}}\mathbf{L} = \mathbf{0}$

Substituting this decomposition into the flexibility formulation of the interface (31) yields the following equation set:

$$\mathcal{P}_{\mathbf{L}}\mathbf{F}_{bb}\mathcal{P}_{\mathbf{L}}\boldsymbol{\lambda}_d = \mathcal{P}_{\mathbf{L}}\mathbf{b} \tag{34}$$

The projected residual is finally given by

$$\mathbf{r} = \mathcal{P}_{\mathbf{L}}(\mathbf{b} - \mathbf{F}_{bb}\mathcal{P}_{\mathbf{L}}\boldsymbol{\lambda}_d) \tag{35}$$

and it is solved for $\mathcal{P}_{\boldsymbol{\lambda}}\boldsymbol{\lambda}_d$.

Iterate on the projected residual: $\mathbf{r} = \mathcal{P}_{\mathbf{L}}(\mathbf{b} - \mathbf{F}_{bb}\boldsymbol{\lambda})$, using the proposed preconditioned projected bi-conjugate gradient algorithm:

(I) Initialize: $\boldsymbol{\lambda}_0$, $\mathbf{r}_0 = \mathcal{P}_{\mathbf{L}}(\mathbf{b} - \mathbf{F}_{bb}\boldsymbol{\lambda}_0)$, $\mathbf{x}_0 = \mathbf{0}$, $\mathbf{p}_0 = \mathbf{0}$.

(II) Iterate $i = 1, 2, 3...$ until convergence:

- → Compute: $\mathbf{p}_i = \mathbf{r}_{i-1} + \omega_i(\mathbf{p}_{i-1} - \alpha_{i-1}\mathbf{x}_{i-1})$ being $\mathbf{p}_1 = \mathbf{r}_0$, $\beta_i = \mathrm{Re}\{(\mathbf{r}_0^T\mathbf{r}_{i-1})\}$, and $\omega_i = \beta_i\gamma_{i-1}/(\alpha_{i-1}\beta_{i-1})$.
- → Precondition: $\mathbf{a}_i = \tilde{\mathbf{F}}_{bb}^{+}\mathbf{p}_i$.
- → Projection: $\mathbf{z}_i = \mathcal{P}_{\mathbf{L}}\mathbf{a}_i$.
- → Compute: $\mathbf{v}_i = \mathbf{r}_{i-1} - \gamma_i\mathbf{x}_i$, being $\mathbf{b}_i = \mathbf{F}_{bb}\mathbf{z}_i$, $\mathbf{x}_i = \mathcal{P}_{\mathbf{L}}\mathbf{b}_i$, and $\gamma_i = \beta_i/\mathrm{Re}\{(\mathbf{x}_i^T\mathbf{r}_0)\}$.
- → Precondition: $\mathbf{c}_i = \tilde{\mathbf{F}}_{bb}^{+}\mathbf{v}_i$.
- → Projection: $\mathbf{y}_i = \mathcal{P}_{\mathbf{L}}\mathbf{c}_i$.
- → Update solution: $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{i-1} + \gamma_i\mathbf{z}_i + \alpha_i\mathbf{y}_i$, being $\mathbf{g}_i = \mathbf{F}_{bb}\mathbf{y}_i$, $\mathbf{w}_i = \mathcal{P}_{\mathbf{L}}\mathbf{g}_i$, and $\alpha_i = \mathrm{Re}\{(\mathbf{w}_i^T\mathbf{v}_i)\}/\mathrm{Re}\{(\mathbf{w}_i^T\mathbf{w}_i)\}$.

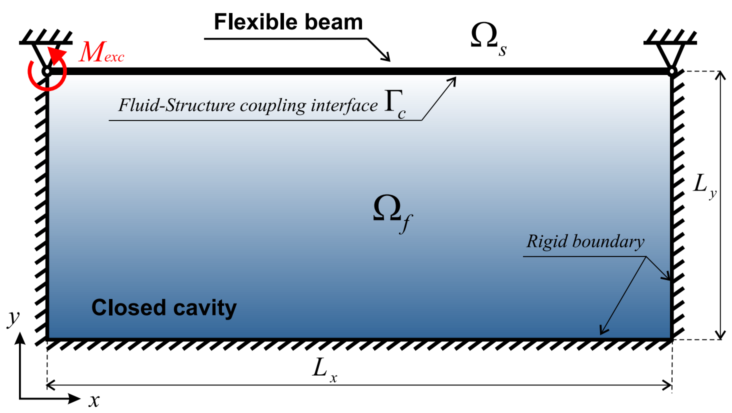


Figure 2: Beam backed by a closed acoustic cavity.

→ Update residual: $\mathbf{r}_i = \mathbf{v}_i - \alpha_i \mathbf{w}_i$.

(III) If $\|\mathbf{r}_i\|/\|\mathbf{r}_0\| > \epsilon$, $i \leftarrow i+1$ go back to (II).

The preconditioner proposed is based on extensions of the well-known lumped and Dirichlet preconditioners in the standard FETI and AFETI algorithms. These preconditioners are defined as

$$\tilde{\mathbf{F}}_{bb}^{+} = \begin{cases} \bar{\mathbf{K}}_{bb} & \text{(FEM)} \\ \bar{\mathbf{A}}_{bb} & \text{(BEM)} \end{cases} \tag{36}$$

where subscript (bb) refers to boundary extraction, i.e. pre and post multiplication by $\mathbf{B}^T$ and $\mathbf{B}$ respectively.

4.2 Benchmark application: flexible wall and acoustic cavity

The coupling possibilities of LLM methodologies are studied in the section, solving a benchmark problem: a two dimensional $L_x \times L_y$ cavity (L_x =10m and L_y =4m), with one flexible side (see Fig. 2). The flexible wall is modeled by the FEM, using beam elements, and simply supported on both edges. The properties of this structural domain are: Young module $E = 2.1 \times 10^{11}$ Pa, beam section inertia I =1.59 $\times 10^{-4}$ m^4 and cross section area A =0.02 m^2, and a mass per unit length m_s =50 kg/m. The remaining edges of the cavity are reverberant walls, i.e. homogeneous Neumann boundary conditions are applied ($v_n = 0$). The acoustics fluid is water being c_f =1500m/s and ρ =1000kg/m^3. This problem is presented in Fig.2 where an oscillatory moment M_{exc} =1 Nm is applied on one edge. In Fig.3 is presented an scheme of the meshes and the coupled BEM-FEM subdomains using LLM.

The *acoustic cavity with a flexible wall* is solved using the nsBETI iterative algorithm. The results agree with the vibration modes of the flexible wall acoustic cavity presented

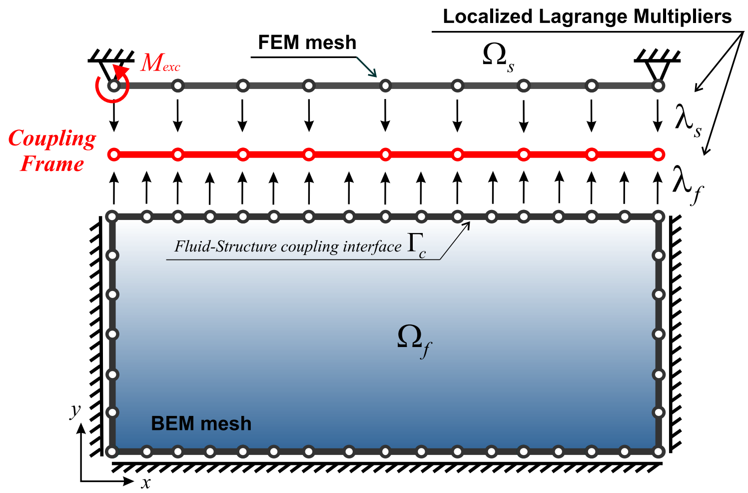
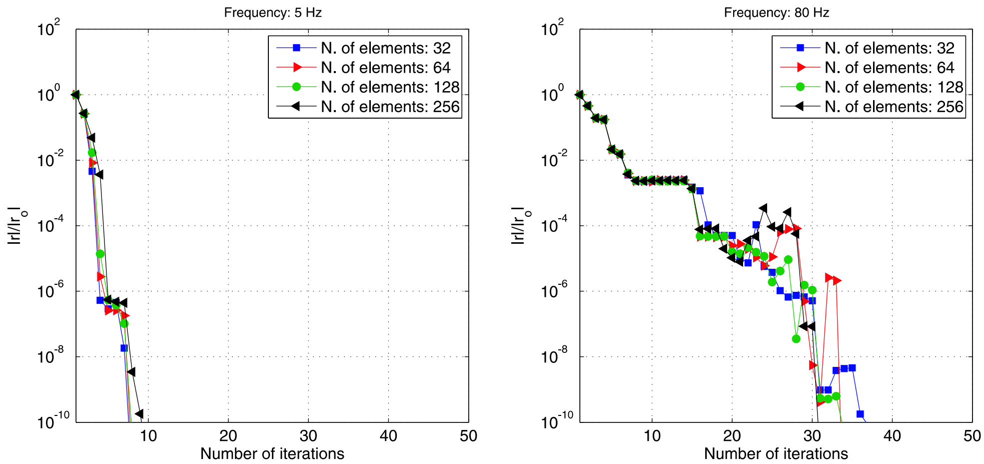


Figure 3: Coupled BEM-FEM subdomains using LLM.

in [13]. The influence of different factors like the number of elements per subdomain, the frequency of the excitation, and the presence of non-matching interfaces are examined in the convergence of nsBETI algorithm. The BiCGSTAB error evolutions in every case are presented in Fig. 4 for different number of elements. It can be observed how the number iteration is not affected by the number of degrees of freedom, but it is very effected by the harmonic excitations frequency. Initially, each subdomain is discretized using BEM-FEM matching meshes with L/h = 32, 64, 128, and 256 divisions at the interface. Figures 4(a) and 4(b) shows the error evolutions for a low frequency excitation of 5 Hz and high frequency of 80 Hz, respectively, with the number of iterations needed by nsBETI to solve these problems with a tolerance of 10^{-10}. It can be observed, for the cases considered, that an exponential increase of the type $H/h = 2^n$ translates into a constant increment of the number of iterations, for every excitation frequency. The difference between the number of iteration in every case (5 Hz and 80 Hz) is due to the differences between their deflections, as Fig.5 shows.

Finally, the non-matching case is considered by changing the discretization of the structure to produce dissimilar meshes at the interface. Figure 6 presents the error evolutions for 5 Hz (Figure 6(a)) and 80 Hz (Figure 6(b)) excitation. The results obtained when the mesh of structure ranged from $(L/h) = 64$ (highly non-matching case) to $(H/h)_{BEM} = 256$ (matching case), maintaining the mesh of fluid fixed with $(L/h) = 256$ divisions. It is noted that the introduction of dissimilar meshes, maintaining a constant $(L/h)_{max}$, increases the number-of-iterations needed by nsBETI to solve the problem for low and high excitation frequencies.

As a summary, we can say that, in the matching case, the convergence of nsBETI algorithm is governed by $(L/h)_{max}$, but the introduction of non-matching interfaces destructs this property, producing a negative effect in the convergence that is controlled by the



(c) (d)

Figure 4: BiCGSTAB error evolution for: (a) 5 Hz and (b) 80 Hz, considering a LLM coupling of matching meshes.

interface mesh-dissimilarity parameter h_{max}/h_{min}.

5 CONCLUSIONS

A FETI-type solution algorithm (nsBETI algorithm) has been extended to treat non-matching and non-symmetrical FE-BE acoustic FSI problems, which enjoys similar scalability properties of those of classical FETI and symmetrical-BETI algorithms. This scheme of resolution is based the LLM methodology which allows to consider non-matching interfaces, and preserves software modularity. The scalability studies have been studied for the cases of dissimilar meshes at the interfaces. In the matching interface case, convergence of nsBETI algorithm is governed by $(L/h)_{max}$, but the introduction of non-matching interfaces produces a negative effect in the convergence that is controlled by the interface mesh-dissimilarity parameter h_{max}/h_{min}.

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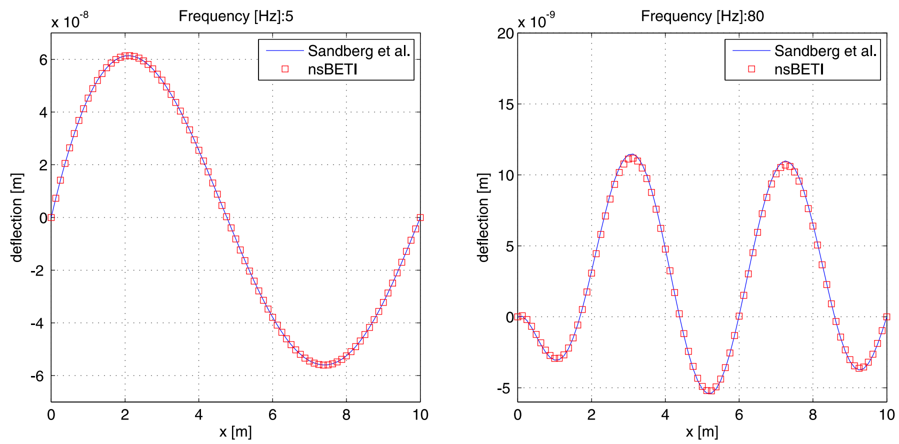
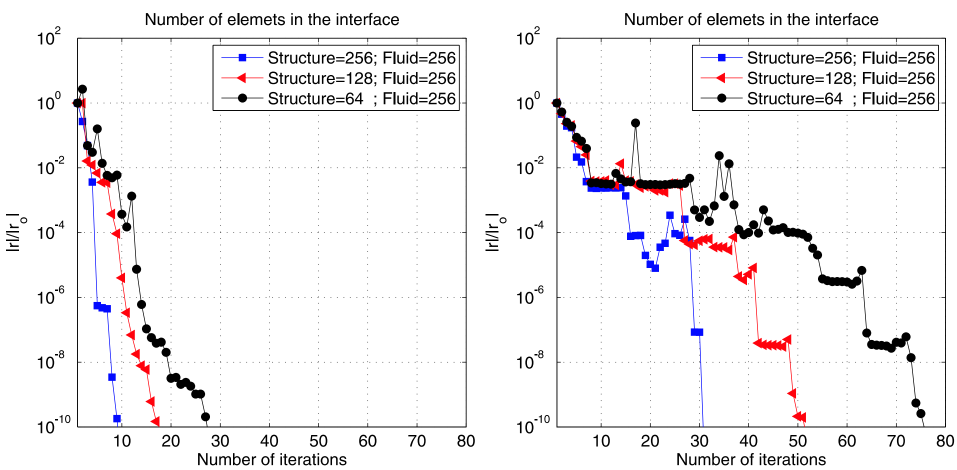


(a) (b)

Figure 5: Beam deflection due to harmonic excitations: (a) 5 Hz and (b) 80 Hz, considering a LLM coupling of matching meshes.



(a) (b)

Figure 6: BiCGSTAB error evolution considering non-matching meshes and harmonic excitation of: (a) 5 Hz and (b) 80 Hz.

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